Binary Frequency Shit Keying Modulation

Frequency Shit Keying (FSK) modulation is a popular form of digital modulation used in low-cost application for transmitting data at moderator or low rate over wired as well as wireless channels. In general, an M-ary FSK modulation are becoming efficiency modulation scheme and several forms of M-ary FSK modulation are becoming popular for spread spectrum communications and other wireless applications. In this lesson, our discussion will be limited to binary frequency shift keying(BFSK).

Two carrier frequencies are used for binary frequency shift keying modulation. One frequency is called the „mark‟ frequency (f2) and the other as the frequency (f1). By convention, the „mark‟ frequency indicates the higher of the two carriers used. If Tb indicates the duration of one information but, the two time- limited signals can be expressed as:

The binary scheme uses two carriers and for special relationship between the two frequencies one can also define two orthonormal basis functions as shown below.

Generation and Coherent Detection of Binary FSK Signals

The block diagram describes a scheme for generating the binary FSK signal; it consists of two components:

1. *On–off level encoder*, the output of which is a constant amplitude of in response to input symbol 1 and zero in response to input symbol 0.

2. *Pair of oscillators*, whose frequencies *f*1 and *f*2 differ by an integer multiple of the bit rate 1/*T*b in accordance with (7.152). The lower oscillator with frequency *f*2 is preceded by an inverter. When in a signaling interval, the input symbol is 1, the upper oscillator with frequency *f*1 is switched on and signal *s*1(*t*) is transmitted,

while the lower oscillator is switched off. On the other hand, when the input symbol is 0, the upper oscillator is switched off, while the lower oscillator is switched on and signal *s*2(*t*) with frequency *f*2 is transmitted. With phase continuity as a requirement, the two oscillators are *synchronized* with each other. Alternatively, we may use a voltage-controlled oscillator, in which case phase continuity is automatically satisfied.

To coherently detect the original binary sequence given the noisy received signal *x*(*t*),

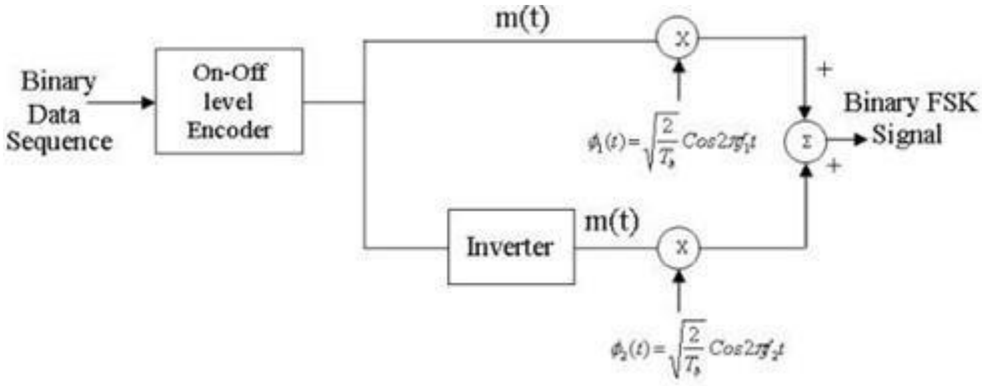


Fig:4.3.1Coherent Binary FSK transmitter
(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 397)

we may use the receiver shown in Figure 7.26b. It consists of two correlators with a common input, which are supplied with locally generated coherent reference signals □1(*t*) and □2(*t*). The correlator outputs are then subtracted, one from the other; the resulting difference *y* is then compared with a threshold of zero. If *y* □ 0, the receiver decides in favor of 1. On the other hand, if *y* □ 0, it decides in favor of 0. If *y* is exactly zero, the receiver makes a random guess (i.e., flip of a fair coin) in favor of 1 or 0.

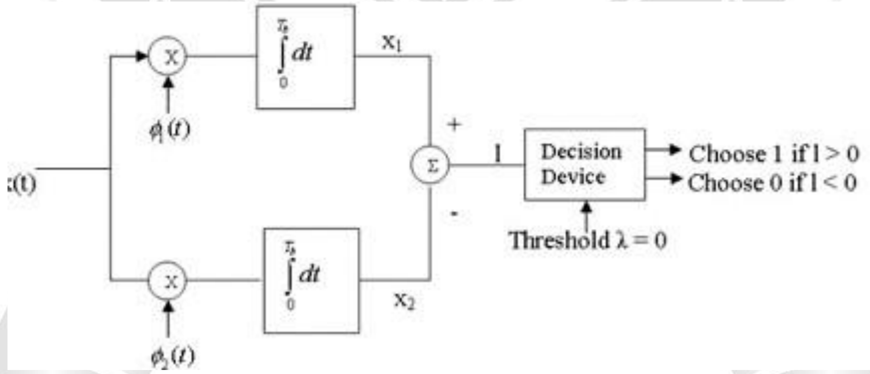


Fig:4.3.2 Coherent Binary FSK receiver
(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page- 397)

A binary FSK Transmitter is as shown, the incoming binary data sequence is applied to on-off level encoder. The output of encoder is √ volts for symbol 1 and 0 volts for symbol "0‟. When we have symbol 1 the upper channel is

switched on with oscillator frequency f1, for symbol "0‟, because of inverter the lower channel is switched on with oscillator frequency f2. These two frequencies are combined using an adder circuit and then transmitted. The transmitted signal is nothing but required BFSK signal. The detector consists of two correlators. The incoming noisy BFSK signal x(t) is common to both correlator. The Coherent reference signal $\phi_1(t)$&$\phi_2(t)$ are supplied to upper and lower correlators respectively.

The correlator outputs are then subtracted one from the other and resulting a random vector “l” (l=x1 - x2). The output “l” is compared with threshold of zero volts.

If l > 0, the receiver decides in favour of symbol 1. l< 0, the receiver decides in favour of symbol 0.

FSK Bandwidth:

- Limiting factor: Physical capabilities of the carrier
- Not susceptible to noise as much as ASK

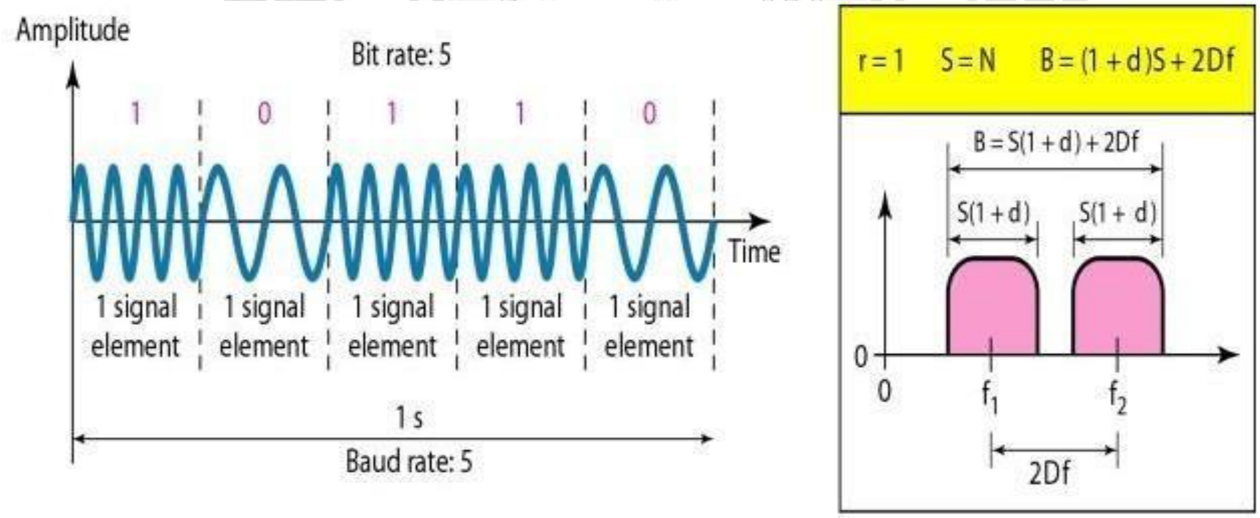


Fig 4.3.3 FSK Waveform (Source:Brainkart)

Error Probability of Binary FSK

The observation vector **x** has two elements $x1$ and $x2$ that are defined by, respectively,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt$$

where $x(t)$ is the received signal, whose form depends on which symbol was transmitted. Given that symbol 1 was transmitted, $x(t)$ equals $s1(t) + w(t)$, where $w(t)$ is the sample function of a white Gaussian noise process of zero mean and power spectral density $N0$□2. If, on the other hand, symbol 0 was transmitted, $x(t)$ equals $s2(t) + w(t)$.

Now, applying the decision rule of (7.57) assuming the use of coherent detection at the receiver, we find that the observation space is partitioned into two decision regions, labeled $Z1$ and $Z2$ in Figure 7.25. The decision boundary, separating region $Z1$ from region $Z2$, is the perpendicular bisector of the line joining the two message points. The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector **x** falls inside region $Z1$. This occurs when $x1$ □ $x2$. If, on the other hand, we have $x1$ □ $x2$, the received signal point falls inside region $Z2$ and the receiver decides in favor of symbol 0. On the decision boundary, we have $x1 = x2$, in which case the receiver makes a random guess in favor of symbol 1 or 0. To proceed further, we define a new Gaussian random variable Y whose sample value y is equal to the difference between $x1$ and $x2$; that is, $y = x1 – x2$. The mean value of the random variable Y depends on which binary symbol was transmitted. Given that symbol 1 was sent, the Gaussian random variables $X1$ and $X2$, whose sample values are denoted by $x1$ and $x2$, have mean values equal to and zero, respectively. Correspondingly, the conditional mean of the random variable Y given that symbol 1 was sent is

Applications

– On voice-grade lines, used up to 1200bps

– Used for high-frequency (3 to 30 MHz) radio transmission

– used at higher frequencies on LANs that use coaxial cable.

Therefore Binary FSK system has 2 dimensional signal space with two messages S1(t) and S2(t), [N=2 , m=2] they are represented.

QUADRATURE PHASE - SHIFT KEYING (QPSK)

The provision of reliable performance, exemplified by a very low probability of error, is one important goal in the design of a digital communication system. Another important goal is the efficient utilization of channel bandwidth. In this subsection we study a *bandwidth-conserving modulation scheme* known as *quadriphase-shift keying* (QPSK), using coherent detection. In a sense, QPSK is an expanded version from binary PSK where in a symbol consists of two bits and two orthonormal basis functions are used. A group of two bits is often called a “dibit”. So, four dibits are possible. Each symbol carries same energy. Let, E: Energy per Symbol and T: Symbol duration = 2.* Tb, where Tb: duration of 1 bit.

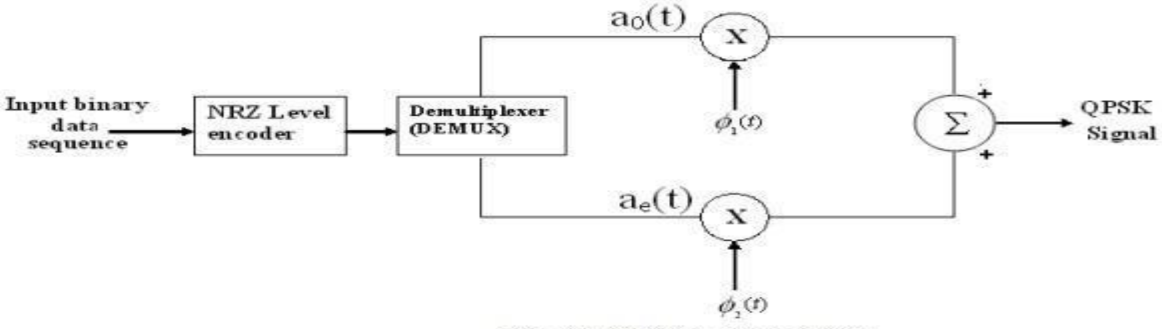


Fig 4.3.4 QPSK

(Source:Ece4uplp)

In QPSK system the information carried by the transmitted signal is contained in the phase. Expanding on the binary PSK transmitter of Figure 7.14a, we may build on (7.113) to construct the QPSK transmitter shown in Figure 7.18a. A distinguishing feature of the QPSK transmitter is the block labeled *demultiplexer*. The function of the demultiplexer is to divide the binary wave produced by the polar NRZ-level encoder into two separate binary waves, one of which represents the odd-numbered dibits in the incoming binary sequence and the other represents the even-numbered dibits.

Accordingly, we may make the following statement:

The QPSK transmitter may be viewed as two binary PSK generators that work in parallel, each at a bit rate equal to one-half the bit rate of the original binary sequence at the QPSK transmitter input.

QPSK Receiver:-

The QPSK receiver consists of a pair of correlators with a common input and supplied with a locally generated pair of coherent reference signals ϕ1(t) &ϕ2(t)as shown in fig(b).The correlator outputs x1 and x2 produced in response to the received signal x(t) are each compared with a threshold value of zero.

Expanding on the binary PSK receiver of Figure 7.14b, we find that the QPSK receiver is

structured in the form of an *in-phase path* and a *quadrature path*, working in parallel as

depicted in Figure 7.18b. The functional composition of the QPSK receiver is as follows:

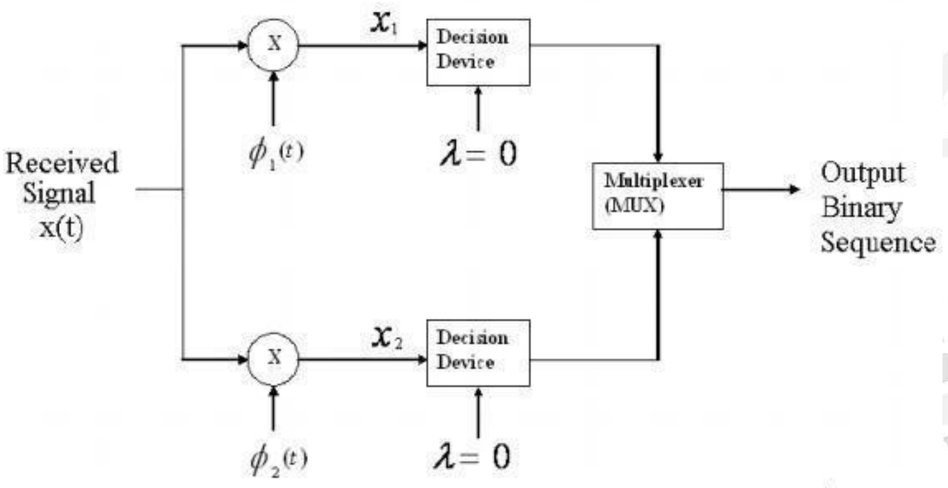


Fig 4..3.5 QPSK Receiver

(Source:Ece4uplp)

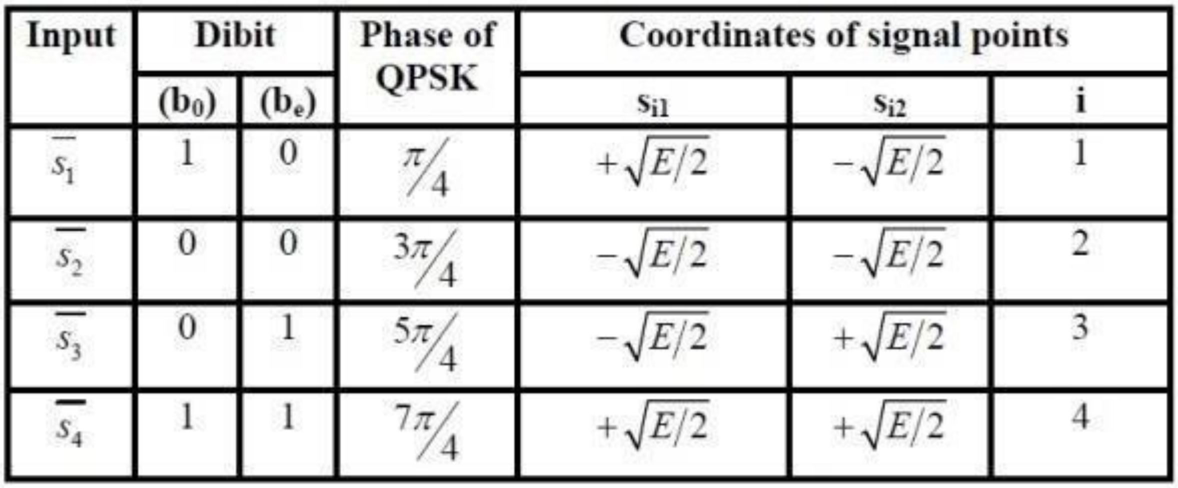
1. *Pair of correlators*, which have a common input *x*(*t*). The two correlators are supplied with a pair of *locally generated orthonormal basis functions* □1(*t*) and □2(*t*), which means that the receiver is synchronized with the transmitter. The correlator outputs, produced in response to the received signal *x*(*t*), are denoted by *x*1 and *x*2, respectively.
2. *Pair of decision devices*, which act on the correlator outputs *x*1 and *x*2 by comparing each one with a zero-threshold; here, it is assumed that the symbols 1 and 0 in the original binary stream at the transmitter input are equally likely. If *x*1 > 0, a decision is made in favor of symbol 1 for the in-phase channel output; on the other hand, if *x*1 0□, then a decision is made in favor of symbol 0. Similar binary decisions are made for the quadrature channel.
3. *Multiplexer*, the function of which is to combine the two binary sequences produced by the pair of decision devices. The resulting binary sequence so produced provides an *estimate* of the original binary stream at the transmitter input.

The in-phase channel output:

If x1> 0 a decision is made in favour of symbol 1 x1< 0 a decision is made in favour of symbol 0.

Similarly quadrature channel output:

If x2>0 a decision is made in favour of symbol 1 and x2<0 a decision is made in favour of symbol 0 Finally these two binary sequences at the in phase and quadrature channel outputs are combined in a multiplexer (Parallel to Serial) to reproduce the original binary sequence.



Input	Dibit		Phase of QPSK	Coordinates of signal points		
	(b_0)	(b_e)		s_{i1}	s_{i2}	i
$\bar{s}_1$	1	0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	1
$\bar{s}_2$	0	0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	2
$\bar{s}_3$	0	1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	3
$\bar{s}_4$	1	1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	4

Signal-Space Diagram of QPSK Signal

Using a well-known trigonometric identity, we may expand (7.112) to redefine the transmitted signal in the canonical form:

where i = 1, 2, 3, 4. Based on this representation, we make two observations:

1. There are two orthonormal basis functions, defined by a pair of quadrature carriers:

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad 0 \le t \le T$$

2.

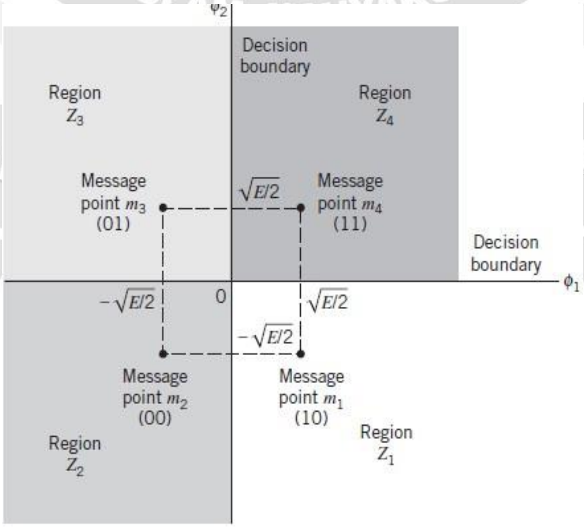


Fig:4.3.6 Signal-space diagram of QPSK system.

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page-358)

Accordingly, a QPSK signal has a two-dimensional signal constellation (i.e., $N = 2$) and four message points (i.e., $M = 4$) whose phase angles increase in a counterclockwise direction, as illustrated ,As with binary PSK, the QPSK signal has *minimum average energy*.

Probability of error:-

A QPSK system is in fact equivalent to two coherent binary PSK systems working arallel and using carriers that are in-phase and quadrature. The in- phase channel output x1 and the Q-channel output x2 may be viewed as the individual outputs of the two coherent binary PSK systems.

Thus the two binary PSK systems may be characterized as follows.

- The signal energy per bit √E/2**Offset QPSK** embodies all the possible phase transitions that can arise in the generation of a QPSK
signal. More specifically, examining the QPSK waveform illustrated
we may make three observations:

1. The carrier phase changes by □180° whenever both the in-phase and quadrature components of the QPSK signal change sign. An example of this situation is illustrated in Figure 7.17 when the input binary sequence switches from dibit 01 to

- dibit 10.

- The carrier phase changes by □90° whenever the in-phase or quadrature component changes sign. An example of this second situation is illustrated in Figure 7.17 when the input binary sequence switches from dibit 10 to dibit 00, during which the inphase component changes sign, whereas the quadrature component is unchanged. 3. The carrier phase is unchanged when neither the in-phase component nor the quadrature component changes sign. This last situation is illustrated in Figure 7.17 when dibit 10 is transmitted in two successive symbol intervals.

- Situation 1 and, to a much lesser extent, situation 2 can be of a particular concern when the QPSK signal is filtered during the course of transmission, prior to detection. Specifically, the 180° and 90° shifts in carrier phase can result in changes in the carrier amplitude (i.e.,
- envelope of the QPSK signal) during the course of transmission over the channel, thereby causing additional symbol errors on detection at the receiver.
- To mitigate this shortcoming of QPSK, we need to reduce the extent of its amplitude fluctuations. To this end, we may use *offset QPSK*.4 In this variant of QPSK, the bit stream responsible for generating the quadrature component is delayed (i.e., offset) by half a symbol interval with respect to the bit stream responsible for generating the in-phase component. Specifically, the two basis functions of offset QPSK are defined by

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \qquad \frac{T}{2} \le t \le \frac{3T}{2}$$

- (

Accordingly, unlike QPSK, the phase transitions likely to occur in offset QPSK are confined to □90°, as indicated in the signalspace diagram of Figure 7.20b. However, □ 90° phase transitions in offset QPSK occur twice as frequently but with half the intensity encountered in QPSK. Since, in addition to □90° phase transitions, □180° phase transitions also occur in QPSK, we find that amplitude fluctuations in offset QPSK due to filtering have a smaller amplitude than in the case of QPSK.

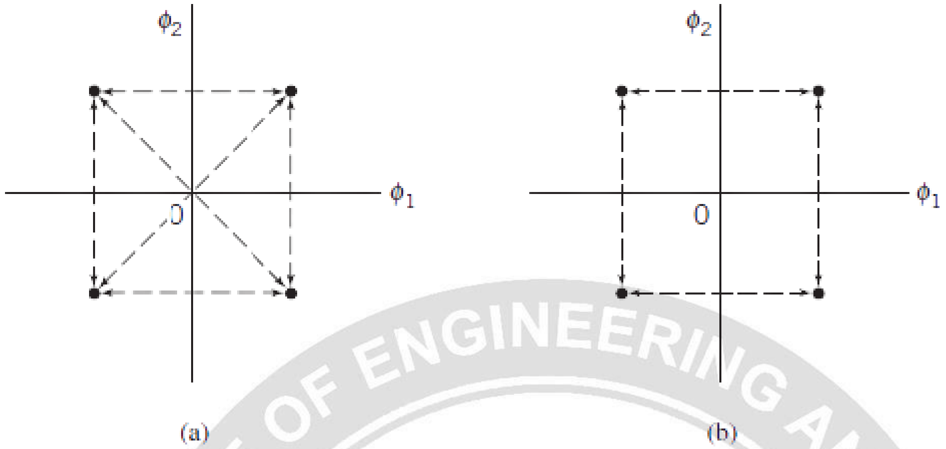


Fig 4.3.7: Possible paths for switching between the message points in (a) QPSK and (b) offset QPSK.

(Source: S. Haykin, ―Digital Communications‖, John Wiley, 2005-Page-358)

the offset QPSK has exactly the same probability of symbol error in an AW GN channel as QPSK. The equivalence in noise performance between these PSK schemes assumes the use of coherent detection at the receiver. The reason for the equivalence is that the statistical independence of the in-phase and quadrature components applies to both QPSK and offset QPSK. We may, therefore, say that Equation (7.123) for the average probability of symbol error applies equally well to the offset QPSK.