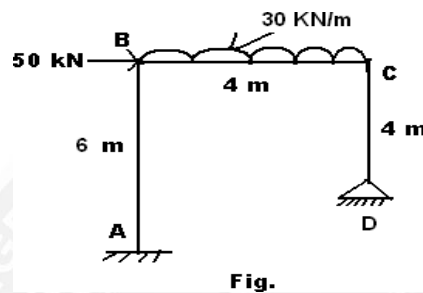


#### 4.4. ANALYSIS OF INDETERMINATE RIGID FRAMES BY FLEXIBILITY METHOD.

##### 4.4.1. NUMERICAL PROBLEMS ON RIGID FRAMES;

##### PROBLEM NO:01

Analysis the rigid portal frame ABCD shown in fig, by using Flexibility method.



##### Solution:

- Static indeterminacy:**

$$\text{Degree of redundancy} = (3 + 2) - 3 = 2$$

Release at B and C by apply hinge

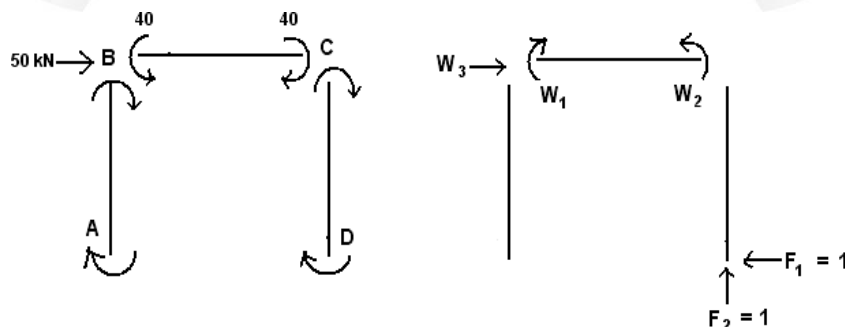
Apply a unit force at B joint.

- Fixed End Moments:**

$$MF_{BC} = -Wl^2/12 = -30 \times 4^2/12 = -40 \text{ kNm}$$

$$MF_{CB} = Wl^2/12 = 30 \times 4^2/12 = 40 \text{ kNm}$$

- Equivalent Joint Loads:**



- Flexibility Co-efficient Matrix (B):**

$$B = B_W \cdot B_X$$

$$B = \begin{bmatrix} 0 & 0 & -6 & -2 & 4 \\ 0 & 0 & 0 & 4 & -4 \\ 1 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

• **Flexibility Matrix (F):**

$$F = \frac{L}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}$$

$$F_x = B_x^T \cdot F \cdot B_x$$

$$= \frac{1}{EI} \begin{bmatrix} -2 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 4 & -4 \\ -4 & 4 \\ 4 & 0 \\ -4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI} \begin{bmatrix} 141.28 & -104 \\ -104 & 117.28 \end{bmatrix}$$

$$\underline{F_x}^{-1} = EI \begin{bmatrix} 0.0204 & 0.0181 \\ 0.0181 & 0.0246 \end{bmatrix}$$

$$\mathbf{F}_w = \mathbf{B}_x^T \cdot \mathbf{F} \cdot \mathbf{B}_w$$

$$= \frac{1}{EI} \begin{bmatrix} -2 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 0 & -6 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{F}_w = \frac{1}{EI} \begin{bmatrix} -8 & -5.32 & 48 \\ 5.32 & 0 & -72 \end{bmatrix}$$

- **Displacement Matrix ( X ):**

$$\mathbf{X} = -\mathbf{F}_x^{-1} \cdot \mathbf{F}_w \cdot \mathbf{W}$$

$$= -\frac{EI}{EI} \begin{bmatrix} 0.0204 & 0.0181 \\ 0.0181 & 0.0246 \end{bmatrix} \begin{bmatrix} -8 & -5.32 & 48 \\ 5.32 & 0 & -72 \end{bmatrix} \begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

$$= - \begin{bmatrix} 0.0669 & 0.1085 & 0.3240 \\ 0.0139 & 0.0963 & 0.9024 \end{bmatrix} \begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

$$= - \begin{bmatrix} -14.536 \\ -41.824 \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 14.536 \\ 41.824 \end{bmatrix}$$

- **Internal Force ( P ):**

$$\mathbf{P} = [ ]_{\mathbf{X}}$$

$$= \begin{bmatrix} 0 & 0 & -6 & -2 & 4 \\ 0 & 0 & 0 & 4 & -4 \\ 1 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 40 \\ -40 \\ 50 \\ 14.536 \\ 41.824 \end{bmatrix}$$

$$P = \begin{bmatrix} -161.776 \\ -109.152 \\ 149.152 \\ 58.144 \\ -98.144 \\ 0 \end{bmatrix}$$

• **Final Moments ( M ):**

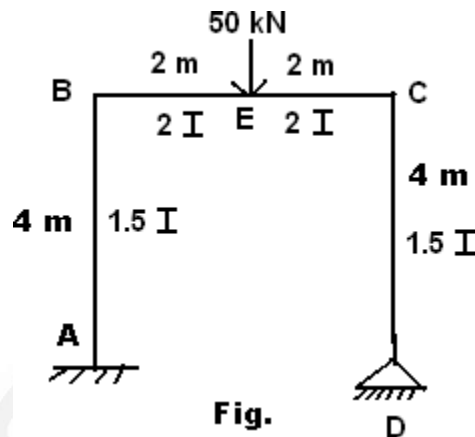
$$M = \mu + P$$

$$= \begin{bmatrix} 0 \\ 0 \\ -40 \\ 40 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -161.776 \\ -109.152 \\ 149.152 \\ 58.144 \\ -98.144 \\ 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -161.776 \\ -109.152 \\ 109.152 \\ 98.144 \\ -98.144 \\ 0 \end{bmatrix}$$

**PROBLEM NO:02**

Analysis the rigid portal frame ABCD shown in fig, by using Flexibility method. And sketch the bending moment diagram.

**Solution:**

- Static indeterminacy:**

$$\text{Degree of redundancy} = (3 + 2) - 3 = 2$$

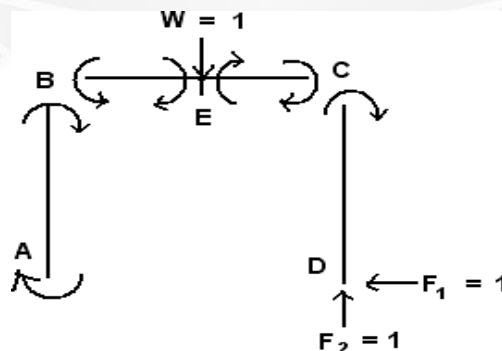
Release at D by apply horizontal and vertical supports

Apply a unit force at E joint.

- Fixed End Moments:**

$$MF_{AB} = MF_{BA} = MF_{BC} = MF_{CB} = MF_{CD} = MF_{DC} = 0$$

- Equivalent Joint Loads:**



- Flexibility Co-efficient Matrix (B):**

$$B = B_W \cdot B_X$$

$$B = \begin{bmatrix} -2 & 0 & 4 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \\ 0 & 4 & -2 \\ 0 & -4 & 2 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

• **Flexibility Matrix (F):**

$$F = \frac{L}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI} \begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix}$$

$$F_X = B_X^T \cdot F \cdot B_X$$

$$= \frac{1}{EI} \begin{bmatrix} 0 & 4 & -4 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 4 & -4 \\ -4 & 4 \\ 4 & -2 \\ -4 & 2 \\ 4 & 0 \\ -4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI} \begin{bmatrix} 60.48 & -37.28 \\ -37.28 & 53.2 \end{bmatrix}$$

$$F_x^{-1} = EI \begin{bmatrix} 0.0291 & 0.0203 \\ 0.0203 & 0.033 \end{bmatrix}$$

$$F_w = B_x^T \cdot F \cdot B_w$$

$$= \frac{1}{EI} \begin{bmatrix} 0 & 4 & -4 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_w = \frac{1}{EI} \begin{bmatrix} 14.64 \\ -24.60 \end{bmatrix}$$

- Displacement Matrix ( X ):**

$$X = - F_x^{-1} \cdot F_w \cdot W$$

$$\begin{aligned}
 &= -\frac{EI}{EI} \begin{bmatrix} 0.0291 & 0.0203 \\ 0.0203 & 0.033 \end{bmatrix} \begin{bmatrix} 14.64 \\ -24.60 \end{bmatrix} 50 \\
 &= - \begin{bmatrix} -0.0734 \\ -0.5146 \end{bmatrix} 50 \\
 &= - \begin{bmatrix} -3.67 \\ -25.73 \end{bmatrix} \\
 X &= \begin{bmatrix} 3.67 \\ 25.73 \end{bmatrix}
 \end{aligned}$$

• **Internal Force ( P ):**

$$\begin{aligned}
 P &= \begin{bmatrix} \\ \\ X \end{bmatrix} \\
 &= \begin{bmatrix} -2 & 0 & 4 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \\ 0 & 4 & -2 \\ 0 & -4 & 2 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 3.67 \\ 25.73 \end{bmatrix} \\
 P &= \begin{bmatrix} 2.92 \\ 11.76 \\ -11.76 \\ -36.78 \\ 36.78 \\ 14.68 \\ -14.68 \\ 0 \end{bmatrix}
 \end{aligned}$$

The final moments also same, since there are no external forces acting on the members.