1. Superposition Theorem

Statement: In linear, active, bilateral network current flowing through any component is the algebraic sum of currents due to individual sources taking one at a time, replacing remaining sources with their internal impedances.

Proof: Consider the network shown in fig(1), in which Z_1, Z_2, Z_3 are impedances and V_1 and V_2 are applied sources.

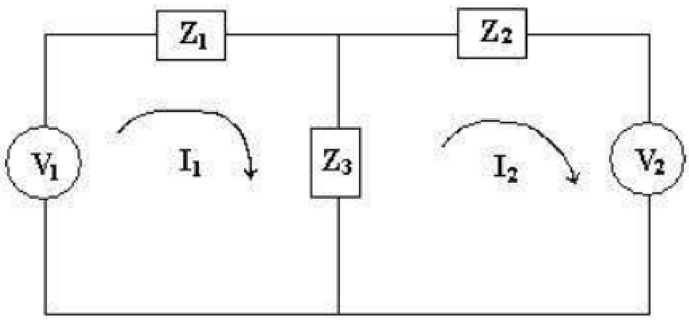


Fig. 1

Let I_1, I_2 be the mesh currents, writing mesh equations for fig(1).

$$(Z_1 + Z_3)\, I_1 - Z_3 I_2 = V_1 \qquad \text{-----------} \ (1)$$

$$-Z_3\, I_1 + (Z_2 + Z_3)\, I_2 = V_2 \qquad \text{------------}(2)$$

Now keeping the source V_1 and replacing V_2 with its internal impedance. The modified ckt is shown in fig(2).

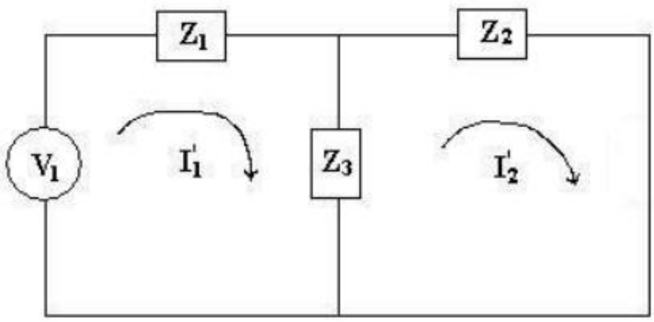


Fig. 2

Let I'_1, I'_2 be mesh currents, mesh equations for fig(2)

$$(Z_1 + Z_3)I'_1 - Z_3 I'_2 = V_1 \qquad \text{----------- (3)}$$

$$- Z_3 I'_1 + (Z_2 + Z_3) I'_2 = 0 \qquad \text{------------(4)}$$

Consider the V_2 and replace V_1 with its internal impedance as shown in fig(3)

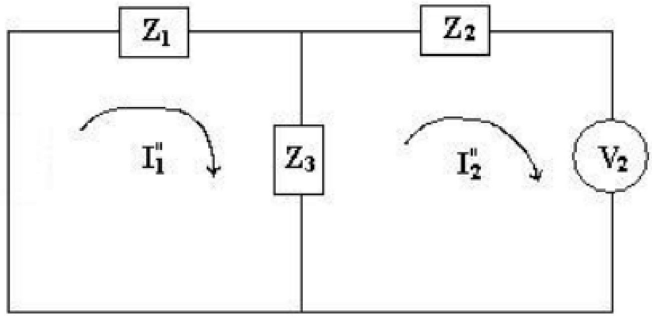


Fig. 3

Let I''_1, I''_2 be the mesh currents , mesh equations for fig(3),

$$(Z_1 + Z_3) I''_1 - Z_3 I''_2 = 0 \qquad \text{----------- (5)}$$

$$- Z_3 I''_1 + (Z_2 + Z_3) I''_2 = V_2 \qquad \text{------------(6)}$$

Adding eq(3) & eq(5) and eq(4) & eq(6), we get

$$(Z_1 + Z_3)(I'_1 + I''_2) - Z_3 (I'_2 + I''_2) = V_1 \qquad \text{-------------(7)}$$

$$- Z_3 (I'_1 + I''_1) + (Z_2 + Z_3) (I'_2 + I''_2) = V_2 \qquad \text{-------------(8)}$$

Comparing eq(1) & eq(7) and eq(2) & eq(8), we have

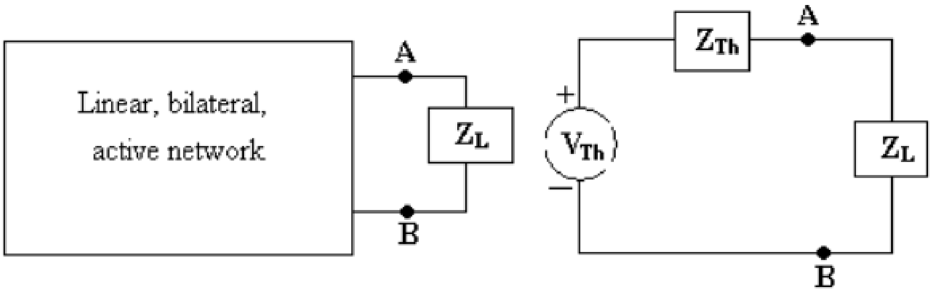
$$I_1 = (I'_1 + I''_2)$$

$$I_2 = (I'_2 + I''_2)$$

It is clear from the above equations that the current flowing through any component is algebraic sum of currents due to individual sources taking on at a time. Hence **Superposition theorem** is proved.

2. THEVENIN'S THEOREM

Statement: Any linear, bilateral, active network connected between two terminal A, B can be replaced by a voltage source supplying voltage V_{TH} in series with an impedance Z_{TH}. Where V_{TH} known as Thevenin's voltage and is the open terminal voltage across A, B terminals. Z_{TH} is known as Thevenin's impedance and is measured across open terminals A, B by replacing all energy sources with their internal impedance.



Proof: Consider the network shown in fig(1), in which Z_1, Z_2 and Z_3 are impedances, V is the applied voltage and Z_L is the load impedance.

Let I_1, I_L be the mesh currents, mesh equations for fig(1)

$$(Z_1 + Z_3)\, I_1 - Z_3 I_L = V \qquad \text{-----------------} \ (1)$$

$$-Z_3 I_1 + (Z_2 + Z_3 + Z_L)\, I_L = 0 \qquad \text{-----------------}(2)$$

From eq(2), we have

$$Z_3 I_1 = (Z_2 + Z_3 + Z_L)\, I_L$$

$$I_1 = \frac{(Z_2 + Z_3 + Z_L)\, I_L}{Z_3} \qquad \text{———} \ (3)$$

Substituting I_1 value from eq(3) in eq(1) we get

$$(Z_1 + Z_3)\frac{(Z_2 + Z_3 + Z_L)\, I_L}{Z_3} - Z_3 I_L = V$$

$$\frac{(Z_1 + Z_3)(Z_2 + Z_3 + Z_L)\, I_L - Z_3^2 I_L}{Z_3} = V$$

$$(Z_1 + Z_3)(Z_2 + Z_3 + Z_L)\, I_L - Z_3^2 I_L = V Z_3$$

$$I_L\left[(Z_1 + Z_3)(Z_2 + Z_3 + Z_L) - Z_3^2\right] = V Z_3$$

$$I_L = \frac{V Z_3}{\left[(Z_1 + Z_3)(Z_2 + Z_3 + Z_L) - Z_3^2\right]} \qquad \text{———} \ (4)$$

$$I_L = \frac{V Z_3}{\left[Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_L + Z_3 Z_2 + Z_3^2 + Z_3 Z_L - Z_3^2\right]}$$

$$I_L = \frac{V Z_3}{(Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1) + Z_L (Z_1 + Z_3)} \qquad \text{———} \ (5)$$

Thevenin's Voltage (V_{Th})

To measure Thevenin's voltage make the terminals A, B open by removing the load impedance connected between them. The modified circuit diagram is shown in fig(2).

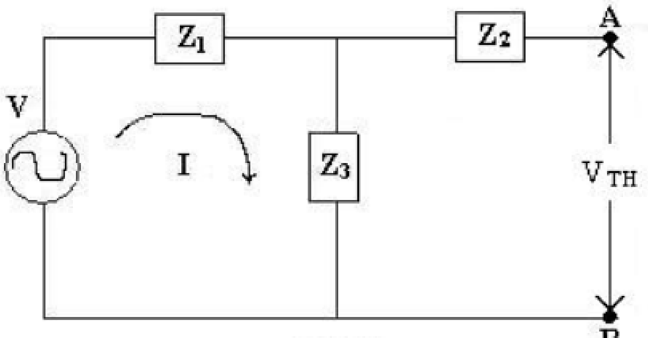


Fig. 2

voltage across terminals AB (V_{TH}) = voltage across Z_3

$$V_{TH} = I \times Z_3 \quad \text{-------- (6)}$$

But from fig(2)

$$I = \frac{V}{Z_1 + Z_3}$$

Substituting I value in eq(6) we get

$$V_{TH} = \frac{VZ_3}{Z_1 + Z_3} \quad \text{———— (7)}$$

Thevenin's Impedance (Z_{TH})

Thevenin's impedance is measured across open terminals AB by replacing all energy sources with their internal impedances. The modified circuit diagram is shown in fig(3).

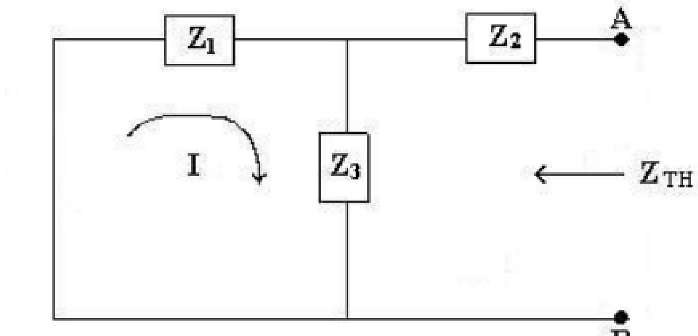


Fig. 3

$$Z_{TH} = (Z_1 \parallel Z_3) + Z_2$$

$$Z_{TH} = \frac{Z_1 Z_3}{Z_1 + Z_3} + Z_2$$

$$Z_{TH} = \frac{Z_1 Z_3 + Z_2(Z_1 + Z_3)}{Z_1 + Z_3}$$

$$Z_{TH} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1 + Z_3} \quad \text{———— (8)}$$

Thevenin's Equivalent Circuit

Thevenin's equivalent circuit can be constructed by connecting voltage source supplying voltage V_{TH} in series with an impedance Z_{TH} as shown in fig.(4). Finally connect the load impedance which was removed from the circuit.

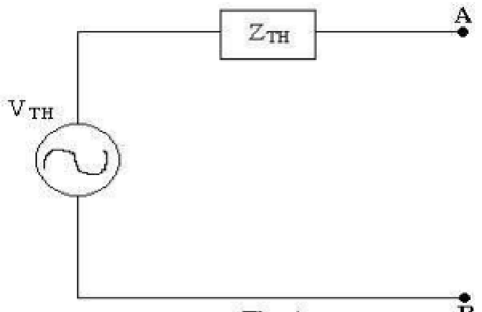


Fig. 4

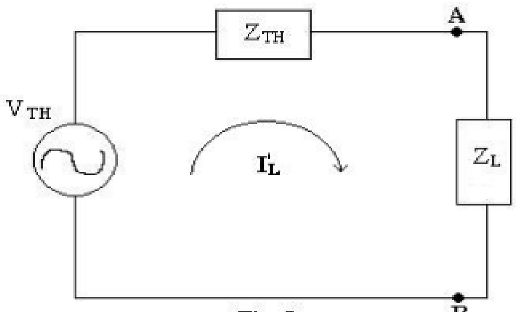


Fig. 5

The current in the Thevenin's equivalent circuit is given by

$$I' = \frac{V_{TH}}{Z_{TH} + Z_L} \qquad \text{------}(9)$$

Substituting V_{TH} and Z_{TH} values in eq (9), we get

$$I'_L = \frac{\dfrac{VZ_3}{Z_1 + Z_3}}{\dfrac{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)}{(Z_1 + Z_3)} + Z_L}$$

$$I'_L = \frac{\dfrac{VZ_3}{Z_1 + Z_3}}{\dfrac{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1 + Z_3)}{(Z_1 + Z_3)}}$$

$$I'_L = \frac{VZ_3}{(Z_1Z_2 + Z_2Z_3 + Z_3Z_1) + Z_L(Z_1 + Z_3)} \qquad \text{———} \; (10)$$

Comparing eq(5) & eq(10), we get

$$I_L = I'_L$$

That is the current flowing through load resistor in a given circuit is equal to the current flowing through the load resistor in Thevenin's equivalent circuit. Hence Thevenin's theorem is proved.