ROHINI college of engineering
\& TECHNOLOGY

DEPARTMENT OF MATHEMATICS
BA4201 / QUANTITATIVE TECHNIQUES FOR DECISION MAKING

### 1.4 BIG - M - METHOD

## INTRODUCTION

The Big M method is a version of the Simplex Algorithm that first finds a best feasible solution by adding "artificial" variables to the problem. The objective function of the original LP must, of course, be modified to ensure that the artificial variables are all equal to 0 at the conclusion of the simplex algorithm. The iterative procedure of the algorithm is given below.

Step-1 : Modify the constraints so that the RHS of each constraint is non-negative (This requires that each constraint with a negative RHS be multiplied by -1 . Remember that if any negative number multiplies an inequality, the direction of the inequality is reversed). After modification, identify each constraint as a <, >, or = constraint.

Step-2 : Convert each inequality constraint to standard form (If a constraint is $\mathrm{a} \leq$ constraint, then add a slack variable Xi ; and if any constraint is $\mathrm{a} \geq$ constraint, then subtract an excess variable Xi , known as surplus variable).

Step-3: Add an artificial variable al to the constraints identified as ' $\geq$ ' or with ' $=$ ' constraints at the end of Step2. Also add the sign restriction $a_{i} \geq 0$.

Step-4: Let M denote a very large positive number. If the LP is a minimization problem, add (for each artificial variable) Mai to the objective function. If the LP is a maximization problem, add (for each artificial variable) -Mai to the objective function.

Step-5: Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from row 0 before beginning the simplex.

## Problem :1

Solve the following LPP by using Big -M Method
Maximize $\mathrm{Z}=6 \mathrm{X} 1+4 \mathrm{X} 2$
Subject to constraints:
$2 \mathrm{X} 1+3 \mathrm{X} 2<=30,3 \mathrm{X} 1+2 \mathrm{X} 2<=24, \mathrm{X} 1+\mathrm{X} 2>=3$

## Solution

Introducing slack variables $S 1>=0, S 2>=0$ to the first and second equations in order to convert <= type to equality and add surplus variable to the third equation $\mathrm{S} 3>=0$ to convert $>=$ type to equality. Then the standard form of LPP is

MAX Z $=6 \mathrm{X} 1+4 \mathrm{X} 2+0 \mathrm{~S} 1+0 \mathrm{~S} 2+0 \mathrm{~S} 3$
Subject to constraints
$2 \mathrm{X} 1+3 \mathrm{X} 2+\mathrm{S} 1=30,3 \mathrm{X} 1+2 \mathrm{X} 2+\mathrm{S} 2=24, \mathrm{X} 1+\mathrm{X} 2-\mathrm{S} 3=3$
Clearly there is no initial basic feasible solution. So an artificial variable A1>=0 is added in the third equation. Now the standard form will be
$\mathrm{MAX} \mathrm{Z}=6 \mathrm{X} 1+4 \mathrm{X} 2+0 \mathrm{~S} 1+0 \mathrm{~S} 2+0 \mathrm{~S} 3+\mathrm{A} 1$
Subject to constraints
$2 \mathrm{X} 1+3 \mathrm{X} 2+\mathrm{S} 1=30,3 \mathrm{X} 1+2 \mathrm{X} 2+\mathrm{S} 2=24$
$\mathrm{X} 1+\mathrm{X} 2-\mathrm{S} 3+\mathrm{A} 1=3$

|  |  |  | 6 | 4 | 0 | 0 | 0 | -M | Xbi/ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cb | YBa | Xba | Y 1 | Y 2 | Y 3 | Y 4 | Y 5 | Y 6 |
| Yi |  |  |  |  |  |  |  |  |  |$|$

It clearly shows the net evaluations for each column variable; from these calculations, it is clear that X 1 is the entering variable and A 1 , the artificial variable leaves the basis. Introduce Y1 and drop Y6. Then, in the usual row operations, we modify this table and the new table is arrived.

|  |  |  | 6 | 4 | 0 | 0 | 0 | -M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cb | YBa | Xba | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |  |
| 0 | Y3 | $\mathrm{X} 3=24$ | 0 | 1 | 1 | 0 | 2 | -2 | $24 / 2=12$ |
| 0 | Y4 | $\mathrm{X} 4=15$ | 0 | -1 | 0 | 1 | 3 | -3 | $15 / 3=5$ |
| 6 | Y1 | $\mathrm{X} 1=3$ | 1 | 1 | 0 | 0 | -1 | 1 | - |
| $\mathrm{Zj}-\mathrm{Cj}$ |  |  | 0 | 2 | 0 | 0 | -6 | 6 |  |

Introduce Y5 and drop Y4 from the basis; the new modified table-3 is given below;

|  |  |  | 6 | 4 | 0 | 0 | 0 | $-M$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cb | YBa | Xba | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |  |
| 0 | Y 3 | $\mathrm{X} 3=14$ | 0 | $5 / 3$ | 1 | $-2 / 3$ | 0 | 0 |  |


| 0 | Y 5 | $\mathrm{X} 5=5$ | 0 | $-1 / 3$ | 0 | $1 / 3$ | 1 | -1 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | Y 1 | $\mathrm{X} 1=8$ | 1 | $2 / 3$ | 0 | $1 / 3$ | 0 | 0 |  |
| $\mathrm{Zj}-\mathrm{Cj}$ |  | 0 | 0 | 0 | 2 | 0 | 0 |  |  |

Since all $\mathrm{Zj}-\mathrm{Cj}>=0$, an optimum solution has reached. Thus an optimum basic feasible solution to the LPP is $\mathrm{X} 1=8, \mathrm{X} 2=0, \mathrm{MAX} \mathrm{Z}=48$

## Problem: 2

Use big $M$ method to solve a given LPP. Minimize $Z=5 X 1-6 X 2-7 X 3$
Subject to constraints
$\mathrm{X} 1+5 \mathrm{X} 2-3 \mathrm{X} 3>=15,5 \mathrm{X} 1-6 \mathrm{X} 2+10 \mathrm{X} 3<=20, \mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3=5$
$\mathrm{X} 1, \mathrm{X} 2 \& \mathrm{X} 3>=0$

## Solution

Introducing slack variables $\mathrm{X} 4>=0$ to the first equations in order to convert $<=$ type to equality and add surplus variable $\mathrm{X} 5>=0$ to the second equation in order to convert $>=$ type to equality.

Then the standard form of LPP is
MIN Z=5X1-6X2-7X3+X4-X5
Subject to constraints
$5 \mathrm{X} 1-6 \mathrm{X} 2+10 \mathrm{X} 3+\mathrm{X} 4=20, \mathrm{X} 1+5 \mathrm{X} 2-3 \mathrm{X} 3-\mathrm{X} 5=15, \mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3=5$
Clearly there is no initial basic feasible solution. So two artificial variables A1>=0 and A2>=0 are added in the second and third equation. Now the standard form will be

MIN Z=5X1-6X2-7X3+X4-X5+A1+A2
Subject to constraints
$5 \mathrm{X} 1-6 \mathrm{X} 2+10 \mathrm{X} 3+\mathrm{X} 4=20, \mathrm{X} 1+5 \mathrm{X} 2-3 \mathrm{X} 3-\mathrm{X} 5+\mathrm{A} 1=15, \mathrm{X} 1+\mathrm{X} 2+\mathrm{X} 3+\mathrm{A} 2=5$

## Simplex Table-1

|  |  |  | 5 | -6 | -7 | 0 | 0 | -M | -M | Xbi/ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cb | YBa | Xba | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 | Yi1 |
| 0 | Y4 | $\mathrm{X} 4=20$ | 5 | -6 | 10 | 1 | 0 | 0 | 0 | - |
| M | Y6 | $\mathrm{X} 6=15$ | 1 | 5 | -3 | 0 | -1 | 1 | 0 | $\begin{gathered} 15 / 5 \\ =3 \end{gathered}$ |
| M | Y7 | $\mathrm{X} 7=5$ | 1 | 1 | 1 | 0 | 0 | 0 | 1 | $5 / 1=5$ |
| Zj-Cj |  |  | 2M-5 | $6 \mathrm{M}+6$ | $2 \mathrm{M}+7$ | 0 | -M | 0 | 0 |  |

From the calculations related to entering column and leaving variable, which is summarized in the above table, it is clear that introduces X 2 and drop X 6 from the basis. The new simplex table is given below.

## Simplex Table-2

| 5 |  |  | -6 | -7 | 0 | 0 | M | M | $\mathrm{Xbi} /$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cb | YBa | Xba | Y 1 | Y 2 | Y 3 | Y 4 | Y 5 | Y 6 | Y 7 | Yi 1 |
| 0 | Y 4 | $\mathrm{X} 4=38$ | $31 / 5$ | 0 | $32 / 5$ | 1 | $-6 / 5$ | $6 / 5$ | 0 | $190 / 31$ |
| -6 | Y 2 | $\mathrm{X} 6=3$ | $1 / 5$ | 1 | $-3 / 5$ | 0 | $-1 / 5$ | $1 / 5$ | 0 | - |
| M | Y 7 | $\mathrm{X} 7=2$ | $4 / 5$ | 0 | $8 / 5$ | 0 | $1 / 5$ | $-1 / 5$ | 1 | $10 / 8$ |
| Zj -Cj |  |  |  |  |  |  |  |  |  | $(-31+$ <br> $4 \mathrm{M}) / 5$ |

Now, we see that we have to includ 13 and drop X7 from the basis.
The new modified table- 3 is given below

## Simplex Table- 3

|  |  |  | 5 | -6 | -7 | 0 | 0 | M | M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cb | YBa | Xba | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Y7 |  |
| 0 | Y4 | $\mathrm{X} 4=30$ | 31/5 | 0 | 0 | 1 | -2 | 2 | -4 |  |
| -6 | Y2 | $\mathrm{X} 2=15 / 4$ | 1/5 | 1 | 0 | 0 | -1/8 | 1/8 | 3/8 |  |
| -7 | Y3 | $\mathrm{X} 3=5 / 4$ | 0 | 0 | 1 | 0 | -1/8 | 1/8 | 5/8 |  |
| Zj-Cj |  |  | -31/5 | 0 | 0 | 0 | -1/8 | - | - |  |

Since all $\mathrm{Zj}-\mathrm{Cj}<=0$, an optimum solution has reached. Thus an optimum basic feasible solution to the LPP is $\mathrm{X} 3=5 / 4, \mathrm{X} 2=15 / 4$,
$\operatorname{MIN} Z=(5 * 0)+(-6 * 15 / 4)+(-7 * 5 / 4)=(-90 / 4)+(-35 / 4)=-125 / 4$

## Problem : 3

Use penalty (or Big 'M') method to
Minimize $z=4 \mathbf{x}_{i}+\mathbf{3} \mathbf{x}_{2}$
subject to the constraints :

$$
\begin{aligned}
& 2 x_{1}+x_{2} \geq 10, \quad-3 x_{1},+2 x_{2} \leq 6 \\
& x_{1}+x_{2} \geq 6, \quad x_{1} \geq 0 \text { and } x_{2} \geq 0 .
\end{aligned}
$$

Solution. Introducing surplus (negative slack) variables $x_{3} \geq 0, x_{5} \geq 0$ and slack variable $\mathrm{x}_{4} \geq 0$ in the constraint inequations, the problem becomes

Maximize $\mathrm{z}^{*}=-4 \mathrm{x}_{1}-3 \mathrm{x}_{2}+0 . \mathrm{x}_{3}+0 . \mathrm{x}_{4}+0 . \mathrm{x}_{5}$
subject to the constraints :

$$
\begin{aligned}
2 x_{1}+x_{2}-x_{3} & =10, \quad-3 x_{1}+2 x_{2}+x_{4}=6 \\
x_{1}+x_{2}-x_{5} & =6, \quad x_{j} \geq 0 Q(j=1,2,3,4,5)
\end{aligned}
$$

Clearly, we do not have a ready basic feasible solution. The surplus variables carry negative coefficients ( -1 ). We introduce two new variables $\mathrm{A}_{1} \geq 0$ and $\mathrm{A}_{2} \geq 0$ in the first and third equations respectively. These extraneous variables, commonly termed as artificial variables, play the same role as that of slack variables in providing a starting basic feasible solution.

We assign a very high penalty cost (say $-M, M \geq 0$ ) to these variables in the objective function so that they may be driven to zero while reaching optimality.

Now the following initial basic feasible solution is available :

$$
\mathrm{A}_{1}=10, \mathrm{x}_{4}=6 \text { and } \mathrm{A}_{2}=6
$$

with $\mathbf{B}=\left(\mathbf{a}_{6}, \mathbf{a}_{4}, \mathbf{a}_{7}\right)$ as the basis matrix. The cost matrix corresponding to basic feasible solution is $\mathbf{c}_{\boldsymbol{B}}=(-M, 0,-M)$

Now, corresponding to the basic variables $\mathrm{A}_{1}, \mathrm{x}_{4}$ and $\mathrm{A}_{2}$. the matrix $\mathbf{Y}=\mathbf{B}^{-1} \mathbf{A}$ and the net evaluations $\mathrm{z}_{\mathrm{j}}-\mathrm{c}_{\mathrm{j}}(\mathrm{j}=1,2, \ldots .7)$ are computed. The initial basic feasible solution is displayed in the following simplex table :

| Initial Iteration. |  |  | $\begin{array}{ccc} \hline \text { Introduce } y_{1} \text { and drop } \mathrm{y}_{6} . \\ -4 & -3 & 0 \end{array}$ |  |  | 0 | 0 | -M | -M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{c}_{\mathrm{B}}$ | $\mathrm{yb}_{\text {B }}$ | $\mathrm{x}_{\mathrm{B}}$ | $\mathrm{y}_{1}$ | $\mathrm{y}_{2}$ | $\mathrm{y}_{3}$ | $\mathrm{y}_{4}$ | $\mathrm{y}_{5}$ | $\mathrm{y}_{6}$ | $\mathrm{y}_{7}$ |
| -M | $\mathrm{y}_{6}$ | 10 | 2* | 1 | -1 | 0 | 0 | 1 | 0 |
| 0 | $\mathrm{y}_{4}$ | 6 | -3 | 2 | 0 | 1 | 0 | 0 | 0 |
| $-M$ | $\mathrm{y}_{7}$ | 6 | 1 | 1 | 0 | 0 | -1 | 0 | 1 |
|  | $z^{*}$ | -16M | $-3 \mathrm{~W}+4$ | $-2 \mathrm{M}+3$ | M | 0 | M | 0 | 0 |

We observe that the most negative $\mathrm{zj}-\mathrm{cj}$ is $4-3 \mathrm{M}\left(=\mathrm{z}_{1}-\mathrm{c}_{1}\right)$. The corresponding column vector $\mathrm{y}_{1}$, therefore, enters the basis. Further, since min. $=5$; the element $\mathrm{y}_{11}(=2)$ becomes the leading element for the first iteration

First Iteration: Introduce $\mathrm{y}_{2}$ and drop $\mathrm{y}_{7}$.

| $\mathrm{c}_{\mathrm{B}}$ | $\mathrm{y}_{\mathrm{B}}$ | $\mathrm{x}_{\mathrm{B}}$ | $\mathrm{y}_{1}$ | $\mathrm{y}_{2}$ | $\mathrm{y}_{3}$ | $\mathrm{y}_{4}$ | $\mathrm{y}_{5}$ | $\mathrm{y}_{7}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -4 | $\mathrm{y}_{1}$ | 5 | 1 | $1 / 2$ | $-1 / 2$ | 0 | 0 | 0 |
| 0 | $\mathrm{y}_{4}$ | 21 | 0 | $7 / 2$ | $-3 / 2$ | 1 | 0 | 0 |
| -M | $\mathrm{y}_{7}$ | 0 | 0 | $1 / 2^{*}$ | $1 / 2$ | 0 | -1 | 0 |
|  | $\mathrm{z}^{*}$ | $-\mathrm{M}-20$ | 0 | $\frac{-M}{2}+1$ | $\frac{-M}{2}+2$ | 0 | M | 0 |
|  |  |  |  |  |  |  |  |  |

In the above table, we omitted all entries of column vector $\mathbf{y}_{6}$, because the artificial variables $A_{l}$ has left the basis and we would not like it to re-enter in any subsequent iterations.

Now since the most negative $\left(\mathrm{Z}_{\mathrm{j}}-\mathrm{c}_{\mathrm{j}}\right)$ is $\mathrm{Z}_{2}-\mathrm{c}_{2}$; the non-basic vector $\mathbf{y}_{2}$ enters the basis. Further, since min is 2 which occurs for the element $y_{32}(=1 / 2)$, the corresponding
basis vector $y_{7}$ leaves the basis and the element $y_{32}$ becomes the leading element for the next iteration.

Final Iteration: Optimum Solution,

| $\mathrm{c}_{\mathrm{B}}$ | $\mathrm{y}_{\mathrm{B}}$ | $\mathrm{x}_{\mathrm{B}}$ | $\mathrm{y}_{1}$ | $\mathrm{y}_{2}$ | $\mathrm{y}_{3}$ | $\mathrm{y}_{4}$ | $\mathrm{y}_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -4 | $\mathrm{y}_{1}$ | 4 | 1 | 0 | -1 | 0 | 1 |
| 0 | $\mathrm{y}_{4}$ | 14 | 0 | 0 | -5 | 1 | 7 |
| -3 | $\mathrm{y}_{2}$ | 2 | 0 | 1 | 1 | 0 | -2 |
|  | $\mathrm{z}^{*}$ | -22 | 0 | 0 | 1 | 0 | 2 |

It is clear from the table that all $\mathrm{z}_{\mathrm{j}}-\mathrm{c}_{\mathrm{j}}$ are positive. Therefore an optimum basic feasible solution has been attained which is given by

$$
\mathrm{x}_{1}=4, \mathrm{x}_{2}=2, \text { maximum } \mathrm{z}=22
$$

## Problem : 4

Maximize $z=\mathbf{3} x_{1},+2 \mathbf{x}_{2} \quad$ subject to the constraints :

$$
2 \mathrm{x}_{1}+\mathrm{x}_{2} \leq 2,3 \mathrm{x}_{1}+4 \mathrm{x}_{2} \geq 12, \mathrm{x}_{1}, \mathrm{x}_{2} \geq 0
$$

## Solution:

Introducing slack variable $\mathrm{x}_{3} \geq 0$, surplus variable $\mathrm{x}_{5} \geq 0$ and an artificial variable $A_{1} \geq 0$, the reformulated L.P.P. can be written as :

Maximize $\mathrm{z}=3 \mathrm{x}_{1}+2 \mathrm{x}_{2}+0 . \mathrm{x}_{3}+0 . \mathrm{x}_{4}-\mathrm{MA}_{1}$
subject to the constraints :
$2 \mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{x}_{3}=2$,
$3 \mathrm{x}_{1}+4 \mathrm{x}_{2}-\mathrm{x}_{4}+\mathrm{A}_{1}=12$
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{x}_{4} \geq 0$ and $\mathrm{A}_{1} \geq 0$.
starting basic feasible solution is :
$\mathrm{x} 3=2$ and $\mathrm{A}_{1}=12$.
The iterative simplex tables are :

Initial Iteration: Introduce $\mathrm{y}_{2}$ and drop $\mathrm{y}_{3}$.

|  |  | 3 | 2 | 0 | 0 | -M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{c}_{\mathrm{B}}$ | $\mathrm{y}_{\mathrm{B}}$ | $\mathrm{x}_{\mathrm{B}}$ | $\mathrm{y}_{1}$ | $\mathrm{y}_{2}$ | $\mathrm{y}_{3}$ | $\mathrm{y}_{4}$ | $\mathrm{y}_{5}$ |
| 0 | $\mathrm{y}_{3}$ | 2 | 2 | $1^{*}$ | 1 | 0 | 0 |
| -M | $\mathrm{y}_{5}$ | 12 | 3 | 4 | 0 | -1 | 1 |
|  | z | $-12 \mathrm{M}-2$ | $-3 \mathrm{M}-3$ | -4 M | 0 | M | 0 |

Final Iteration. No solution.

| $\mathrm{c}_{\mathrm{B}}$ | $\mathrm{y}_{\mathrm{B}}$ | $\mathrm{x}_{\mathrm{B}}$ | $\mathrm{y}_{1}$ | $\mathrm{y}_{2}$ | $\mathrm{y}_{3}$ | $\mathrm{y}_{4}$ | $\mathrm{y}_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $\mathrm{y}_{2}$ | 2 | 2 | 1 | 1 | 0 | 0 |
| -M | $\mathrm{y}_{5}$ | 4 | -5 | 0 | -4 | -1 | 1 |
|  | z | $4 \mathrm{M}+4$ | $5 \mathrm{M}+1$ | 0 | $4 \mathrm{M}+2$ | M | 0 |

Here the coefficient of $M$ in each $z_{j}-c_{j}$ is non-negative and an artificial vector appears in the basis, not at the zero level. Thus the given L.P.P. does not possess any feasible solution.

## Problem 5

$\operatorname{Max} Z=-2 \mathrm{x}_{1}-\mathrm{X}_{2}$
Subject to

$$
3 x_{1}+x_{2}=3, \quad 4 x_{1}+3 x_{2} \geq 6, \quad x_{1}+2 x_{2} \leq 4
$$

$$
\text { and } \mathrm{x}_{1} \geq 0, \mathrm{x}_{2} \geq 0
$$

## Solution

$\operatorname{Max} Z=-2 \mathrm{x}_{1}-\mathrm{x}_{2}+0 \mathrm{~s}_{1}+0 \mathrm{~s}_{2}-\mathrm{Ma}_{1}-\mathrm{Ma} \mathrm{a}_{2}$
Subject to

$$
\begin{aligned}
& 3 x_{1}+x_{2}+a_{1}=3 \\
& 4 x_{1}+3 x_{2}-s_{1}+a_{2}=6 \\
& x_{1}+2 x_{2}+s_{2}=4 \\
& x_{1}, x_{2}, s_{1}, s_{2}, a_{1}, a_{2} \geq 0
\end{aligned}
$$

| $\mathrm{C}_{\mathrm{j}} \rightarrow$ |  | -2 | -1 | 0 | 0 | -M | -M |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Basic | $\mathrm{C}_{\mathrm{B}}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ | Min ratio <br> $\mathrm{X}_{\mathrm{B}} / \mathrm{X}_{\mathrm{k}}$ |
| Variables | -M | 3 | 3 | 1 | 0 | 0 | 1 | 0 | $3 / 3=1 \rightarrow$ |
| $\mathrm{a}_{1}$ | -M | 6 | 4 | 3 | -1 | 0 | 0 | 1 | $6 / 4=1.5$ |
| $\mathrm{a}_{2}$ | 0 | 4 | 1 | 2 | 0 | 1 | 0 | 0 | $4 / 1=4$ |
| $\mathrm{~s}_{2}$ |  |  | $\uparrow$ |  |  |  |  |  |  |
|  |  | $\mathrm{Z}=-9 \mathrm{M}$ | $2-7 \mathrm{M}$ | $1-4 \mathrm{M}$ | M | 0 | 0 | 0 | $\leftarrow \Delta_{\mathrm{j}}$ |
| $\mathrm{x}_{1}$ | -2 | 1 | 1 | $1 / 3$ | 0 | 0 | X | 0 | $1 / 1 / 3=3$ |
| $\mathrm{a}_{2}$ | -M | 2 | 0 | $\boxed{5 / 3}$ | -1 | 0 | X | 1 | $6 / 5 / 3=1.2 \rightarrow$ |
| $\mathrm{~s}_{2}$ | 0 | 3 | 0 | $5 / 3$ | 0 | 1 | X | 0 | $4 / 5 / 3=1.8$ |
|  |  |  |  | $\uparrow$ |  |  |  |  |  |
|  | $\mathrm{Z}=-2-2 \mathrm{M}$ | 0 | $\frac{(-5 \mathrm{M}+1)}{}$ | 0 | 0 | X | 0 | $\leftarrow \Delta_{\mathrm{j}}$ |  |
| $\mathrm{x}_{1}$ | -2 | $3 / 5$ | 1 | 0 | $1 / 5$ | 0 | X | X |  |
| $\mathrm{x}_{2}$ | -1 | $6 / 5$ | 0 | 1 | $-3 / 5$ | 0 | X | X |  |
| $\mathrm{s}_{2}$ | 0 | 1 | 0 | 0 | 1 | 1 | X | X |  |
|  |  |  |  |  | 0 | $1 / 5$ | 0 | X | X |

Since all $\Delta_{\mathrm{j}} \geq 0$, optimal basic feasible solution is obtained
Therefore the solution is $\operatorname{Max} Z=-12 / 5, x_{1}=3 / 5, x_{2}=6 / 5$
Problem : 6
$\operatorname{Max} Z=3 \mathrm{x}_{1}-\mathrm{x}_{2}$
Subject to

$$
\begin{aligned}
& 2 x_{1}+x_{2} \geq 2, \quad x_{1}+3 x_{2} \leq 3, \quad x_{2} \leq 4 \\
& \text { and } x_{1} \geq 0, x_{2} \geq 0
\end{aligned}
$$

Solution
$\operatorname{Max} Z=3 x_{1}-x_{2}+0 s_{1}+0 s_{2}+0 s_{3}-M a_{1} \quad$ Subject to

$$
2 x_{1}+x_{2}-s_{1}+a_{1}=2
$$

$\mathrm{x}_{1}+3 \mathrm{x}_{2}+\mathrm{s}_{2}=3$
$\mathrm{x}_{2}+\mathrm{s}_{3}=4$
$\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{~s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}, \mathrm{a}_{1} \geq 0$

| $\mathrm{C}_{\mathrm{j}} \rightarrow \quad 3$ |  |  |  | -1$\mathrm{X}_{2}$ | 0$S_{1}$ | 0$S_{2}$ | 0 | -M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Basic Variables | $\mathrm{C}_{\mathrm{B}}$ | $\mathrm{X}_{\mathrm{B}}$ | $\mathrm{X}_{1}$ |  |  |  | $\mathrm{S}_{3}$ | $\mathrm{A}_{1}$ | Min ratio $\mathrm{X}_{\mathrm{B}} / \mathrm{X}_{\mathrm{k}}$ |
| $\mathrm{a}_{1}$ |  |  | 2 | 1 | -1 | 0 | 0 | 1 | $2 / 2=1 \rightarrow$ |
| $\mathrm{S}_{2}$ |  | 3 | 1 | 3 | 0 | 1 | 0 | 0 | $3 / 1=3$ |
| $\mathrm{S}_{3}$ | 0 | 4 | 0 | 1 | 0 | 0 | 1 | 0 | - |
|  | $\mathrm{Z}=-2 \mathrm{M}$ |  | $\begin{gathered} \uparrow \\ -2 \mathrm{M}-3 \end{gathered}$ | -M+1 | M | 0 | 0 | 0 | $\leftarrow \Delta_{\mathrm{j}}$ |
| $\mathrm{x}_{1}$ | 3 | 1 | 1 | 1/2 | -1/2 | 0 | 0 | X | - |
| $\mathrm{S}_{2}$ | 0 | 2 | 0 | 5/2 | 1/2 | 1 | 0 | X | $2 / 1 / 2=4 \rightarrow$ |
| $\mathrm{S}_{3}$ | 0 | 4 | 0 | 1 | 0 | 0 | 1 | X | - |
|  | $\mathrm{Z}=3$ |  | 0 | 5/2 | $\begin{gathered} \uparrow \\ -3 / 2 \\ \hline \end{gathered}$ | 0 | 0 | X | $\leftarrow \Delta_{\mathrm{j}}$ |
| $\mathrm{X}_{1}$ | 3 | 3 | 1 | 3 | 0 | 1/2 | 0 | X |  |
| $\mathrm{S}_{1}$ |  | 4 | 0 | 5 | 1 | 2 | 0 | X |  |
| $\mathrm{S}_{3}$ | 0 | 4 | 0 | 1 | 0 | 0 | 1 | X |  |
|  | $\mathrm{Z}=9$ |  | 0 | 10 | 0 | 3/2 | 0 | X |  |

Since all $\Delta_{\mathrm{j}} \geq 0$, optimal basic feasible solution is obtained
Therefore the solution is $\operatorname{Max} Z=9, x_{1}=3, x_{2}=0$

## Problem ; 7

$\operatorname{Max} Z=3 x_{1}+2 x_{2}+x_{3}$
Subject to

$$
\begin{aligned}
& 2 x_{1}+x_{2}+x_{3}=12 \\
& 3 x_{1}+4 x_{2}=11
\end{aligned}
$$

and $x_{1}$ is unrestricted

$$
x_{2} \geq 0, x_{3} \geq 0
$$

## Solution

$\operatorname{Max} Z=3\left(\mathrm{x}_{1}{ }^{\prime}-\mathrm{x}_{1}{ }^{\prime \prime}\right)+2 \mathrm{x}_{2}+\mathrm{x}_{3}-\mathrm{M} \mathrm{a}_{1}-\mathrm{M} \mathrm{a}_{2}$
Subject to

$$
\begin{aligned}
& 2\left(x_{1}^{\prime}{ }^{\prime}-x_{1}{ }^{\prime \prime}\right)+x_{2}+x_{3}+a_{1}=12 \\
& 3\left(x_{1}{ }^{\prime}-x_{1}{ }^{\prime \prime}\right)+4 x_{2}+a_{2}=11 \\
& x_{1}, x_{1}{ }^{\prime \prime}, x_{2}, x_{3}, a_{1}, a_{2} \geq 0
\end{aligned}
$$

$\operatorname{Max} Z=3 \mathrm{x}_{1}{ }^{\prime}-3 \mathrm{x}_{1}{ }^{\prime \prime}+2 \mathrm{x}_{2}+\mathrm{x}_{3}-\mathrm{Ma} \mathrm{a}_{1}-\mathrm{M} \mathrm{a}_{2}$
Subject to

$$
\begin{aligned}
& 2 \mathrm{x}_{1}{ }^{\prime}-2 \mathrm{x}_{1}{ }^{\prime \prime}+\mathrm{x}_{2}+\mathrm{x}_{3}+\mathrm{a}_{1}=12 \\
& 3 \mathrm{x}_{1}-3 \mathrm{x}_{1}+4 \mathrm{x}_{2}+\mathrm{a}_{2}=11 \\
& \mathrm{x}_{1}, \mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{a}_{1}, \mathrm{a}_{2} \geq 0
\end{aligned}
$$



Since all $\Delta_{\mathrm{j}} \geq 0$, optimal basic feasible solution is obtained

$$
\begin{aligned}
& x_{1}{ }^{\prime}=11 / 3, x_{1}{ }^{\prime \prime}=0 \\
& x_{1}=x_{1}{ }^{-}-x_{1}{ }^{\prime}=11 / 3-0=11 / 3
\end{aligned}
$$

Therefore the solution is $\operatorname{Max} Z=47 / 3, x_{1}=11 / 3, x_{2}=0, x_{3}=14 / 3$

