ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MATHEMATICS

BA4201 / QUANTITATIVE TECHNIQUES FOR DECISION MAKING

2.4 LEAST COST METHOD

INTRODUCTION

The **Least Cost Method** is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation. The Least Cost Method is considered to produce more optimal results than the North-west Corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost. The procedure is given below:

Step 1: Balance the problem i.e. $\sum \text{Supply} = \sum \text{Demand}$
Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand.
i.e. Identifying the lowest cell value in this entire matrix.
Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix
Step 4: Repeat the procedure until all the allocations are over

i.e. Repeat the same procedure of allocation of the smallest value in the new generated matrix
Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

i.e. Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost

Problem 1:
A mobile phone manufacturing company has three branches located in three different regions, say Jaipur, Udaipur and Mumbai. The company has to transport mobile phones to three destinations, say Kanpur, Pune and Delhi. The availability from Jaipur, Udaipur and

Mumbai is 40, 60 and 70 units respectively. The demand at Kanpur, Pune and Delhiare 70, 40 and 60 respectively. The transportation cost is shown in the matrix below. Use the Least Cost method to find a basic feasible solution (BFS).

		Destinations			
		Kanpur	Pune	Delhi	Supply
	Jaipur	4	5	1	40
sources	Udaipur	3	4	3	60
	Mumbai	6	2	8	70
	Demand	70	40	60	170

Solution

Step 1: Balance the problem : Σ Supply=Σ Demand

→ The given transportation problem is balanced.

		Destinations			
		Kanpur	Pune	Delhi	Supply
	Jaipur	4	5	1	40
sources	Udaipur	3	4	3	60
	Mumbai	6	2	8	70
	Demand	70	40	60	170

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand.

Identifying the lowest cell value in this entire matrix. Here, in this matrix we have 1 (For cell: Jaipur-Delhi) as the lowest value. So, moving with that cell, and allocating the minimum of demand or supply, i.e. allocating 40 here (as supply value is 40, whereas demand is of 60). Subtracting allocated value (i.e. 40) from corresponding supply and demand.

			Destinations		
		Kanpur	Pune	Delhi	Supply
	Jaipur	~~4~~	~~5~~	~~1~~ (40)	~~40~~ 0
	Udaipur	3	4	3	60
sources	Mumbai	6	2	8	70
	Demand	70	40	~~60~~ 20	

Step 3:

Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

			Destinations		
		Kanpur	Pune	Delhi	Supply
	Udaipur	3	4	3	60
sources	Mumbai	6	2	8	70
	Demand	70	40	20	

Step 4: Repeat the procedure until all the allocations are over

Repeat the same procedure of allocation of the smallest value in the new generated matrix and check out demand or supply based on the smallest value (of demand or supply) as shown below, until all allocations are over.

			Destinations		
		Kanpur	Pune	Delhi	Supply
	Udaipur	3	~~4~~	3	60
sources	Mumbai	6	~~2~~ (40)	8	~~70~~ 30
	Demand	70	~~40~~ 0	20	

		Destinations		
		Kanpur	Delhi	Supply
sources	Udaipur	3	3 **(20)**	~~60~~ **40**
	Mumbai	6	8	30
	Demand	70	~~20~~ **0**	

		Destinations	
		Kanpur	Supply
sources	Udaipur	3 **(40)**	~~40~~ **0**
	Mumbai	6 **(30)**	~~30~~ **0**
	Demand	~~70~~ ~~**30**~~ **0**	

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost as follows:

		Destinations			
		Kanpur	Pune	Delhi	Supply
sources	Jaipur	4	5	1 **(40)**	40
	Udaipur	3 **(40)**	4	3 **(20)**	60
	Mumbai	6 **(30)**	2 **(40)**	8	70
	Demand	70	40	60	

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Therefore, Transportation Cost = (1*40) + (3*40) + (3*20) + (6*30) + (2*40) = Rs. 480.

Problem : 2
Find Solution using Least Cost method

To / Source	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150

Solution

Balance the problem : Σ Supply=Σ Demand

→ The given transportation problem is balanced.

In the given matrix, the supply of each source A, B, C is given Viz. 50units, 40 units, and 60 units respectively. The weekly demand for three retailers D, E, F i.e. 20 units, 95 units and 35 units is given respectively. The shipping cost is given for all the routes.

The minimum transportation cost can be obtained by following the steps given below:

The minimum cost in the matrix is Rs 3, but there is a tie in the cell BF, and CD, now the question arises in which cell we shall allocate. Generally, the cost where maximum quantity can be assigned should be chosen to obtain the better initial solution. Therefore, 35 units shall be assigned to the cell BF. With this, the demand for retailer F gets fulfilled, and only 5 units are left with the source B. Again the minimum cost in the matrix is Rs 3. Therefore, 20 units shall be assigned to the cell CD. With this, the demand of retailer D gets fulfilled. Only 40 units are left with the source C.

The next minimum cost is Rs 4, but however, the demand for F is completed, we will move to the next minimum cost which is 5. Again, the demand of D is completed. The next minimum cost is 6, and there is a tie between three cells. But however, no units can be assigned to the cells BD and CF as the demand for both the retailers D and F are saturated. So, we shall assign 5 units to Cell BE. With this, the supply of source B gets saturated.

The next minimum cost is 8, assign 50 units to the cell AE. The supply of source A gets saturated.

The next minimum cost is Rs 9; we shall assign 40 units to the cell CE. With his both the demand and supply of all the sources and origins gets saturated.

Source \ To	D	E	F	Supply
A	5	8 50	4	50
B	6	6 5	3 35	40
C	3 20	9 40	6	60
Demand	20	95	35	150

The total cost can be calculated by multiplying the assigned quantity with the concerned cost of the cell. Therefore,

Total Cost = 50*8 + 5*6 + 35*3 +20*3 +40*9 = Rs 955.

Problem : 3 Find Solution using Least Cost method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:
Problem Table is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10	7
$S2$	70	30	40	60	9
$S3$	40	8	70	20	18
Demand	5	8	7	14	

Σ Supply=Σ Demand

$\longrightarrow$ The given transportation problem is balanced.
The smallest transportation cost is 8 in cell $S3$ $D2$.
The allocation to this cell is min(18,8) = 8.
This satisfies the entire demand of $D2$ and leaves 18 - 8 = 10 units with $S3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10	7
$S2$	70	30	40	60	9
$S3$	40	8(8)	70	20	10
Demand	5	0	7	14	

The smallest transportation cost is 10 in cell $S1D4$

The allocation to this cell is min(7,14) = 7.

This exhausts the capacity of $S1$ and leaves 14 - 7 = 7 units with $D4$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10(7)	0
$S2$	70	30	40	60	9
$S3$	40	8(8)	70	20	10
Demand	5	0	7	7	

The smallest transportation cost is 20 in cell *S*3*D*4

The allocation to this cell is min(10,7) = 7.

This satisfies the entire demand of *D*4 and leaves 10 - 7 = 3 units with *S*3

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	19	30	50	10(7)	0
*S*2	70	30	40	60	9
*S*3	40	8(8)	70	20(7)	3
Demand	5	0	7	0	

The smallest transportation cost is 40 in cell *S*2*D*3

The allocation to this cell is min(9,7) = 7.

This satisfies the entire demand of *D*3 and leaves 9 - 7 = 2 units with *S*2

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	19	30	50	10(7)	0
*S*2	70	30	40(7)	60	2
*S*3	40	8(8)	70	20(7)	3
Demand	5	0	0	0	

The smallest transportation cost is 40 in cell *S*3*D*1

The allocation to this cell is min(3,5) = 3.

This exhausts the capacity of *S*3 and leaves 5 - 3 = 2 units with *D*1

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	19	30	50	10(7)	0
*S*2	70	30	40(7)	60	2
*S*3	40(3)	8(8)	70	20(7)	0
Demand	2	0	0	0	

The smallest transportation cost is 70 in cell $S2D1$

The allocation to this cell is min(2,2) = 2.

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10(7)	0
$S2$	70(2)	30	40(7)	60	0
$S3$	40(3)	8(8)	70	20(7)	0
Demand	0	0	0	0	

Initial feasible solution is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10 (7)	7
$S2$	70 (2)	30	40 (7)	60	9
$S3$	40 (3)	8 (8)	70	20 (7)	18
Demand	5	8	7	14	

The minimum total transportation cost $=10\times7+70\times2+40\times7+40\times3+8\times8+20\times7=814$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate.

Problem : 4 Find Solution using Least Cost method

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:
Problem Table is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11	13	17	14	250
$S2$	16	18	14	10	300
$S3$	21	24	13	10	400
Demand	200	225	275	250	

Σ Supply=Σ Demand, $\longrightarrow$ The given transportation problem is balanced.

The smallest transportation cost is 10 in cell $S3D4$

The allocation to this cell is min(400,250) = **250**.

This satisfies the entire demand of $D4$ and leaves 400 - 250 = 150 units with $S3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11	13	17	14	250
$S2$	16	18	14	10	300
$S3$	21	24	13	10**(250)**	150
Demand	200	225	275	0	

The smallest transportation cost is 11 in cell $S1D1$

The allocation to this cell is min(250,200) = **200**.

This satisfies the entire demand of $D1$ and leaves 250 - 200 = 50 units with $S1$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11**(200)**	13	17	14	50
$S2$	16	18	14	10	300
$S3$	21	24	13	10**(250)**	150
Demand	0	225	275	0	

The smallest transportation cost is 13 in cell *S*3*D*3

The allocation to this cell is min(150,275) = **150**.

This exhausts the capacity of *S*3 and leaves 275 - 150 = 125 units with *D*3

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	11**(200)**	13	17	14	50
*S*2	16	18	14	10	300
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	225	125	0	

The smallest transportation cost is 13 in cell *S*1*D*2

The allocation to this cell is min(50,225) = **50**.

This exhausts the capacity of *S*1 and leaves 225 - 50 = 175 units with *D*2

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14	10	300
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	175	125	0	

The smallest transportation cost is 14 in cell *S*2*D*3

The allocation to this cell is min(300,125) = **125**.

This satisfies the entire demand of *D*3 and leaves 300 - 125 = 175 units with *S*2

	*D*1	*D*2	*D*3	*D*4	Supply
*S*1	11**(200)**	13**(50)**	17	14	0
*S*2	16	18	14**(125)**	10	175
*S*3	21	24	13**(150)**	10**(250)**	0
Demand	0	175	0	0	

The smallest transportation cost is 18 in cell $S2D2$

The allocation to this cell is min(175,175) = **175**.

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11**(200)**	13**(50)**	17	14	0
$S2$	16	18**(175)**	14**(125)**	10	0
$S3$	21	24	13**(150)**	10**(250)**	0
Demand	0	0	0	0	

Initial feasible solution is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11 **(200)**	13 **(50)**	17	14	250
$S2$	16	18 **(175)**	14 **(125)**	10	300
$S3$	21	24	13 **(150)**	10 **(250)**	400
Demand	200	225	275	250	

The minimum total transportation cost $=11\times200+13\times50+18\times175+14\times125+13\times150+10\times250=12200$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate

Problem 5 : Find Solution using Least Cost method

	D1	D2	D3	Supply
S1	4	8	8	76
S2	16	24	16	82
S3	8	16	24	77
Demand	72	102	41	

Solution:

Problem Table is

	$D1$	$D2$	$D3$	Supply
$S1$	4	8	8	76
$S2$	16	24	16	82
$S3$	8	16	24	77
Demand	72	102	41	

Here Total Demand = 215 is less than Total Supply = 235. So We add a dummy demand (D4) constraint with 0 unit cost and with allocation 20.

Now, The modified table is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4	8	8	0	76
$S2$	16	24	16	0	82
$S3$	8	16	24	0	77
Demand	72	102	41	20	

The smallest transportation cost is 0 in cell $S1Ddummy$

The allocation to this cell is min(76,20) = **20**.

This satisfies the entire demand of $Ddummy$ and leaves 76 - 20 = 56 units with $S1$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4	8	8	0**(20)**	56
$S2$	16	24	16	0	82
$S3$	8	16	24	0	77
Demand	72	102	41	0	

The smallest transportation cost is 4 in cell $S1D1$

The allocation to this cell is min(56,72) = **56**.

This exhausts the capacity of $S1$ and leaves 72 - 56 = 16 units with $D1$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4**(56)**	8	8	0**(20)**	0
$S2$	16	24	16	0	82
$S3$	8	16	24	0	77
Demand	16	102	41	0	

The smallest transportation cost is 8 in cell $S3D1$

The allocation to this cell is min(77,16) = **16**.

This satisfies the entire demand of $D1$ and leaves 77 - 16 = 61 units with $S3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4**(56)**	8	8	0**(20)**	0
$S2$	16	24	16	0	82
$S3$	8**(16)**	16	24	0	61
Demand	0	102	41	0	

The smallest transportation cost is 16 in cell $S3D2$

The allocation to this cell is min(61,102) = **61**.

This exhausts the capacity of $S3$ and leaves 102 - 61 = 41 units with $D2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4**(56)**	8	8	0**(20)**	0
$S2$	16	24	16	0	82
$S3$	8**(16)**	16**(61)**	24	0	0
Demand	0	41	41	0	

The smallest transportation cost is 16 in cell $S2D3$

The allocation to this cell is min(82,41) = **41**.

This satisfies the entire demand of $D3$ and leaves 82 - 41 = 41 units with $S2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4**(56)**	8	8	0**(20)**	0
$S2$	16	24	16**(41)**	0	41
$S3$	8**(16)**	16**(61)**	24	0	0
Demand	0	41	0	0	

The smallest transportation cost is 24 in cell $S2D2$

The allocation to this cell is min(41,41) = **41**.

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4**(56)**	8	8	0**(20)**	0
$S2$	16	24**(41)**	16**(41)**	0	0
$S3$	8**(16)**	16**(61)**	24	0	0
Demand	0	0	0	0	

Initial feasible solution is

	$D1$	$D2$	$D3$	$Ddummy$	Supply
$S1$	4 **(56)**	8	8	0 **(20)**	76
$S2$	16	24 **(41)**	16 **(41)**	0	82
$S3$	8 **(16)**	16 **(61)**	24	0	77
Demand	72	102	41	20	

The minimum total transportation cost $=4\times56+0\times20+24\times41+16\times41+8\times16+16\times61=2968$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

$\therefore$ This solution is non-degenerate