2.6 STEADY STATE ERROR

The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as e_{ss}. We can find steady state error using the final value theorem as follows.

$$e_{ss} = \lim_{t\to\infty} e(t)$$

$$e_{ss} = \lim_{s\to 0} sE(s)$$

where, E(s) is the Laplace transform of the error signal, e(t)

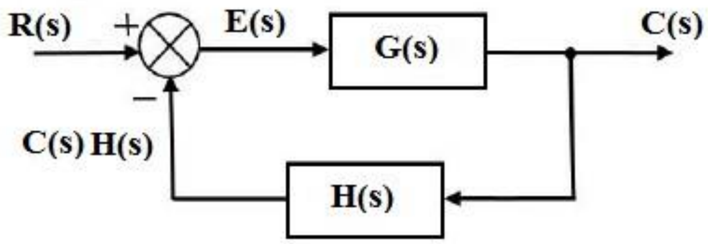

Figure 2.6.1 Closed loop control system

[Source: "Control Systems Engineering" by I J Nagrath, M Gopal, Page: 213]

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - C(s)H(s) = R(s) - G(s)E(s)H(s)$$

$$E(s)(1 + G(s)H(s)) = R(s)$$

$$E(s) = \frac{R(s)}{(1 + G(s)H(s))}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{(1 + G(s)H(s))}$$

When a control system is excited with standard input signal, the steady state error may be zero, constant or infinity. Its value depends on the type number and input signal.

a) Type-0 system will have a constant steady state error when the input is step signal
b) Type-1 system will have a constant steady state error when the input is ramp signal
c) Type-2 system will have a constant steady state error when the input is parabolic signal

For unit step input, $e_{ss} = \frac{1}{1+K_p}$

For unit ramp input, $e_{ss} = \frac{1}{K_v}$

For unit parabolic input, $e_{ss} = \frac{1}{K_a}$

Static error constants for various type number of systems

Error constants	**Type number of system**			
	0	**1**	**2**	**3**
K_p	Constant	∞	∞	∞
K_v	0	Constant	∞	∞
K_a	0	0	Constant	∞

Steady state error for various types of input

Input signal	**Type number of system**			
	0	**1**	**2**	**3**
K_p	$\frac{1}{1+K_p}$	0	0	0
K_v	∞	$\frac{1}{K_v}$	0	0
K_a	∞	∞	$\frac{1}{K_a}$	0