1.1 INTRODUCTION

SIGNAL

Signal is one that carries information and is defined as a physical quantity that varies with one or more independent variable.

Example: Music, speech

The signal may depend on one or more independent variables. If a signal depends on only one variable, then it is known as one dimensional signal. Example: AC power signal, speech signal, ECG etc.

When a signal is represented as a function of two or more variable, it is said to be multidimensional signal Example: An image represented as F(x,y). Here x & y represents the horizontal and vertical co-ordinates. The intensity of the image varies at each co-ordinate.

SIGNAL MODELING

The representation of a signal by mathematical expression is known as signal modeling.

ANALOG SIGNAL

A signal that is defined for every instants of time is known as analog signal. Analog signals are continuous in amplitude and continuous in time. It is denoted by x(t). It is also called as Continuous time signal. Example for Continuous time signal is shown in Figure 1.1.1

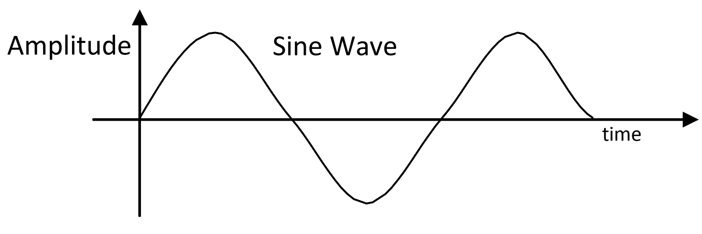


Figure 1.1.1 Continuous time signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

DIGITAL SIGNAL

The signals that are discrete in time and quantized in amplitude is called digital signal (Figure 1.1.2)

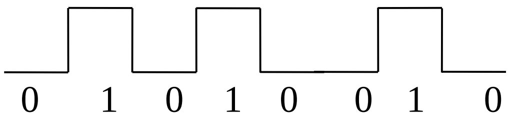


Figure 1.1.2 Digital Signal

[https://drive.google.com/file/d/1baseARsD-geFLoR-QrEbV5hVsYKKLzGM/view]

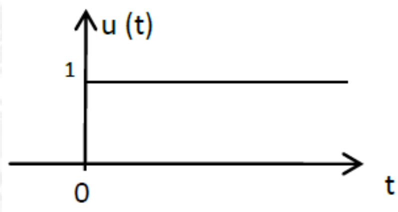
BASIC (ELEMENTARY OR STANDARD) CONTINUOUS TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(t) = 1 \;\; for\; t \geq 0$$

$$= 0 \; for \; t < 0$$



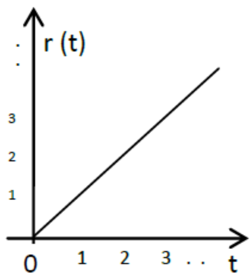
Unit step signal

Ramp signal

Unit ramp signal is defined as

$$r(t) = t \;\; for \; t \geq 0$$

$$= 0 \; for \; t < 0$$

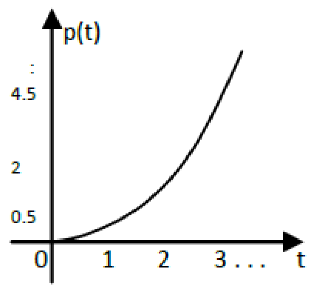


Unit ramp signal

Parabolic signal

Unit Parabolic signal is defined as

$$x(t) = \frac{t^2}{2} \quad for\ t \geq 0$$
$$= 0\ for\ t < 0$$



Unit Parabolic signal

Relation between Unit Step signal, Unit ramp signal and Unit Parabolic signal:

- Unit ramp signal is obtained by integrating unit step signal

$$i.e., \int u(t)dt = \int 1dt = t = r(t)$$

- Unit Parabolic signal is obtained by integrating unit ramp signal

$$i.e., \int r(t)dt = \int t\ dt = \frac{t^2}{2} = p(t)$$

- Unit step signal is obtained by differentiating unit ramp signal

$$i.e., \frac{d}{dt}(r(t)) = \frac{d}{dt}(t) = 1 = u(t)$$

- Unit ramp signal is obtained by differentiating unit Parabolic signal

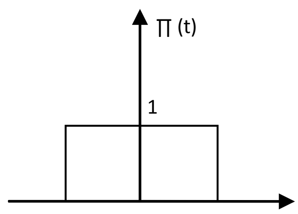
$$i.e., \frac{d}{dt}(p(t)) = \frac{d}{dt}\left(\frac{t^2}{2}\right) = \frac{1}{2}(2t) = t = r(t)$$

Unit Pulse Signal

Unit Pulse signal is defined as

$$\Pi(\mathrm{t}) = 1 \text{ for } |\mathrm{t}| \leq \frac{1}{2}$$

$$= 0 \text{ elsewhere}$$

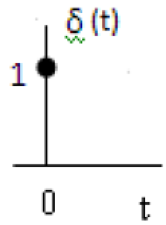


Unit pulse signal

Impulse signal

Unit Impulse signal is defined as

$$\delta(t) = 0 \; for \; t \neq 0$$
$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



Unit Impulse signal

Properties of Impulse signal:

Property 1:

$$\int_{-\infty}^{\infty} \boldsymbol{x(t)\delta(t)\,dt} = \boldsymbol{x(0)}$$

$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)\delta(0) = x(0) \qquad [\because \delta(t) \; exists \; only \; at \; t = 0 \; and \; \delta(0) = 1]$$

Hence proved

Property 2:

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_o)dt = x(t_o)$$

$proof:$

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_o)dt = x(t_o)\delta(t_o - t_o) = x(t_o)\delta(0) = x(t_o)$$

$$\therefore [\delta(t - t_o) \; exists \; onlyat \; t = t_o \; and \; \delta(0) = 1]$$

Hence proved

Sinusoidal signal

Cosinusoidal signal is defined as

$$x(t) = Acos(\Omega t + \Phi)$$

Sinusoidal signal is defined as

$$x(t) = Asin(\Omega t + \Phi)$$

Where $\Omega = 2\pi f = \frac{2\pi}{T}$ *and* Ω *is the angular frequency in rad/sec* f is frequency in cycles/sec or Hertz and

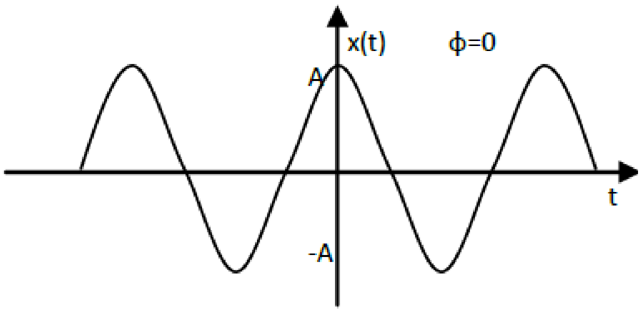
A is amplitude

T is time period in seconds

Φ is phase angle in radians

Cosinusoidal signal

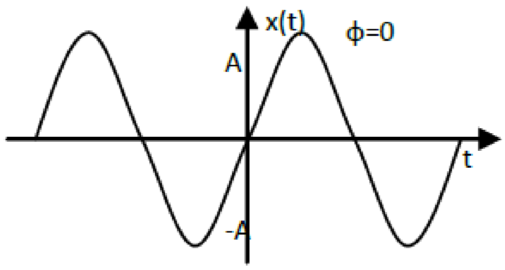
$$when\ \phi = 0,\ x(\ t\ ) = Acos\ (\Omega t)$$



Cosinusoidal signal

Sinusoidal signal

$$when\ \phi = 0,\ x(\ t) = Asin\ (\Omega t)$$

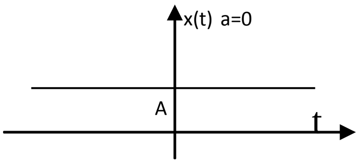


Sinusoidal signal

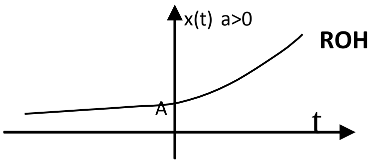
Exponential signal

Real Exponential signal is defined as $x(t) = Ae^{at}$,where A is amplitude

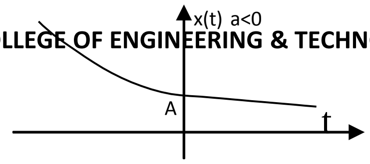
Depending on the value of 'a' we get dc signal or growing exponential signal or decaying exponential signal



DC signal



Exponentially growing signal



Exponentially decaying signal

Complex exponential signal is defined as

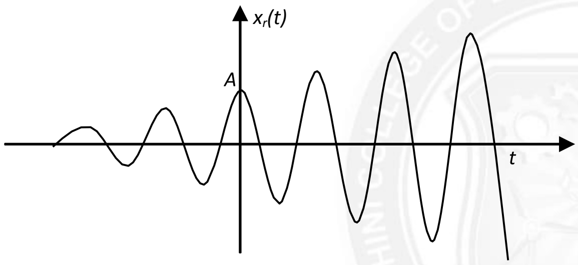
$$x(t) = Ae^{st}$$

where A is amplitude, s is complex variable and $s = \sigma + j\Omega$

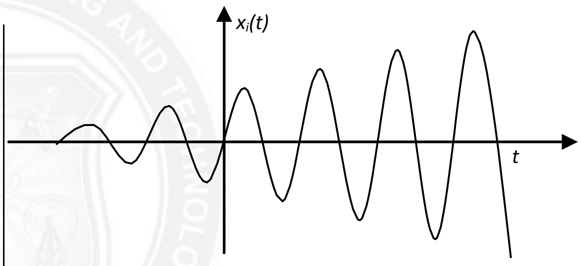
$$x(t) = Ae^{st} = Ae^{(\sigma+j\Omega)t} = Ae^{\sigma t}\, e^{j\Omega t} = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t)$$

$when\ \sigma = +ve,\ tten\ x(t) = Ae^{\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{\sigma t}\,cos\Omega t\ and\ x_i(t) = Ae^{\sigma t}\,sin\Omega t$



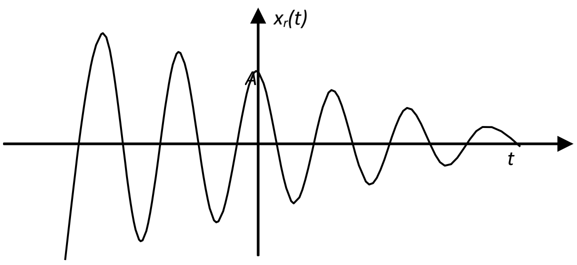
Exponentially growing Cosinusoidal signal



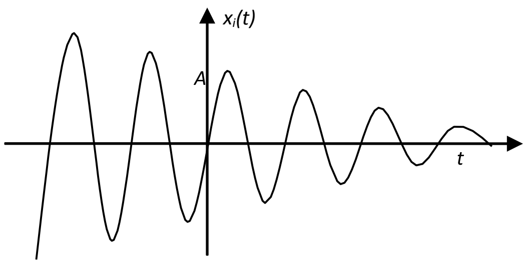
Exponentially growing sinusoidal signal

$when\ \sigma = -ve,\ then\ x(t) = Ae^{-\sigma t}\,(cos\Omega t + jsin\Omega t),$

$where\ x_r(t) = Ae^{-\sigma t}\,cos\Omega t\ and\ x_i(t) = Ae^{-\sigma t}\,sin\Omega t$



Exponentially decaying Cosinusoidal signal



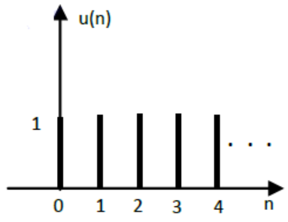
Exponentially decaying sinusoidal signal

BASIC (ELEMENTARY OR STANDARD) DISCRETE TIME SIGNALS

Step signal

Unit Step signal is defined as

$$u(n) = 1 \;\; for\; n \geq 0$$
$$= 0 \; for\; n < 0$$

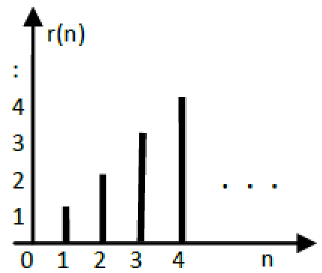


Unit step signal

Unit Ramp signal

Unit Ramp signal is defined as

$$r(n) = n \;\; for\; n \geq 0$$
$$= 0 \quad for\; n < 0$$

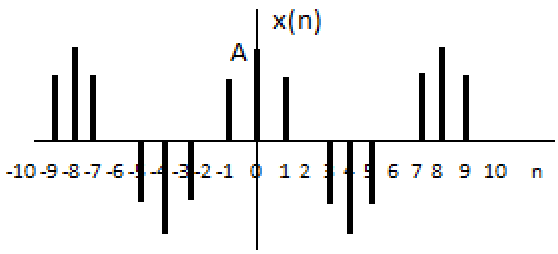


Unit Ramp signal

Sinusoidal signal

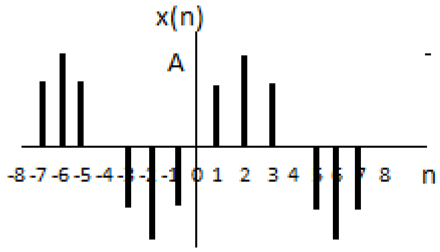
Cosinusoidal signal is defined as

$$x(n) = Acos(\omega n)$$



Sinusoidal signal is defined as

$$x(n) = Asin(\omega n)$$



Sinusoidal signal

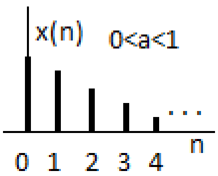
$$where\ \omega = 2\pi f = \frac{2\pi}{N} m\ and\ \omega\ is\ frequency\ in\ radians/sample$$

$m\ is\ the\ smallest\ integer$

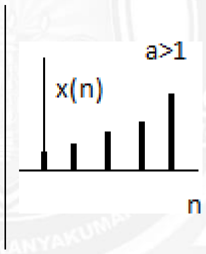
f is frequency in cycles/sample, A is amplitude

Exponential signal

Real Exponential signal is defined as $x(n) = a^n\ for\ n \geq 0$



Decreasing exponential signal

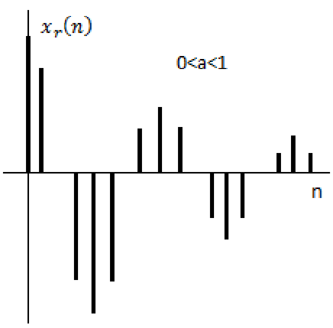


Increasing exponential signal

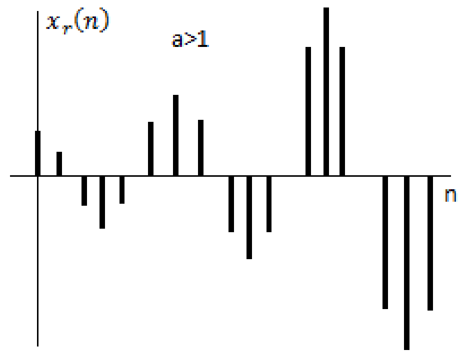
Complex Exponential signal is defined as

$$x(n) = a^n e^{j(\omega_0 n)} = a^n[\cos\omega_0 n + j\sin\omega_0 n]$$

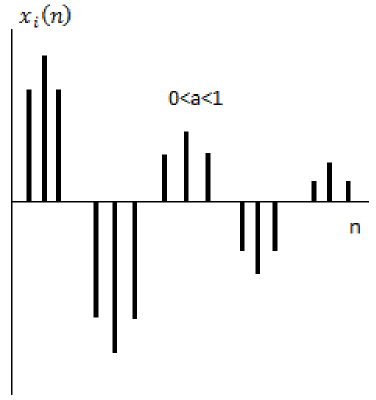
$$where\ x_r(n) = a^n \cos\omega_0 n\ and\ x_i(n) = a^n \sin\omega_0 n$$



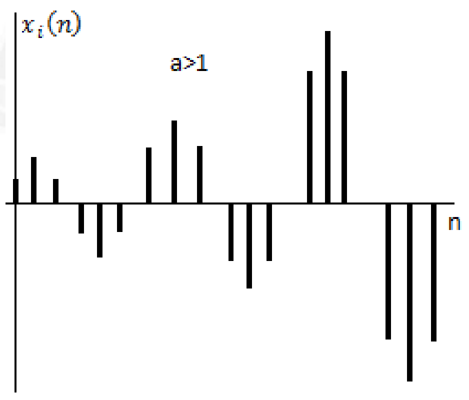
Exponentially decreasing Cosinusoidal signal



Exponentially growing Cosinusoidal signal



Exponentially decreasing sinusoidal signal



Exponentially growing sinusoidal signal