Mutual Inductance [M]

The mutual inductance is defined as the ability of one coil to produce emf in other coil by induction when the current in the first changes.

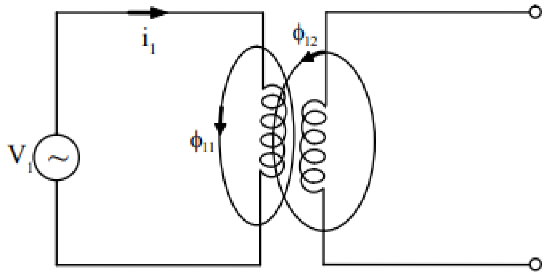

Fig. 5.14 (a) Mutual Inductance

It is also defined as the weber turns in one coil per ampere current in other coil. It is measured in Henry.

Consider the circuit shown in fig. 5.14(a). If a varying voltage source V_1 is connected to coil 1, it produces a varying current i_1. Let the number of turns in coil 1 be N_1. The number of turns in coil 2 be N_2. The varying current i_1, in coil produces a changing flux ϕ_1 in it.

The flux ϕ_1 is divided into two parts, $\phi_1 = \phi_{11} + \phi_{12}$ where $\phi_{11} \rightarrow$ part of flux ϕ_1 which links only with coil1, $\phi_{12} \rightarrow$ part of flux ϕ_1 which links with both coil1 & coil 2.

Let the induced emf in coil 2 be e_2. The induced emf e_2 is proportional to the rate of change of flux in the coil 2.

i.e; $e_2 \alpha \frac{d\phi_{12}}{dt}$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt} \qquad \text{--------(20)}$$

where, $N_2 \rightarrow$ Number of turns in coil 2.

$\frac{d\phi_{12}}{dt} \rightarrow$ rate of change of flux linking coil 1 & coil 2.

The induced emf is also proportional to rate of change of current causing the flux ϕ_{12}. The current causing the flux ϕ_{12} is i_1

$$e_2 \alpha \frac{di_1}{dt}$$

$$e_2 = -M\frac{di_1}{dt} \qquad \text{----------(21)}$$

where, $M \rightarrow$ Mutual inductance between coil 1 & coil 2.

$\frac{di_1}{dt} \rightarrow$ Rate of change of current i_1,

Equating equation (20) & (21), $-N_2\frac{d\phi_{12}}{dt} = -M\frac{di_1}{dt}$

$$\Rightarrow M = N_2\frac{d\phi_{12}}{di_1}$$

For constant permeability, $M = N_2\frac{\phi_{12}}{i_1}$ ----------(22)

If the voltage source is connected to coil 2, the circuit becomes, as shown in fig. 5.14 (b). The varying voltage V_2 produces a varying current i_2. Let the number of turns in coil1 be N_1 & the number of turns in coil 2 be N_2. The varying current i_2 in coil 2 produces a changing flux ϕ_2 in it.

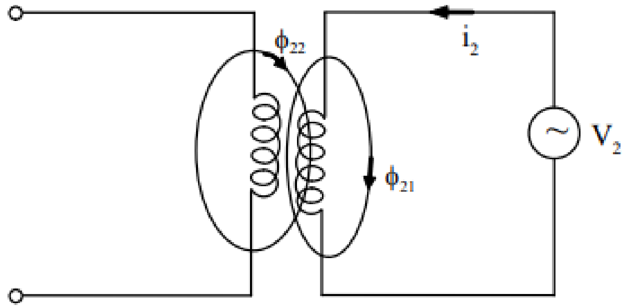

Fig. 5.14 (b) Mutual Inductance with source in the second coil

The flux ϕ_2 is divided into two parts, $\phi_2 = \phi_{22} + \phi_{21}$

where $\phi_{22} \rightarrow$ part of flux ϕ_2 links only with coil 2

$\phi_{21} \rightarrow$ part of flux ϕ_2 links with coil 2 and coil 1.

Let $e_1 \rightarrow$ induced emf in coil 1

The induced emf e_1, is proportional to the rate of change of flux in coil 1.

i.e., $e_1 \alpha \frac{d\phi_{21}}{dt}$

$$\Rightarrow \quad e_1 = -N_1 \frac{d\phi_{21}}{dt} \qquad \text{------------(23)}$$

where, $N_1 \rightarrow$ Number of turns in coil1.

$\frac{d\phi_{21}}{dt} \rightarrow$ Rate of change of flux linking coil 2 and coil 1.

The induced emf is also proportional to the rate of change of current causing the flux ϕ_{21}.

The current causing the flux ϕ_{21} is i_2.

i.e., $e \ \alpha \ \frac{di_2}{dt}$

$$\Rightarrow e = -M \frac{di_2}{dt} \qquad \text{------------(24)}$$

where, $M \rightarrow$ Mutual inductance between coil 2 and coil 1.

$\frac{di_2}{dt} \rightarrow$ Rate of change of current i_2.

Equating eq. (4) and (5) $-N_1 \frac{d\phi_{21}}{dt} = -M \frac{di_2}{dt}$

$$\Rightarrow \quad M = N_1 \frac{d\phi_{21}}{di_2}$$

when permeability is constant, $M = \frac{N_1 \phi_{21}}{i_2}$ -------------(25)

Coupling Coefficient [K]

It is also known as coefficient of coupling or magnetic coupling coefficient or coefficient of magnetic coupling.

It is defined as the fraction of the total flux produced by one coil linking the other coil.

i.e., $K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$ -------------(26)

Multiplying equation (22) & (25), $M^2 = \left(\frac{N_2\phi_{12}}{i_1}\right)\left(\frac{N_1\phi_{21}}{i_2}\right)$ -------------(27)

From equation (26), substitute $\phi_{12} = k\phi_1$; $\phi_{21} = k\phi_2$ in equation (27)

$$M^2 = \left(\frac{N_2 k\phi_1}{i_1}\right)\left(\frac{N_1 k\phi_2}{i_2}\right) \Rightarrow M^2 = K^2\left(\frac{N_1\phi_1}{i_1}\right)\left(\frac{N_2\phi_2}{i_2}\right) \quad \text{-------------(28)}$$

we know that $L = \frac{N\phi}{i} \Rightarrow L_1 = \frac{N_1\phi_1}{i_1}$ and $L_2 = \frac{N_2\phi_2}{i_2}$

Substituting L_1 and L_2 in (28) $M^2 = K^2 L_1 L_2$

$$\Rightarrow M = K\sqrt{L_1 L_2}$$

K value depends on the spacing between coils. As spacing increases K decreases. It is also depends on coil orientation and permeability. K is always positive and its maximum value is 1.

The maximum mutual inductance ocurs when K = 1

$$\text{i.e. } M_{max} = 1\sqrt{L_1 L_2} = \sqrt{L_1 L_2}$$

For iron-core coupled circuits, K may be as high as 0.99 and is between 0.4 to 0.8 for air core coupled circuit.