EC 3352 – DIGITAL SYSTEM DESIGN

UNIT – I : BASIC CONCEPTS

1.5 BOOLEAN FUNCTIONS:

Minimization and Implementation of Boolean expressions:

The Boolean expressions can be simplified by applying properties, laws andtheorems of Boolean algebra.

Simplify the following Boolean functions to a minimum number of literals:

1. x (x’+y)

= xx’+ xy [x. x’= 0]
= 0 + xy [x+ 0 = x]
= xy.

2. x+ x’y

= x + xy + x’y [x+ xy= x][x+ x’ = 1]
= x+ y (x+x’)
= x+ y (1)
= x+ y.

3. (x+ y) (x+ y’)

= x.x+ xy’+ xy+ yy’
= x+ xy’+ xy+ 0 [x. x= 0]; [y. y’=0]
= x (1+ y’+ y) [1+y= 1]
= x(1)
= x.

4.xy + x’z + yz.

= xy + x’z + yz(x+ x’) [x+ x’= 1]
= xy + x’z + xyz + x’yz
Re-arranging,
= xy + xyz + x’z +x’yz
= xy (1+ z) + x’z (1+y)
= xy+ x’z.

5.xy+ yz+ y’z

= xy+ z (y+ y’)

= xy+ z (1) **[1+y= 1]**

= xy+ z.

[y+ y' = 1]

6. (x+ y) (x'+ z) (y+ z)

= (x+ y) (x'+ z) **[** dual form of consensus theorem**,**
(A+ B) (A'+ C) (B+ C) = (A+ B) (A'+ C)]

7. x'y+ xy+ x'y'

= y (x'+ x) + x'y' **[x (y+ z) = xy+ xz]**

= y (1) + x'y'[x+ x' = 1] = y+ x'y' **[x+ x'y' = x+ y']**

= y+ x'.

8. x+ xy'+ x'y = x (1+ y')+ x'y

= x (1) + x'y **[1+ x = 1]**

= x+ x'y **[x+ x'y = x+ y]**

= x+ y.

9. AB + (AC)' + AB'C (AB + C)

= AB + (AC)' + AAB'BC + AB'CC

= AB + (AC)' + 0+ AB'CC **[B.B' = 0]**

= AB + (AC)' + AB'C **[C.C = 1]**

= AB + A' + C' +AB'C **[(AC)' = A' + C']**

= AB + A' + C' + AB' **[C' + AB'C = C' + AB']**

= A' + B+ C'+ AB' **[A' + AB = A' + B]**

Re- arranging,

= A' + AB'+ B+ C' **[A' + AB = A' + B]**

= A' + B'+ B+ C' **[B'+ B= 1]**

= A' +1+ C' **[A+ 1= 1]**

= 1

10. (x'+ y) (x+ y)

= x'.x+ x'y+ yx+ y.y

= 0+ x'y+ xy+ y **[x.x'= 0]; [x. x= x]**

= y (x'+ x+ 1)

= y(1) **[1+ x = 1]**

= y.

11.xy+ xyz+ xy (w+ z)

= xy (1+ z+ w+ z)

= xy (1) **[1+ x = 1]**

= xy.

12.xy+ xyz+ xyz'+ x'yz = xy (1+ z+ z')+ x'yz

= xy (1) + x'yz **[1+ x = 1]**

= xy+ x'yz

= y (x+ x'z) **[x+ x'y = x+ y]**

= y (x+ z).

13.xyz+ xy'z+ xyz' = xy (z+ z') + xy'z

= xy+ xy'z **[x+ x'= 1]**

= x(y+ y'z) **[x+ x'y = x+ y]**

= x(y+ z)

14.x'y'z'+ x'yz'+ xy'z'+ xyz'

= x'z' (y'+ y)+ xz' (y'+ y) **[x+ x'= 1]**

= x'z'+ xz'

= z' (x'+ x)

= z'

15.w'xyz'+ xyz'+ xy'z'+ xy'z

= xyz' (w'+ 1) + xy'z'+ xy'z

= xyz'+ xy'z'+ xy'z
= xz' (y+ y') + xy'z [x+ x'= 1]
= xz'+ xy'z
= x (z'+ y'z)
= x (z'+ y').
[1+ x = 1

16. w'xy'z+ w'xyz+ wxz
= w'xz (y'+ y)+ wxz][x+ x'=
= w'xz (1)+ wxz
= w'xz+ wxz 1]
= xz (w'+ w)
[x'+ xy' = x'+ y']

[x+ x'= 1]
= xz. [x+ x'= 1]

17. x'y'z'+ x'y'z+ x'yz'+ x'yz+ xy'z'
= x'y' (z'+z) + x'y (z'+z)+ xy'z'
= x' y' (1) + x'y (1)+ xy'z' [x+ x'= 1]
= x'y' + x'y + xy'z'
= x'(y'+y) + xy'z'
= x' (1) + [x+ x'= 1]
xy'z'
= x' + xy'z'

$= x' + y'z'.$ $[x' + xy' = x' + y']$

18. $w'y\,(w'xz)' + w'xy'z' + wx'y$

$= w'y\,(w'' + x' + z') + w'xy'z' + wx'y$

$= w'y\,(w + x' + z') + w'xy'z' + wx'y$ $[x'' = x]$

$= w'yw + w'y\,x' + w'y\,z' + w'xy'z' + wx'y$

$= 0 + w'x'y + w'y\,z' + w'xy'z' + wx'y$ $[x.\,x' = 0]$

Re-arranging,

$= w'x'y + wx'y + w'y\,z' + w'xy'z' =$

$x'y\,(w' + w) + w'z'\,(y + xy')$

$= x'y\,(1) + w'z'\,(y + xy')$ $[x + x' = 1]$

$= x'y + w'z'\,(y + x)$ $[x + x'y = x + y]$

19. $xy + x\,(y + z) + y\,(y + z)$

$= xy + xy + xz + yy + yz$

$= xy + xz + y + yz$ $[x + x = x]; [x.\,x = x]$

$= xy + xz + y$ $[x + xy = x]$

$= y + xz$ $[x + xy = x]$

20. $[\,xy'\,(z + wy) + x'y'\,]\,z$

$= [\,xy'z + xy'wy +$

$x'y'\,]\,z$ $[x.\,x' = 0]$

$= [\,xy'z + 0 + x'y'\,]\,z$

$= xy'z.\,z + x'y'z$ $[x.\,x = x]$

$= xy'z + x'y'z$

$= y'z\,(x + x')$ $[x + x' = 1]$

$= y'z\,(1)$

$= y'z.$

21. $x'yz + xy'z' + x'y'z' + xy'z + xyz$

$= yz(x'+x) + xy'z' + x'y'z' + xy'z$

$= yz(1) + y'z'(x+x') + xy'z$ $\quad [x+x'=1]$

$= yz + y'z'(1) + xy'z$ $\quad [x+x'=1]$

$= yz + y'z' + xy'z$

$= yz + y'(z'+xz)$

$= yz + y'(z'+x)$ $\quad [x'+xy = x'+y]$

$= yz + y'z' + xy'$

22. $[(xy)' + x' + xy]'$

$= [x' + y' + x' + xy]'$

$= [x' + y' + xy]'$ $\quad [x+x=x]$

$= [x' + y' + x]'$ $\quad [x'+xy = x'+y]$

$= [y' + 1]'$ $\quad [x+x'=1]$

$= [\ 1\ ]'$ $\quad [1+x=1]$

$= 0.$

23. $[xy + xz]' + x'y'z$

$= (xy)'.(xz)' + x'y'z$

$= (x'+y').(x'+z') + x'y'z$

$= x'x' + x'z' + x'y' + y'z' + x'y'z$ $\quad [x+x=x]$

$= x' + x'z' + x'y' + y'z' + x'y'z$

$= x' + x'z' + x'y' + y'[z' + x'z]$

$= x' + x'z' + x'y' + y'[z' + x']$ $\quad [x'+xy = x'+y]$

$= x' + x'y' + y'[z' + x']$ $\quad [x+xy = x]$

$= x' + x'y' + y'z' + x'y'$

$= x' + y'z' + x'y'$ $\quad [x+xy = x]$

= x'+ y'z'.

24. xy+ xy'(x'z')'
= xy+ xy' (x''+ z'')
= xy+ xy' (x+ z)
= xy+ xy'x+ xy'z
= xy+ xy'+ xy'z
= xy+ xy' [1+ z]
= xy+ xy' [1]
= xy+ xy'
= x(y+ y')
= x [1]
= x.

25. [(xy'+ xyz)'+ x (y+ xy')]'
= [x(y'+yz)'+ x (y+ xy')]'
= [x(y'+z)'+ x (y+ x)]'
= [x(y'+z)'+ xy+ x.x)]'
[x+ xy = x]
[x'' = x]
[x. x= x]
[1+ x = 1]
[x+ x'= 1]
[x'+ xy = x'+ y]; [x+ x'y = x+y]
= [(xy'+xz)'+ xy+ x)]' [x. x= x]
= [(xy'+xz)'+ x)]'
= [(xy')'. (xz)'+ x]'
= [(x'+y''). (x'+z')+ x]'
[x+ xy = x]
= [(x'+y). (x'+z')+ x]' [x'' = x]

= [(x'+ yz')+ x]'
= [x'+ yz'+ x]'

[(x+ y) (x+ z)= x+ yz]

= [1+ yz']' [x+ x'= 1]
= [1]' [1+ x = 1]
= 0.

26. [(xy+ z') ((x+ y)'+z)]'

$= [(xy+ z') ((x'. y')+z)]'$

$= [xy. x'y'+ xy. z+ z'. x'y'+ z'. z]'$

$= [0+ xyz+ x'y'z'+ 0]'$ $\quad$ $[x. x'= 0]$

$= [xyz+ x'y'z']'$

$= (xyz)'. (x'y'z')'$

$= (x'+ y'+ z'). (x''+ y''+ z'')$

$= (x'+ y'+ z'). (x+ y+ z).$ $\quad$ $[x'' = x]$

27. $(x+ y) (x'z'+ z) (y'+ xz)'$

$= (x+ y) (x'z'+ z) (y''. (xz)')$

$= (x+ y) (x'+ z) (y. (xz)')$ $\quad$ $[x+ x'y = x+ y]; [x'' = x]$

$= (x+ y) (x'+ z) (y. (x'+z'))$

$= (x.x'+ xz+ x'y+ yz) (x'y+ yz')$

$= (0+ xz+ x'y+ yz) (x'y+ yz')$

$= (xz+ x'y+ yz) (x'y+ yz')$

$= xz. x'y+ xz. yz'+ x'y. x'y+ x'y. yz'+ yz. x'y+ yz. yz'$

$= 0+ 0+ x'y+ x'yz'+ x'yz+ 0$ $\quad$ $[x. x'= 0]; [x. x= x]$

$= x'y+ x'yz'+ x'yz$

$= x'y (1+ z'+ z)$

$= x'y (1)$ $\quad$ $[1+ x = 1]$

$= x'y.$

28. $Y= \sum m (1, 3, 5, 7)$

$= x'y'z+ x'yz+ xy'z+ xyz$

$= x'z(y'+y) + xz(y'+y)$

$= x'z (1)+ xz (1)$ $\quad$ $[x+ x'= 1]$

$= x'z+ xz$

$= z(x'+ x)$

$= z (1)$ $\quad$ $[x+ x'= 1]$

$= z.$

COMPLEMENT OF A FUNCTION :

The complement of a function F is F’ and is obtained from an interchange of 0’s for 1’s and 1’s for 0’s in the value of F. The complement of a function may be derived algebraically through DeMorgan’s theorem.

DeMorgan's theorems for any number of variables resemble in form the two-variable case and can be derived by successive substitutions similar to the method used in the preceding derivation. These theorems can be generalized as

$$\mathbf{(A+B+C+D+\ldots+F)' = A'\,B'\,C'\,D'\ldots F'}$$

$$\mathbf{(A\,B\,C\,D\ldots F)' = A'+B'+C'+D'+\ldots+F'.}$$

Find the complement of the following functions,

1. **$F= x'yz'+ x'y'z$** $F'= (x'yz'+ x'y'z)'$

$= (x''+ y'+ z'') . (x''+ y''+z') =$
$(x+ y'+ z). (x+ y+ z')$.

2. **$F= (xy + y'z + xz)\,x$.**

$F' = [(xy + y'z + xz)\,x]'$

$= (xy + y'z + xz)' + x'$

$= [(xy)' . (y'z)'. (xz)'] + x'$

$= [(x'+y'). (y+z'). (x'+z')] + x'$

$= [(x'y+ x'z'+ 0+ y'z') (x'+z')] + x'$

$= x'x'y+ x'x'z'+ x'y'z'+ x'yz'+ x'z'z'+ y'z'z'+ x'$

$= x'y+ x'z'+ x'y'z'+ x'yz'+ x'z'+ y'z'+ x'$ [x+ x = x], [x. x = x]

$= x'y+ x'z'+ x'z' (y'+ y) + y'z'+ x'$ [x+ x'= 1]

$= x'y+ x'z'+ x'z' (1) + y'z'+ x'$

$= x'y+ x'z'+ y'z'+ x'$

$= x'y+ x'+ x'z'+ y'z'$

$= x'(y+1) + x'z+ y'z'$ [y+1= 1] $= x' (1+z) + y'z'$ [y+1= 1]

$= x'+ y'z'$

3. **$F= x (y'z'+ yz)$** $F'= [x\,(y'z'+ yz)]'$

$= x'+ (y'z'+ yz)'$

$= x'+ (y'z')'. (yz)' = x'+ (y''+ z'') . (y'+ z') = x'+$

(y+ z) . (y’+ z’).

4. F= xy’+ x’y F’=

(xy’+ x’y)’

= (xy’)’. (x’y)’

= (x’+y) (x+y’) =

x’x+ x’y’+ yx+ yy’

= x’y’+ xy.

5. f = wx’y + xy’+

wxz f’ = (wx’y +

xy’+ wxz)’

= (wx’y)’ (xy’)’ (wxz)’

= (w’+x+ y’) (x’+ y) (w’+ x’+ z’)

= (w’x’+ w’y+ xx’+ xy+ x’y’+ yy’) (w’+ x’+ z’)

= (w’x’+ w’y+ xy+ x’y’) (w’+ x’+ z’)

= w’x’. w’+ w’y. w’+ xy. w’+ x’y’. w’+ w’x’. x’+w’y. x’+ xy. x’+ x’y’.

x’+ w’x’.z’+ w’y. z’+ xy. z’+ x’y’.z’

= w’x’+ w’y+ w’xy+ w’x’y’+ w’x’+ w’x’y+ 0 + x’y’+ w’x’z’+ w’yz’+ xyz’+ x’y’z’

= w’x’+ w’y+ w’xy+ w’x’y’+ w’x’y+ x’y’+ w’x’z’+ w’yz’+ xyz’+ x’y’z’

= w’x’(1+ y’+ y+ z’)+ w’y(1+ x+ z’)+ x’y’(1+ z’)+ xyz’

= w’x’(1)+ w’y(1)+ x’y’(1)+ xyz’

= w’x’+ w’y+ x’y’+ xyz’