

Frequency Sampling Method:(April/May-15)

Discuss the design procedure of FIR filters using frequency sampling method. [May/June-2013]

Generally, FIR filter can be specified by giving impulse response coefficients $h(n)$ (or) DFT coefficients $H(k)$.

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j2\pi kn/N} \text{-----> (1)}$$

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N} \text{-----> (2)}$$

$H(k) = \text{DFT samples}$

$$H(k) = H(z) \Big|_{z=e^{j2\pi k/N}} \text{-----> (3)}$$

and

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \text{-----> (4)}$$

Put (1) in (4)

$$H(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j2\pi kn/N} \right] z^{-n}$$

If $z = e^{j\omega}$

$$H(e^{j\omega}) = H(z)$$

$$\omega_k = \left(\frac{2\pi k}{N} \right)$$

where $\omega_k \text{---> sampling frequency.}$

General steps to design FIR filter using frequency sampling method [type-I design]:

Step 1: Draw the filter graph, as in FIR design using window function.

Step 2: Draw the unit circle and mark the points, if $k=0,1,\dots,N-1$.

$$\theta_k = \frac{360^\circ}{N} * k$$

$$\text{if } k=1; \quad \theta_1 = \frac{360^\circ}{N} * 1$$

Step 3: To find $H(k)$, replace ω by $\frac{2\pi k}{N}$ is the given equation.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & , 0 \leq \omega \leq \omega_c \\ 0 & \text{for otherwise} \end{cases}$$

(Or)

$$H(k) = \begin{cases} e^{-\frac{j(N-1)\pi k}{N}} & , \text{for } k = 0 \text{ pass band values} \\ 0 & \text{for } k = \text{stop band values.} \end{cases}$$

Note: In type-II design replace $\omega = \frac{2\pi}{N} \left(k + \frac{1}{2} \right)$

Step 4: Find $h(n)$

If 'N' = odd

$$h(n) = \frac{1}{N} \left\{ H(0) + \sum_{k=1}^{N-1} 2 \operatorname{Re} \left(H(k) e^{j2\pi kn/N} \right) \right\} \quad \therefore H(0) = 1$$

If 'N' = Even :

$$h(n) = \frac{1}{N} \left\{ H(0) + \sum_{k=1}^{N-1} 2 \operatorname{Re} \left(H(k) e^{j2\pi kn/N} \right) \right\}$$

Step 5: Find $H(z)$:

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

Design LPF which has the following specifications, $N=7$ using frequency sampling Technique. [Nov/Dec-2016][Nov/Dec-15]

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & , 0 \leq \omega \leq \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} \leq \omega \leq \pi. \end{cases}$$

Given:
$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & , 0 \leq \omega \leq \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} \leq \omega \leq \pi. \end{cases}$$

Step 1: From a unit circle, mark points from

$$\theta_k = \frac{360^\circ}{N} * k$$

$$\theta_1 = 51.42^\circ$$

$$\theta_2 = 102.8^\circ$$

$$\theta_3 = 154.26^\circ$$

$$\theta_4 = 205.68^\circ$$

$$\theta_5 = 257.1^\circ$$

$$\theta_6 = 308.52^\circ$$

Step 2:

$$H(k) = \begin{cases} e^{-j\left(\frac{N-1}{N}\right)\pi k} & \text{for } k = 0, 1, 6 \\ 0 & \text{for otherwise} \end{cases}$$

$N=7$.

$$H(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } k = 0, 1, 6 \\ 0 & \text{for } k = 2, 3, 4, 5 \end{cases}$$

$$h(n) = \frac{1}{N} \left\{ H(0) + \sum_{k=1}^{\frac{N-1}{2}} 2 \operatorname{Re} \left(H(k) e^{\frac{j2\pi kn}{N}} \right) \right\} \quad \therefore H(0) = 1$$

$$= \frac{1}{7} \left\{ 1 + \sum_{k=1}^3 2 \operatorname{Re} \left(e^{-j\frac{6\pi k}{7}} \cdot e^{\frac{j2\pi kn}{7}} \right) \right\}$$

$$= \frac{1}{7} \left\{ 1 + 2 \sum_{k=1}^3 \operatorname{Re} \left(e^{\frac{j\pi k(2n-6)}{7}} \right) \right\}$$

$$h(n) = \frac{1}{7} \left\{ 1 + 2 \sum_{k=1}^3 \cos \left(\frac{\pi k(2n-6)}{7} \right) \right\}$$

$$h(n) = \frac{1}{7} \left\{ 1 + 2 \left(\cos \left(\frac{\pi(2n-6)}{7} \right) \right) \right\} \quad \text{for } k = 1.$$

$$h(0) = -0.114; h(1) = 0.079; h(2) = 0.321; h(3) = 0.4286; h(4) = h(2); h(5) = h(1); h(6) = h(0)$$

Find coefficient of LP FIR with $N=15$ and it has symmetric unit sample response. It satisfies the following condition.

Solution:**Step 1:**

$$H(k) = \begin{cases} 1.e^{-j\left(\frac{N-1}{N}\right)\pi k} & , k = 0,1,2,3 \\ 0.4e^{-j\left(\frac{N-1}{N}\right)\pi k} & , k = 4 \\ 0 & , k = 5,6,7 \end{cases}$$

$$H(k) = \begin{cases} 1.e^{-j\frac{14\pi k}{15}} & , k = 0,1,2,3 \\ 0.4e^{-j\frac{14\pi k}{15}} & , k = 4 \\ 0 & , k = 5,6,7 \end{cases}$$

Step 2:

$$h(n) = \frac{1}{N} \left\{ H(0) + \sum_{k=1}^{\frac{N-1}{2}} 2 \operatorname{Re} \left\{ H(k) e^{\frac{j2\pi kn}{N}} \right\} \right\} \quad \therefore H(0) = 1$$

$$h(n) = \frac{1}{15} \left\{ 1 + \sum_{k=1}^7 2 \operatorname{Re} \left\{ \left(e^{-j\frac{14\pi k}{15}} + 0.4e^{-j\frac{14\pi k}{15}} e^{\frac{j2\pi kn}{15}} \right) \right\} \right\}$$

$$= \frac{1}{15} \left\{ 1 + 2 \operatorname{Re} \left[\sum_{k=1}^3 e^{\frac{j2\pi k(n-7)}{15}} \right] + 2(0.4) \sum_{k=4}^7 e^{\frac{j2\pi k(n-7)}{15}} \right\}$$

$$= \frac{1}{15} \left\{ 1 + 2 \left[\cos \frac{2\pi(n-7)}{15} + \cos \frac{4\pi(n-7)}{15} + \cos \frac{6\pi(n-7)}{15} \right] + 0.8 \cos \left(\frac{8\pi(n-7)}{15} \right) \right\}$$

$$h(0) = -0.0141; h(1) = -0.0195; h(2) = 0.04; h(3) = 0.0122$$

$$h(4) = -0.0913; h(5) = -0.01809; h(6) = 0.313; h(7) = 0.52.$$

Determine the coefficients $\{h(n)\}$ of a linear phase FIR filter of length $M=15$ has a symmetric unit sample response and a frequency response that satisfies the condition $H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & \text{for } k = 0,1,2,3 \\ 0 & \text{for } k = 4,5,6,7 \end{cases}$ (May/June-13) (April/May-11)(Nov/Dec-09)

Solution:

$$\begin{aligned} |\overline{H}(k)| &= 1 && \text{for } 0 \leq k \leq 3 \text{ and } 12 \leq k \leq 14 \\ &= 0 && \text{for } 4 \leq k \leq 11 \end{aligned}$$

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$$

$$= -14/15 \pi k \quad 0 \leq k \leq 7$$

and

$$\theta(k) = 14\pi - \frac{14\pi k}{15} \quad \text{for } 8 \leq k \leq 14$$

$$H(k) = e^{-j14\pi k/15} \quad \text{for } k=0,1,2,3$$

$$= 0 \quad \text{for } 4 \leq k \leq 11$$

$$= e^{-j14\pi(k-15)/15} \quad \text{for } 12 \leq k \leq 14$$

$$h(n) = \frac{1}{n} \left[H(0) + 2 \sum_{k=1}^{\frac{N-1}{2}} \text{Re}(H(k)e^{j2\pi nk/15}) \right]$$

$$= \frac{1}{5} \left[1 + 2 \sum_{k=1}^7 \text{Re}(e^{-j14\pi k/15} e^{j2\pi nk/15}) \right]$$

$$= \frac{1}{15} \left[1 + 2 \sum_{k=1}^3 \cos \frac{2\pi k(7-n)}{15} \right]$$

$$= \frac{1}{15} \left[1 + 2 \cos \frac{2\pi(7-n)}{15} + 2 \cos \frac{4\pi(7-n)}{15} + 2 \cos \frac{6\pi(7-n)}{15} \right]$$

$$h(0) = h(14) = -0.05; \quad h(1) = h(3) = 0.041; \quad h(4) = h(10) = -0.1078$$

$$h(2) = h(12) = 0.0666; \quad h(3) = h(11) = -0.0365; \quad h(5) = h(9) = 0.034$$

$$h(6) = h(8) = 0.3188; \quad h(7) = 0.466$$

Using frequency sampling method, design BPF with the following specifications. [May/June-2016]

Sampling frequency $F=8000\text{Hz}$

Cut off frequencies $f_{c1}=1000\text{Hz}$

Cut off frequencies $f_{c2}=3000\text{Hz}$ Determine the filter coefficients for $N=7$.

Solution:

$$\omega_{c_1} = 2\pi f_{c_1} T = \frac{2\pi f_{c_1}}{F} = \frac{2\pi(1000)}{8000} = \frac{\pi}{4}$$

$$\omega_{c_2} = 2\pi f_{c_2} T = \frac{2\pi f_{c_2}}{F} = \frac{2\pi(3000)}{8000} = \frac{3\pi}{4}$$

$$H(K) = H_d \left(e^{j\omega} \right)_{\omega = \frac{2\pi k}{7}} \quad k=0,1,\dots,6$$

$$|H(k)| = \begin{cases} 0 & \text{for } k = 0,3 \\ 1 & \text{for } k = 1,2 \end{cases}$$

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi \quad \text{for } 0 \leq k \leq \frac{N-1}{2}$$

$$= -\frac{6}{7}\pi k \quad \text{for } 0 \leq k \leq 3$$

$$|H(k)| = \begin{cases} 0 & \text{for } k = 0,3 \\ e^{-\frac{j6\pi k}{7}} & \text{for } k = 1,2 \end{cases}$$

The filter coefficients are given by

$$h(n) = \frac{1}{N} \left[H(0) + \sum_{k=1}^{\frac{N-1}{2}} 2 \cdot \text{Re} \left[H(k) \cdot e^{\frac{j2\pi kn}{N}} \right] \right]$$

$$= \frac{1}{7} \left[2 \sum_{k=1}^{\frac{N-1}{2}} \text{Re} \left[e^{\frac{j6\pi k}{7}} e^{\frac{j2\pi kn}{7}} \right] \right]$$

$$= \frac{1}{7} \left[2 \sum_{k=1}^{\frac{N-1}{2}} \cos \frac{2\pi k}{7} ((3-n)) \right]$$

$$= \frac{2}{7} \left[\cos \frac{2\pi}{7} (3-n) + \cos \frac{4\pi}{7} (3-n) \right]$$

$$h(0) = h(6) = -0.07928.$$

$$h(1) = h(5) = -0.321$$

$$h(2) = h(4) = -0.11456$$

$$h(3) = 0.57$$

