Boolean Algebra:

A complemented distributive lattice is called Boolean Algebra.

A Boolean algebra is distributive lattice with "0" element and "1" element in which every element has a complement.

A Boolean algebra is a non empty set with 2 binary operations \land and \lor and is satisfied by the following conditions. $\forall a, b, c \in L$

- 1. $L_1: a \wedge a = a$ and $a \vee a = a$
- 2. $L_2: a \wedge b = b \wedge a$ and $a \vee b = b \vee a$
- 3. $L_3: a \land (b \land c) = (a \land b) \land c$ and $a \lor (b \lor c) = (a \lor b) \lor c$
- 4. $L_4: a \land (a \lor b) = a$ and $a \lor (a \land b) = a$
- 5. $D_1: a \lor (b \land c) = (a \lor b) \land (a \lor c)$
- 6. $D_2: a \land (b \lor c) = (a \land b) \lor (a \land c)$

7. There exist between "0" and "1" such that $a \wedge 0 = 0$, $a \vee 0 = a$, $a \wedge 1 = a$ and

- $a \lor 1 = 1 \forall a$
- 8. $\forall a \in L$, there exist corresponding element a' in L such that $a \wedge a' = 0$ and

$$a \lor a' = 1$$

Note:

Boolean Sum is defined as 1 + 1 = 1, 1 + 0 = 1, 0 + 1 = 1, 0 + 0 = 0Boolean Product is defined as $1 \cdot 1 = 1$, $1 \cdot 0 = 0$, $0 \cdot 1 = 0$, $0 \cdot 0 = 0$ Absorption law in Boolean Algebra NEER/NG 1. Prove that a + ab = a**Solution:** LHS = a + ab= a(1+b) (Distributive law) = a(1)(1+a) = 1a + ab = a $(a \cdot 1 = a)$ 2. Prove that $a + \overline{a}b = a + b$ (4.4M, KANYAKUMA Solution: OBSERVE OPTIMIZE OUTSPREAD $LHS = a + \bar{a}b$ (a = a + ab) $= a + ab + \overline{a}b$ $= a + b(a + \overline{a})$ (Distributive law) = a + b(1) $(a + \overline{a}) = 1$ $(a \cdot 1 = a)$

= RHS

3. Prove that (a + b)(a + c) = a + bc

Solution:

LHS =
$$(a + b)(a + c)$$

= $aa + ac + ab + bc$ (Distributive law)
= $a + ac + ab + bc$ ($a \cdot a = a$)
= $a(1 + c) + ab + bc$ (Distributive law)
= $a + ab + bc$ ($1 + a = 1$)
= $a + bc$ ($a + ab = a$)
= RHS

4. In any Boolean Algebra, show that $a = b \Leftrightarrow a\overline{b} + \overline{a}b = 0$

Proof:

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Let $(B, \cdot, +, 0, 1)$ be any Boolean Algebra.

Let $a, b \in B$ and a = b ...(1)

Claim: $a\overline{b} + \overline{a}b = 0$

Now $a\overline{b} + \overline{a}b = a \cdot \overline{b} + \overline{a}.b$

$$= a \cdot \overline{a} + \overline{a} \cdot a \qquad \text{using (1)}$$

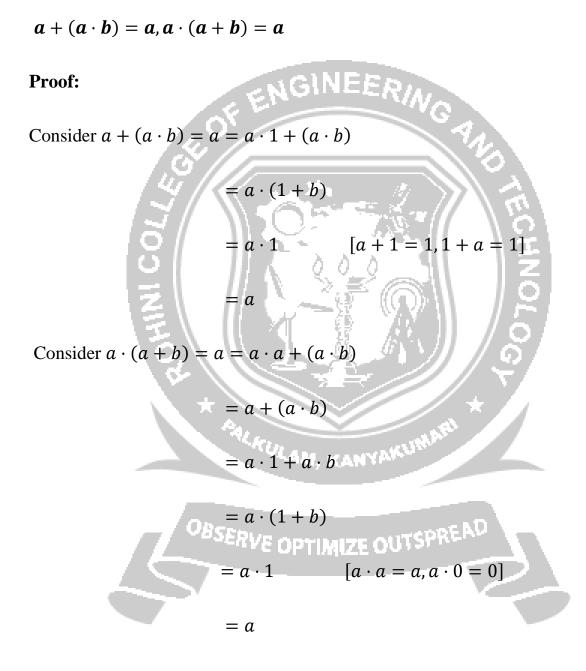
$$= 0 + 0 \qquad (\text{since } a \cdot \overline{a} = 0)$$

$$= 0$$
Conversely, assume $a\overline{b} + \overline{a}b = 0$ **GINEER**

$$\Rightarrow a + a\overline{b} + \overline{a}b = a$$
(Left Cancellation law)
$$\Rightarrow a + a\overline{b} = a$$
(Absorption law)
$$\Rightarrow (a + \overline{a}) \cdot (a + b) = a$$
(Distributive law)
$$\Rightarrow 1 \cdot (a + b) = a$$
($a + \overline{a} = 1$)
$$\Rightarrow (a + b) = a$$
($a + 1 = a$)
(Right Cancellation law)
$$\Rightarrow a\overline{b} + b = b$$
(Right Cancellation law)
$$\Rightarrow a\overline{b} + b = b$$
(Right Cancellation law)
$$\Rightarrow (a + b) \cdot (b + \overline{b}) = b$$
(Distributive law)
$$\Rightarrow (a + b) \cdot (b + \overline{b}) = b$$
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(Distributive law)
$$\Rightarrow (a + b) \cdot 1 = a$$
($b + \overline{b} = 1$)
$$\Rightarrow (a + b) = b$$
($b \cdot 1 = b$)
(b)
From (a) and (b) we get $a = a + b = b$

Hence a = b

5. If *a* and *b* are two elements of a Boolean algebra, then prove that



Hence the proof.

6. Prove that in a Boolean algebra, the complement of any element is unique.

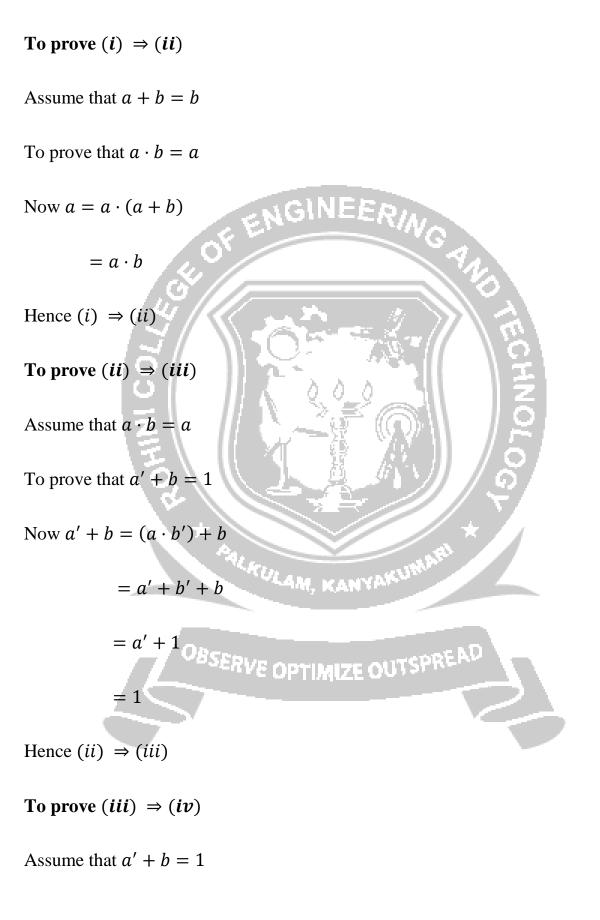
Proof:

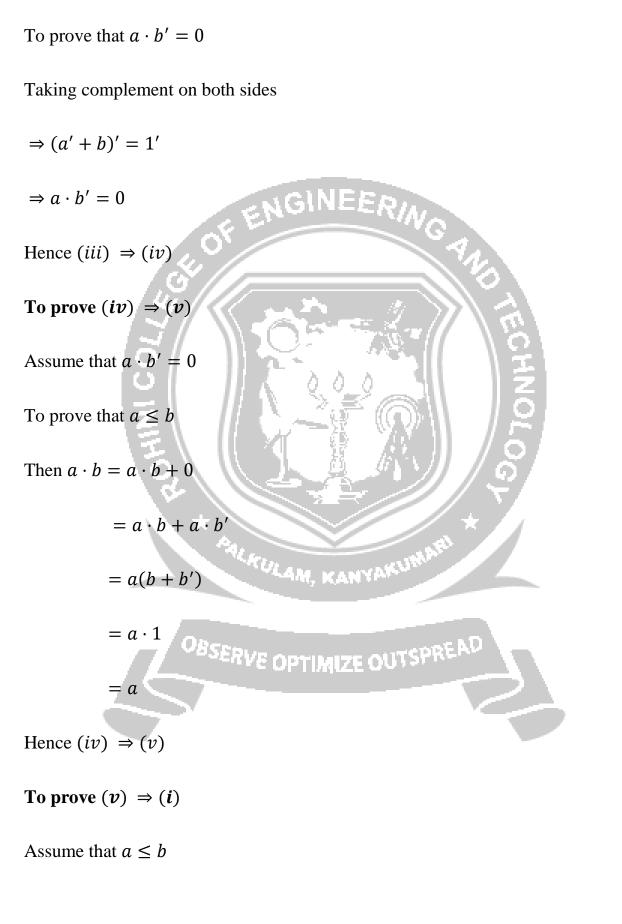
Let b and c be the complements of the element a.

Then b + a = 1, $b \cdot a = 0$ GINEERINGA $a + c = 1, a \cdot c = 0$ Consider $b = 1 \cdot b$ $= (a + c) \cdot b$ $= a \cdot b + c \cdot b$ $= 0 + c \cdot b$ $= a \cdot c + c \cdot b$ $= c \cdot (a + b)$ PALKULAM, KANYAKUMA $= 1 \cdot c$ = cOBSERVE OPTIMIZE OUTSPREAD Hence the complement is unique.

7. In a Boolean algebra show that the following statements are equivalent. For any a and b (i) a + b = b (ii) $a \cdot b = a$ (iii) a' + b = 1 (iv) $a \cdot b' = 0$ (v) $a \le b$

Proof:





To prove that a + b = bWe have $a \cdot b = b$ $\Rightarrow a + b = (a \cdot b) + b$ NEERINGA $= a \cdot b + 1 \cdot b$ $= (a+1) \cdot b$ $= 1 \cdot b$ = bHence the proof. 8. Prove that in a Boolean algebra $(a+b)\cdot(a'+c) = ac + a'b = ac + a'b + bc$ KULAM, KANYAKUN **Proof:** Now, $(a + b) \cdot (a' + c) = (a + b) \cdot a' + (a + b) \cdot c$ ERVE OPTIMIZE OUTSPREAD $= a' \cdot (a+b) + (a+b) \cdot c$ = aa' + a'b + ac + bc= 0 + a'b + ac + bc= a'b + ac + bc

$$= ac(b + b') + a'b(c + c') + bc(a + a')$$

$$= abc + ab'c + a'bc + a'bc' + abc + a'bc$$

$$= abc + ab'c + a'bc + a'bc'$$

$$= abc + ab'c + a'b(c + c')$$

$$= ac(b + b') + a'b(c + c')$$

$$= ac(1) + a'b(1)$$

$$= ac + a'b$$

$$= RHS$$

9. Show that in a Boolean algebra the law of the double complement holds.

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(or) Prove the involution law (a')' = a

Solution:

It is enough to prove that a' + a = 1 and $a \cdot a' = 0$ BSERVE OPTIMIZE OUTSPREP

By domination laws of Boolean algebra, we get

a' + a = 1 and $a \cdot a' = 0$

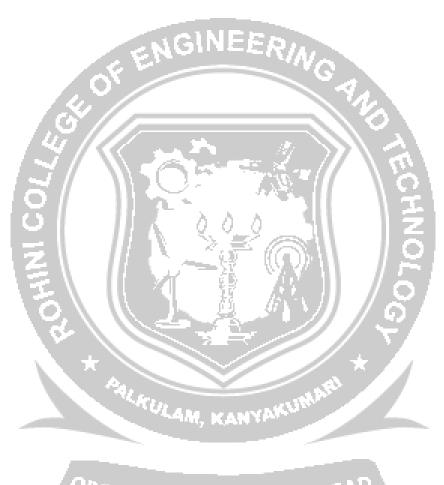
By commutative law, we get a' + a = 1 and $a \cdot a' = 0$

Therefore complement of a' is a

$$\Rightarrow (a')' = a$$

$$\Rightarrow a'' = a$$

Hence the proof.



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