1. 1. Proportional Logic

Proposition:

A proposition (or statement) is a declarative sentence which is either true or false but not both.

Notations:

If a proposition is true then it's truth value is denoted by T.

If a proposition is false then it's truth value is denoted by F.

P, Q, R, S, . . . are used to denote propositions.

Connectives:

Connective is an operation which is used to connect two or more than two statements. Simply it is called sentential connectives. It is also known as Logical Connectives or Logical Operators.

Compound Statement:

Statements which contain one or more primary statements and some connectives are called compound or molecular or composite statements.

Example:

Let $p: 5 + 10 = 20$ be the statement

$\neg p$: It is false that 5 + 10 = 20

Hence $\neg p$ is a compound statement with primary statement as p and connective as $\neg p$

Five Basic Connectives

	Logical Connectives	Name	Symbols	Type of Operator
1	Not	Negation	$\neg$	Unary
2	And	Conjunction	$\wedge$	Binary
3	Or	Disjunction	$\vee$	Binary
4	If . . . then	Conditional (or) Implication	$\rightarrow$	Binary
5	If and only if (iff)	Biconditional	$\leftrightarrow (or) \rightleftharpoons$	Binary

Statement Formula:

A statement formula is an expression which is a string consisting of variables (capital letters with or without subscripts), parenthesis and connective symbols.

Truth Tables:

The truth value of proposition is either true (T) or false (F).

A truth table is a table that shows the truth value of a compound proposition for all possible cases.

Negation:

If a statement is **TRUE**, then its negation is **FALSE**. (And if a statement is **FALSE**, then its negation is **TRUE**).

P	**¬p**
T	F
F	T

Conjunction:

A conjunction is a compound statement formed by joining two statements with the connector AND. The conjunction "*p* and *q*" is symbolized by p, q.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:

A conjunction is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by p, q. A disjunction is FALSE if and only if (iff) both statements are FALSE; otherwise it is TRUE.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional:

A conditional statement, symbolized by $p \rightarrow q$ is an if – then statement in which p is a hypothesis and q is a conclusion. The logical connector in a conditional statement is denoted by the symbol. The conditional is defined to be TRUE unless a TRUE hypothesis leads to a FALSE conclusion.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F

F	T	T
F	F	T

Bi conditional:

A bi -conditional statement is defined to be TRUE whenever both parts have the same truth value. The bi-conditional operator is denoted by a double – headed arrow. The bi-conditional $p \leftrightarrow q$ represents "p if and only if", where p is a hypothesis and q is a conclusion.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Problems under logical connectives

1.Write the following statements in symbolic form, "If either S.Pavithra takes calculus or S. Sharnika takes sociology, then Malathy will take English.

Solution:

P: S.Pavithra takes calculus.

Q: S.Sharnika takes sociology.

R: Malathy takes English.

∴ The logical expression is $(P \vee Q) \rightarrow R$

2. S. Pavithra can accesses the internet from campus only if she is a computer science major or she is not a fresh girl.

Solution:

P: S. Pavithra can accesses the internet from campus.

Q: S. Pavithra is a computer science major.

R: S. Pavithra is a fresh girl.

$\neg P$: S. Pavithra is not a fresh girl.

∴ The logical expression is $P \rightarrow (Q \vee \neg R)$

3. How can this English sentence be translated into logical expression.

"You can access the internet from campus only if you are computer science major or you are not a freshman".

Solution:

P: You can access the internet from campus.

Q: You are computer science major.

R: You are a freshman.

¬R: you are not a freshman.

∴ The logical expression is $P \rightarrow (Q \vee \neg R)$

4. Write the logical expression for "If tigers have wings then the earth travels round the sun."

Solution:

P: Tigers have wings. (F)

Q: Earth travels round the sun. (F)

The logical expression is $P \rightarrow Q$ (T)

5. Construct the truth table for a)$\neg (P \wedge Q)$ and b) $(\neg P) \vee (\neg Q)$

Solution:

To prove $\neg (P \wedge Q)$ and $(\neg P) \vee (\neg Q)$

P	Q	$(P \wedge Q)$	$\neg (P \wedge Q)$
T	T	T	F
T	F	F	T

F	T	F	T
F	F	F	T

P	Q	¬P	¬Q	(¬P) ∨ (¬Q)
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

6. Construct the truth table for (P ∨ Q) ∨ ¬Q.

Solution:

P	Q	P ∨ Q	¬Q	(P ∨ Q) ∨ ¬Q.
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

7. Construct the truth table for ¬(¬*P* ∨ ¬Q).

Solution:

P	Q	¬P	¬Q	$\neg P \vee \neg Q$	$\neg(\neg P \vee \neg Q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

8. Construct the truth table for S: $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q)$

Solution:

P	Q	$P \wedge Q$	¬P	¬Q	$\neg P \wedge Q$	$P \wedge \neg Q$	$(\neg P \wedge Q) \vee (P \wedge \neg Q)$	S
T	T	T	F	F	F	F	F	T
T	F	F	F	T	F	T	T	T
F	T	F	T	F	T	F	T	T
F	F	F	T	T	F	F	F	F

9. Construct the truth table for i) R: $\neg(\neg P \vee \neg Q)$. ii) $\neg(\neg P \wedge \neg Q)$.

Solution:

P	Q	¬P	¬Q	$\neg P \vee \neg Q$	$\neg P \wedge \neg Q$	R	S

T	T	F	F	F	F	T	T
T	F	F	T	T	F	F	T
F	T	T	F	T	F	F	T
F	F	T	T	T	T	F	F

10. Construct the truth table for $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

Solution:

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Tautology:

A Tautology is a statement that is always TRUE, no matter what. If you construct a truth table for a statement and all the column values for the statement are TRUE, then the statement is a Tautology because it's always TRUE.

Contradiction:

A statement that is always FALSE is known as a Contradiction.

i.e, The last column values contains all FALSE values.

Results:

Tautology	**Contradiction**
In the result column all the entries are T	In the result column all the entries are F
T	F
T	F
T	F
T	F

Problems under Tautology and contradiction

1.Show that the proposition $P \vee \neg (P \wedge Q)$ is a tautology.

Solution:

P	Q	$P \wedge Q$	$\neg (P \wedge Q)$	$P \vee \neg (P \wedge Q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

2.Show that $(Q \vee (P \wedge \neg Q)) \vee (\neg P \wedge \neg Q)$ is a tautology.

Solution:

Let S = $(Q \vee (P \wedge \neg Q)) \vee (\neg P \wedge \neg Q)$

P	Q	¬P	¬Q	P∧ ¬Q	¬P ∧ ¬Q	Q ∨ (P∧ ¬Q)	S
T	T	F	F	F	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	T	F	T

3. Show that $\neg P \rightarrow (P \rightarrow Q)$ is a tautology.

Solution:

P	Q	¬P	P →Q	¬P → (P →Q)
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

4. Show that $(P \wedge Q) \wedge \neg (P \vee Q)$ is a contradiction.

Solution:

P	Q	$P \wedge Q$	$P \vee Q$	$\neg (P \vee Q)$	$(P \wedge Q) \wedge \neg (P \vee Q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F