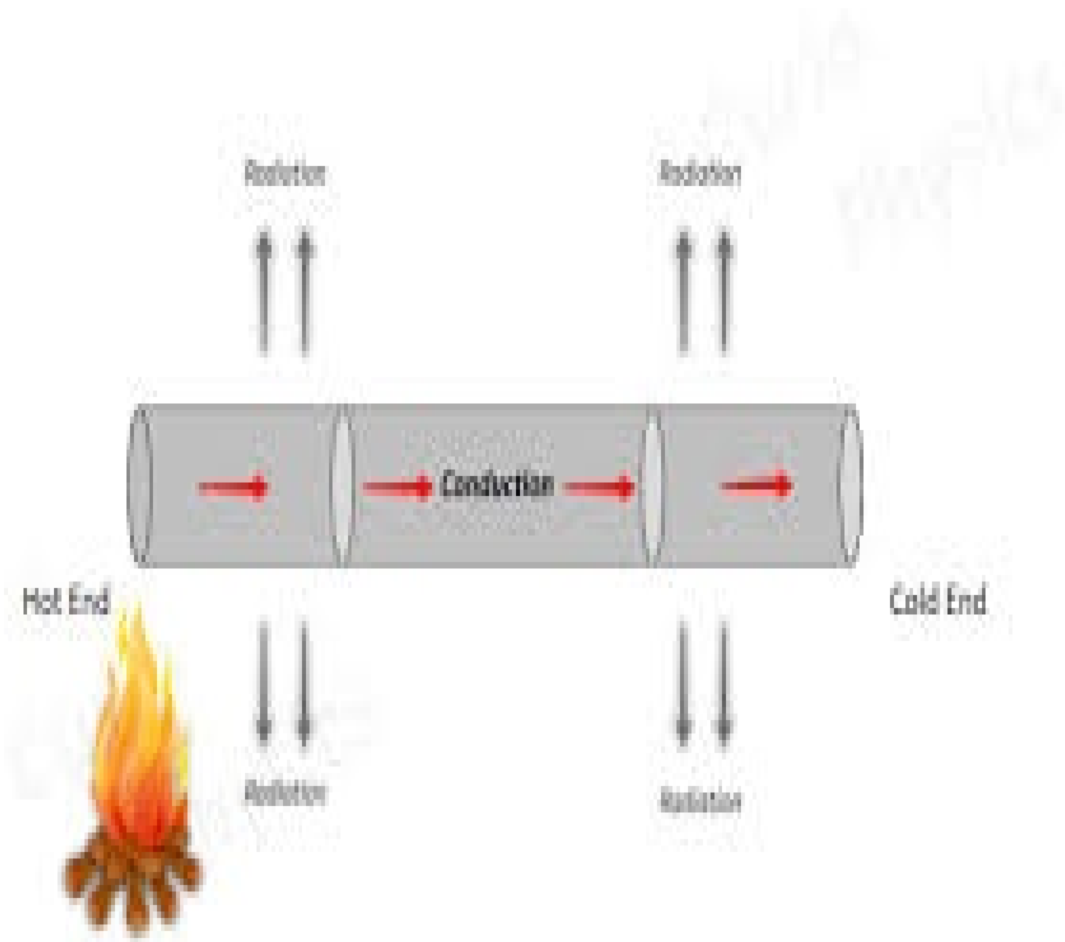
2. STEADY STATE CONDUCTION ONE DIMENSION

2.1 The General Heat Conduction Equation for an Isotropic Solid with Constant Thermal Conductivity

Any physical phenomenon is generally accompanied by a change in space and time of its physical properties. The heat transfer by conduction in solids can only take place when there



is a variation of temperature, in both space and time. Let us consider a small volume of a solid element as shown in Fig. 1.2. The dimensions are: Δx, Δy, Δz along the X-, Y-, and Z-coordinates.

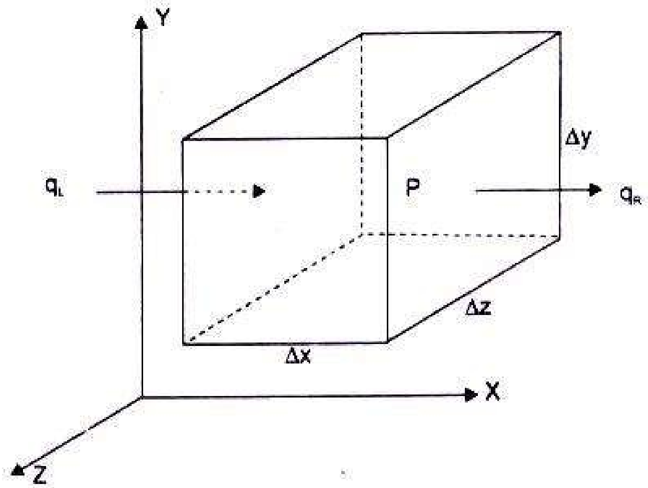


Fig 1.2 Elemental volume in Cartesian coordinates

First we consider heat conduction the X-direction. Let T denote the temperature at the point P (x, y, z) located at the geometric centre of the element. The temperature gradient at the left hand face (x - ~x12) and at the right hand face (x + Δx/2), using the Taylor's series, can be written as:

$$\partial T/\partial x\,|_L = \partial T/\partial x - \partial^2 T/\partial x^2 . \Delta x/2 \; + \text{higher order terms.}$$

$$\partial T/\partial x\,|_R = \partial T/\partial x + \partial^2 T/\partial x^2 . \Delta x/2 + \text{ higher order terms.}$$

The net rate at which heat is conducted out of the element 10 X-direction assuming k as constant and neglecting the higher order terms,

$$\text{we get } -k\Delta y\Delta z\left[\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2}\frac{\Delta x}{2}-\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2}\frac{\Delta x}{2}\right] = -k\Delta y\Delta z\Delta x\left(\frac{\partial^2 T}{\partial x^2}\right)$$

Similarly for Y- and Z-direction,

We have $-k\Delta x\Delta y\Delta z\, \partial^2 T/\Delta y^2$ and $-k\Delta x\Delta y\Delta z\, \partial^2 T/\Delta z^2$.

If there is heat generation within the element as Q, per unit volume and the internal energy of the element changes with time, by making an energy balance, we write

Heat generated within the element — Heat conducted away from the element — Rate of change of internal energy within with the element

or, $\dot{Q}_v(\Delta x\Delta y\Delta z) + k(\Delta x\Delta y\Delta z)\left(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2\right)$

$= \rho c(\Delta x\Delta y\Delta z)\,\partial T/\partial t$

Upon simplification, $\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2 + \dot{Q}_v/k = \frac{\rho c}{k}\partial T/\partial t$

or, $\nabla^2 T + \dot{Q}_v/k = 1/\alpha(\partial T/\partial t)$

where $\alpha = k/\rho\,.\,c$, is called the thermal diffusivity and is seen to be a physical property of the material of which the solid is composed.

The Eq. (2.la) is the general heat conduction equation for an isotropic solid with a constant thermal conductivity. The equation in cylindrical (radius r, axis Z and longitude □) coordinates is written as: Fig. 2.I(b),

$$\partial^2 T/\partial r^2 + (1/r)\partial T/\partial r + \left(1/r^2\right)\partial^2 T/\partial\theta^2 + \partial^2 T/\partial z^2 + \dot{Q}_v/k = 1/\alpha\,\partial T/\partial t \qquad (2.1b)$$

And, in spherical polar coordinates Fig. 2.1(c) (radius, □ longitude, □ colatitudes) is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta\,\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} + \frac{\dot{Q}_v}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.1c)$$

Under steady state or stationary condition, the temperature of a body does not vary with time, i.e. $\partial T/\partial t = 0$. And, with no internal generation, the equation (2.1) reduces to

$$\nabla^2 T = 0$$

It should be noted that Fourier law can always be used to compute the rate of heat transfer by conduction from the knowledge of temperature distribution even for unsteady condition and with internal heat generation.

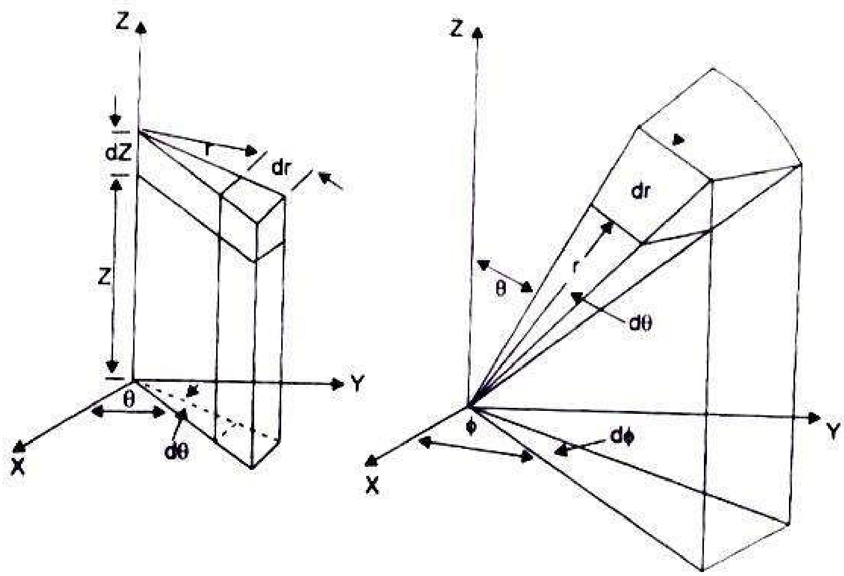


Fig1.3: Elemental volume in cylindrical coordinates (c):spherical coordinates

## .	One-Dimensional Heat Flow

The term 'one-dimensional' is applied to heat conduction problem when:

(i) Only one space coordinate is required to describe the temperature distribution within a heat conducting body;

(ii) Edge effects are neglected;

(iii) The flow of heat energy takes place along the coordinate measured normal to the surface.

## 3.	Thermal Diffusivity and its Significance

Thermal diffusivity is a physical property of the material, and is the ratio of the material's ability to transport energy to its capacity to store energy. It is an essential parameter for transient processes of heat flow and defines the rate of change in temperature. In general, metallic solids have higher value, while non metallics, like paraffin, have a lower value. Materials having large thermal diffusivity respond quickly to changes in their thermal environment, while materials having lower a respond very slowly, take a longer time to reach a new equilibrium condition.