5.5 *Adam's Bash Forth Predictor and Corrector Method*

$$y_{n+1},p = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right]$$

$$y_{n+1},c = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$$

1. *Using Adam's Bash Method Given* $\frac{dy}{dx} = x^2(1+y)$ *with* $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$ *find* $y(1.4)$

solution:

$$Given: y' = x^2(1+y)$$

$x_0 = 1 y_0 = 1$	
$x_1 = 1.1$	$y_1 = 1.233$
$x_2 = 1.2$	$y_2 = 1.548$
$x_3 = 1.3$	$y_3 = 1.979$
$x_4 = 1.4$	$y_4 = ?$

$$h = x_1 - x_0 = 1.1 - 1 = 0.1$$

By Adam's Bash Forth Predictor Method

$$y_{n+1},p = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right]$$

$$y_4,p = y_3 + \frac{h}{24}\left[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right]$$

We have $y' = x^2(1+y)$

$y'_0 = x_0^2(1+y_0)$	$y'_0 = (1)^2(1+1) = 1(2) = 2$
$y'_1 = x_1^2(1+y_1)$	$y'_1 = (1.1)^2(1+1.233) = 1.21 + 2.233 = 3.443$
$y'_2 = x_2^2(1+y_2)$	$y'_2 = (1.2)^2(1+1.548) = 1.44 + 2.548 = 3.988$
$y'_3 = x_3^2(1+y_3)$	$y'_3 = (1.3)^2(1+1.979) = 1.69 + 2.979 = 4.669$

$$y_4,p = y_3 + \frac{h}{24}\left[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right]$$

$$y_4,p = 1.979 + \frac{0.1}{24}[55(4.669) - 59(3.988) + 37(3.443) - 9(2)]$$

$$y_4, p = 1.979 + \frac{0.1}{24}[256.795 - 235.292 + 127.391 - 18]$$

$$y_4, p = 1.979 + \frac{0.1}{24}[130.894]$$

$$y_4, p = 1.979 + 0.5454 = 2.5244$$

By Adam's Bash Forth Corrector Method

$$y_{n+1}, c = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$$

$$y_4, c = y_3 + \frac{h}{24}\left[9y'_4 + 19y'_3 - 5y'_2 + y'_1\right]$$

$$y'_4 = x_4^2(1 + y_4) = (1.4)^2(1 + 2.5244)$$

$$= 1.96 + 3.5244 = 5.4844$$

$$y_4, c = 1.979 + \frac{0.1}{24}[9(5.4844) + 19(4.669) - 5(3.988) + 3.443]$$

$$= 2.452 + \frac{0.2}{3}[49.3596 + 88.711 - 19.94 - 3.443]$$

$$= 2.452 + \frac{0.2}{3}[114.6876]$$

$$= 2.452 + 0.4779$$

$$= 2.9299$$

2. *Using Adam's Bash Method Given* $\frac{dy}{dx} = xy + y^2$ *with* $y(0) = 1, y(0.1) = 1.1169, y(0.2) = 1.2774, y(0.3) = 1.5041$ *find* $y(0.4)$

solution:

Given: $y' = xy + y^2$

$x_0 = 0 \quad y_0 = 1$
$x_1 = 0.1 \quad y_1 = 1.1169$
$x_2 = 0.2 \quad y_2 = 1.2774$
$x_3 = 0.3 \quad y_3 = 1.5041$
$x_4 = 0.4 \quad y_4 = ?$

$$h = x_1 - x_0 = 0.1 - 0 = 0.1$$

By Adam's Bash Forth Predictor Method

$$y_{n+1},p = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right]$$

$$y_4,p = y_3 + \frac{h}{24}\left[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right]$$

$We\ have\ \ y' = xy + y^2$

$y'_0 = x_0y_0 + {y_0}^2$	$y'_0 = (0)(1) + (1)^2 = 0 + 1 = 1$
$y'_1 = x_1y_1 + {y_1}^2$	$y'_1 = (0.1)(1.1169) + (1.1169)^2 = 1.3592$
$y'_2 = x_2y_2 + {y_2}^2$	$y'_2 = (0.2)(1.2774) + (1.2774)^2 = 1.8872$
$y'_3 = x_3y_3 + {y_3}^2$	$y'_3 = (0.3)(1.5041) + (1.5041)^2 = 2.7135$

$$y_4,p = y_3 + \frac{h}{24}\left[55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right]$$

$$y_4,p = 1.5041 + \frac{0.1}{24}[55(2.7135) - 59(1.8872) + 37(1.3592) - 9(1)]$$

$$y_4,p = 1.5041 + 0.33 = 1.8341$$

<u>$By\ Adam's\ Bash\ Forth\ \ Corrector\ \ Method$</u>

$$y_{n+1},c = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right]$$

$$y_4,c = y_3 + \frac{h}{24}\left[9y'_4 + 19y'_3 - 5y'_2 + y'_1\right]$$

$$y'_4 = x_4y_4 + {y_4}^2 = (0.4)(1.8341) + (1.8341)^2 = 4.0976$$

$$y_4,c = 1.5041 + \frac{0.1}{24}[9(4.0976) + 19(2.7135) - 5(1.8872) + 1.3592]$$

$$= 1.5041 + 0.3348$$

$$= 1.8389$$