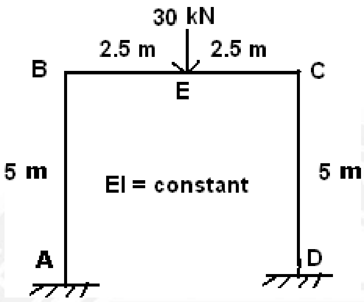
5.4. ANALYSIS OF RIGID FRAMES BY STIFFNESS MATRICES METHOD

5.4.1.NUMERICAL PROBLEMS ON RIGID FRAMES;

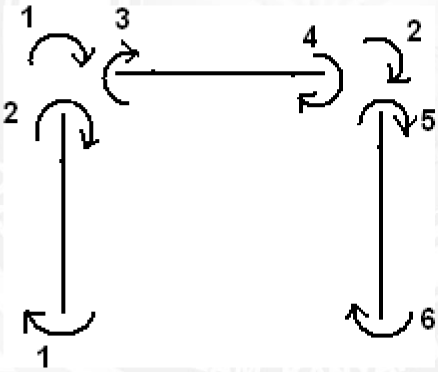
PROBLEM NO:01

Analysis the rigid portal frame ABCD shown in fig,by using Stiffness method.



Solution:

- **Assigned Co-Ordinates:**



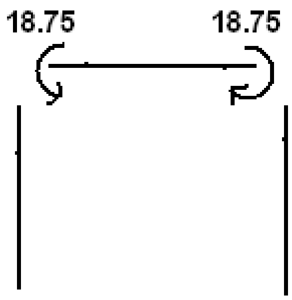
- **Fixed End Moments:**

  MFBC = -Wl/8 = -30x5/8 = -13.75 kNm

  MFCB = Wl/8 = 30x5/8 = 13.75 kNm

  MFAB = MFBA = MFCD = MFDC = 0

- **Fixed End Moments Diagrams:**



$$W^{O} = \begin{bmatrix} -18.75 \\ 18.75 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix(K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

$J = A^{T} . K . A$

$$= EI\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI\begin{bmatrix} 1.6 & 0.4 \\ 0.4 & 1.6 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI}\begin{bmatrix} 0.67 & -0.17 \\ -0.17 & 0.67 \end{bmatrix}$$

- **Displacement Matrix(Δ):**

$$\mathbf{\Delta = J^{-1} . W}$$

$$\mathbf{= J^{-1} [W^{*} - W^{0}]}$$

$$= \frac{1}{EI}\begin{bmatrix} 0.67 & -0.17 \\ -0.17 & 0.67 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -18.75 \\ 18.75 \end{Bmatrix}\right]$$

$$\Delta = \frac{1}{EI}\begin{bmatrix} 15.75 \\ -15.75 \end{bmatrix}$$

- **Element Force (P):**

$$\mathbf{P = K . A . \Delta}$$

$$= \frac{EI}{EI}\begin{bmatrix} 0.8 & 0.4 & 0 & 0 & 0 & 0 \\ 0.4 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0 & 0.8 & 0.4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 15.75 \\ -15.75 \end{bmatrix}$$

$$P = \begin{bmatrix} 6.3 \\ 12.6 \\ 6.3 \\ -6.3 \\ -12.6 \\ -6.3 \end{bmatrix}$$

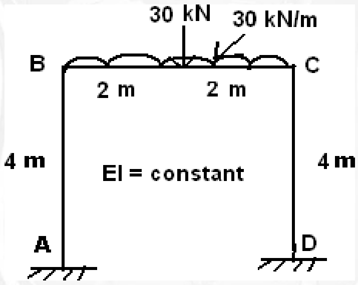
- **Final Moments (M):**

M = μ + P

$$= \begin{bmatrix} 0 \\ 0 \\ -18.75 \\ 18.75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6.3 \\ 12.6 \\ 6.3 \\ -6.3 \\ -12.6 \\ -6.3 \end{bmatrix} = \begin{bmatrix} 6.3 \\ 12.6 \\ -12.5 \\ 12.5 \\ -12.6 \\ -6.3 \end{bmatrix}$$

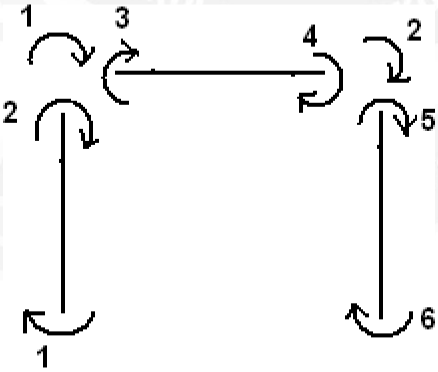
PROBLEM NO:02

Analysis the portal rigid frame ABCD using stiffness method and find the support moments.



Solution:

- **Assigned Co-Ordinates:**



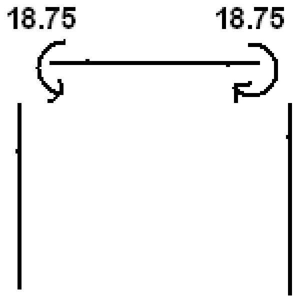
- **Fixed End Moments:**

  MFBC = -[Wl/8 + Wl^2/12] = -[30 x 4/8 + 30 x 4^2/12]= -55 kNm

  MFCB = [Wl/8 + Wl^2/12] = [30 x 4/8 + 30 x 4^2/12] = 55 kNm

  MFAB = MFBA = MFCD = MFDC = 0

- **Fixed End Moments Diagrams:**



$$W^{O} = \begin{bmatrix} -55 \\ 55 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix(K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

$J = A^{T} . K . A$

$$= EI \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI \begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI} \begin{bmatrix} 0.53 & -0.13 \\ -0.13 & 0.53 \end{bmatrix}$$

- **Displacement Matrix(Δ):**

$$\Delta = J^{-1} . W$$

$$= J^{-1} [W^{*} - W^{0}]$$

$$= \frac{1}{EI} \begin{bmatrix} 0.53 & -0.13 \\ -0.13 & 0.53 \end{bmatrix} \left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -55 \\ 55 \end{Bmatrix} \right]$$

$$\Delta = \frac{1}{EI} \begin{bmatrix} 36.3 \\ -36.3 \end{bmatrix}$$

- **Element Force (P):**

$$P = K . A . \Delta$$

$$= \frac{EI}{EI} \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 36.3 \\ -36.3 \end{bmatrix}$$

$$P = \begin{bmatrix} 18.15 \\ 36.3 \\ 18.15 \\ -18.15 \\ -36.3 \\ -18.15 \end{bmatrix}$$

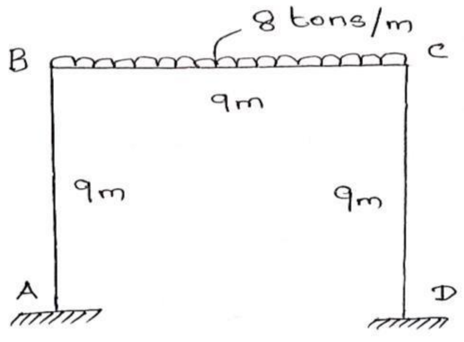
- **Final Moments (M):**

M = μ + P

$$= \begin{bmatrix} 0 \\ 0 \\ -55 \\ 55 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 18.15 \\ 36.3 \\ 18.15 \\ -18.15 \\ -36.3 \\ -18.15 \end{bmatrix} = \begin{bmatrix} 18.15 \\ 36.3 \\ -36.3 \\ 36.45 \\ -36.3 \\ -18.15 \end{bmatrix}$$

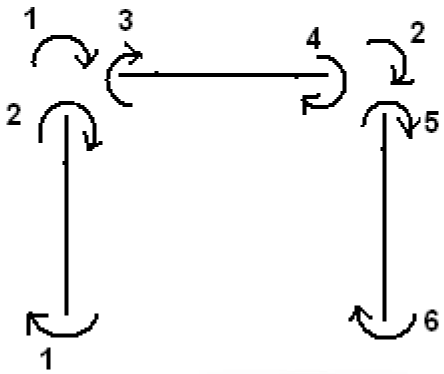
PROBLEM NO:03

A portal frame ABCD with supports A and D are fixed at same level carries a uniformly distributed load of 8 tons/m on the span AB. Span AB = BC = CD = 9 m. EI is constant throughout. Analyse the frame by stiffness matrix method.



Solution:

- **Assigned Co-Ordinates:**



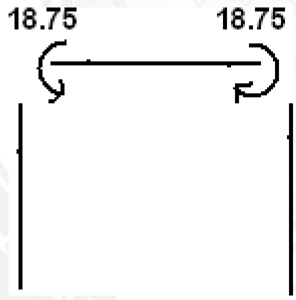
- **Fixed End Moments:**

MFBC = -Wl^2/12 = -8 x 9^2/12 = -54 ton.m

MFCB = Wl^2/12 = -8 x 9^2/12 = 54 ton.m

MFAB = MFBA = MFCD = MFDC = 0

- **Fixed End Moments Diagrams:**



$$W^O = \begin{bmatrix} -54 \\ 54 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix(K):**

$$K = \frac{EI}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} = EI\begin{bmatrix} 0.44 & 0.22 & 0 & 0 & 0 & 0 \\ 0.22 & 0.44 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.44 & 0.22 & 0 & 0 \\ 0 & 0 & 0.22 & 0.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.44 & 0.22 \\ 0 & 0 & 0 & 0 & 0.22 & 0.44 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

J = A^T . K . A

$$= EI\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.44 & 0.22 & 0 & 0 & 0 & 0 \\ 0.22 & 0.44 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.44 & 0.22 & 0 & 0 \\ 0 & 0 & 0.22 & 0.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.44 & 0.22 \\ 0 & 0 & 0 & 0 & 0.22 & 0.44 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI\begin{bmatrix} 0.88 & 0.22 \\ 0.22 & 0.88 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI}\begin{bmatrix} 1.212 & -0.303 \\ -0.303 & 1.212 \end{bmatrix}$$

- **Displacement Matrix(Δ):**

Δ = J^{-1} . W

= J^{-1} [W* - W^0]

$$= \frac{1}{EI}\begin{bmatrix} 1.212 & -0.303 \\ -0.303 & 1.212 \end{bmatrix}\left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -54 \\ 54 \end{Bmatrix}\right]$$

$$\Delta = \frac{1}{EI}\begin{bmatrix} 81.81 \\ -81.81 \end{bmatrix}$$

- **Element Force (P):**

P = K . A . Δ

$$= \frac{EI}{EI}\begin{bmatrix} 0.44 & 0.22 & 0 & 0 & 0 & 0 \\ 0.22 & 0.44 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.44 & 0.22 & 0 & 0 \\ 0 & 0 & 0.22 & 0.44 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.44 & 0.22 \\ 0 & 0 & 0 & 0 & 0.22 & 0.44 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 81.81 \\ -81.81 \end{bmatrix}$$

$$= \begin{bmatrix} 0.22 & 0 \\ 0.44 & 0 \\ 0.44 & 0.22 \\ 0.22 & 0.44 \\ 0 & 0.44 \\ 0 & 0.22 \end{bmatrix}\begin{bmatrix} 81.81 \\ -81.81 \end{bmatrix}$$

$$P = \begin{bmatrix} 18 \\ 36 \\ 18 \\ -18 \\ -36 \\ -18 \end{bmatrix}$$

- **Final Moments (M):**

M = μ + P

$$= \begin{bmatrix} 0 \\ 0 \\ -54 \\ 54 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 18 \\ 36 \\ 18 \\ -18 \\ -36 \\ -18 \end{bmatrix} = \begin{bmatrix} 18 \\ 36 \\ -36 \\ 36 \\ -36 \\ -18 \end{bmatrix}$$