3.1 INTRODUCTION:

Mathematical models of a robot's dynamics provide a description of *why things move* when forces are generated in and applied on the system. They play an important role for both *simulation* and *control*. This chapter presents a concise overview regarding different approaches for modeling the dynamics of a robot and provides insight into model-based control techniques.

3.1 Introduction

For many applications with fixed-based robots we need to find a multi-body dynamics formulated as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau} + \mathbf{J}_c(\mathbf{q})^T\mathbf{F}_c \tag{3.1}$$

consisting of the following components:

$\mathbf{M}(\mathbf{q})$	$\in \mathbb{R}^{n_q \times n_q}$	Generalized mass matrix (orthogonal).
$\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$	$\in \mathbb{R}^{n_q}$	Generalized position, velocity and acceleration vectors.
$\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$	$\in \mathbb{R}^{n_q}$	Coriolis and centrifugal terms.
$\mathbf{g}(\mathbf{q})$	$\in \mathbb{R}^{n_q}$	Gravitational terms.
$\boldsymbol{\tau}$	$\in \mathbb{R}^{n_q}$	External generalized forces.
$\mathbf{F}_c$	$\in \mathbb{R}^{n_c}$	External Cartesian forces (e.g. from contacts).
$\mathbf{J}_c(\mathbf{q})$	$\in \mathbb{R}^{n_c \times n_q}$	Geometric Jacobian corresponding to the external forces.

Please note: For simplicity and compactness, we use here the formulation for fixed base systems with $\ddot{\mathbf{q}}$ instead of $\dot{\mathbf{u}}$ as introduced for floating base systems. Section 3.7 will specifically deal with the dynamics of floating base systems.

In literature, different methods exist to compute the so-called Equations of Motion (EoM) of a given system, i.e., a closed-form mathematical model of the system dynamics. All such methods are usually based on *Newtonian* and/or *Lagrangian* mechanics formulations, but despite the different approaches taken, all methods will result in *equivalent* descriptions of the dynamics.

In this text we present the most common methods used in robotics. The first such approach presented in section 3.3, is the well-known classical *Newton-Euler* method, which essentially applies the principles of conservation of linear and angular momentum for all links of a robot, and considers the motion explicitly in Cartesian space.