ME3491 THEORY OF MACHINES

UNIT II NOTES

2.3 Condition for Constant Velocity Ratio of Toothed Wheels–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 12.6. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure. Let T T be the common tangent and M N be the common normal to the curves at the point of contact Q. From the centres O1 and O2 , draw O1 M and O2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2. Let v1 and v2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

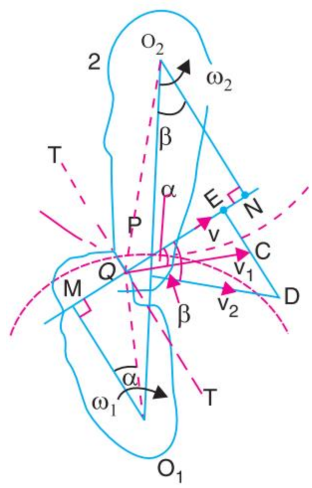


Figure: Law of gearing

$$v_1 \cos \alpha = v_2 \cos \beta$$
$$(\omega_1 \times O_1 Q) \cos \alpha = (\omega_2 \times O_2 Q) \cos \beta$$
$$(\omega_1 \times O_1 Q) \frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q) \frac{O_2 N}{O_2 Q}$$

$$\omega_1 / \omega_2 = O_2N/O_1M$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O1 and O2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

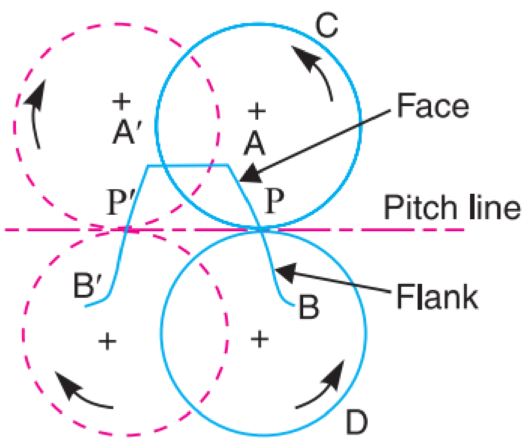
2.3.1 Forms of Teeth

Following are the two types of teeth commonly used : 1. Cycloidal teeth ; and 2. Involute teeth.

Cycloidal Teeth

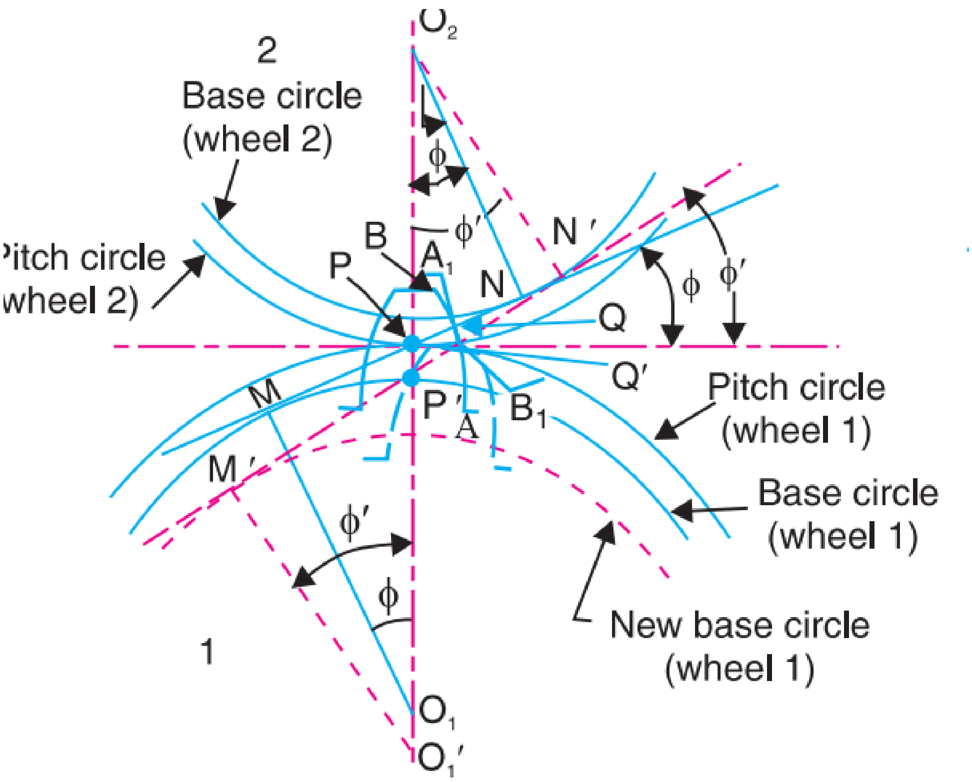
A cycloid is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as epi-cycloid. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called hypo-cycloid.

The fixed line or pitch line of a rack is shown. When the circle C rolls without slipping above the pitch line in the direction as indicated in Figure, then the point P on the circle traces epi-cycloid PA. This represents the face of the cycloidal tooth profile. When the circle D rolls without slipping below the pitch line, then the point P on the circle D traces hypo-cycloid PB, which represents the flank of the cycloidal tooth. The profile BPA is one side of the cycloidal rack tooth. Similarly, the two curves P' A' and P'B' forming the opposite side of the tooth profile are traced by the point P' when the circles C and D roll in the opposite direction.



Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 12.9. In connection with toothed wheels, the circle is known as base circle.



2.3.2.Comparison Between Involute and Cycloidal Gears

In actual practice, the involute gears are more commonly used as compared to cycloidal gears, due to the following advantages :

Advantages of involute gears Following are the advantages of involute gears :

1. The most important advantage of the involute gears is that the centre distance for a pair of involute gears can be varied within limits without changing the

velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.

2. In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

2.3.3 Length of Arc of Contact

Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 12.11, the arc of contact is EPF or GPH. Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as arc of approach and the arc PH is called arc of recess. The angles subtended by these arcs at O1 are called angle of approach and angle of recess respectively.

We know that the length of the arc of approach (arc *GP*)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

and the length of the arc of recess (arc *PH*)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

2.3.4 Contact Ratio

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

Problem

The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum

We know that the circular pitch,

$$p_c = \pi m = \pi \times 6 = 18.85 \text{ mm}$$

∴ Length of arc of contact

$$= 1.75\, p_c = 1.75 \times 18.85 = 33 \text{ mm}$$

ength of path of contact

$$= \text{Length of arc of contact} \times \cos\phi = 33 \cos 20° = 31 \text{ mm}$$

Let $R_A = r_A$ = Radius of the addendum circle of each wheel.

We know that pitch circle radii of each wheel,

$$R = r = m.T/2 = 6 \times 40/2 = 120 \text{ mm}$$

and length of path of contact

$$31 = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

$$= 2\left[\sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi\right] \quad ...(\because R = r, \text{ and } R_A = r_A)$$

$$\frac{31}{2} = \sqrt{(R_A)^2 - (120)^2\cos^2 20°} - 120\sin 20°$$

$$15.5 = \sqrt{(R_A)^2 - 12\,715} - 41$$

$$(15.5 + 41)^2 = (R_A)^2 - 12\,715$$

$$3192 + 12\,715 = (R_A)^2 \quad \text{or} \quad R_A = 126.12 \text{ mm}$$

We know that the addendum of the wheel,

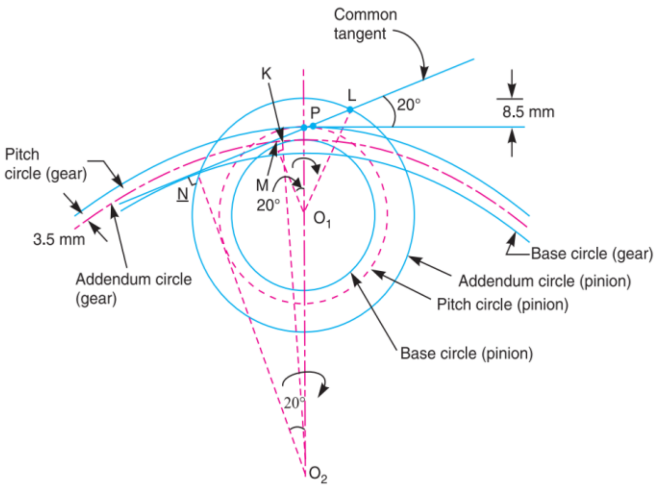
$$= R_A - R = 126.12 - 120 = 6.12 \text{ mm}$$

Problem:A pinion having 18 teeth engages with an internal gear having 72 teeth. If the gears have involute profiled teeth with 20° pressure angle, module of 4 mm and the addenda on pinion and gear are 8.5 mm and 3.5 mm respectively, find the length of path of contact.

Solution

Given : t = 18 ; T = 72 ; φ = 20° ; m = 4 mm ; Addendum on pinion = 8.5 mm ; Addendum on gear = 3.5 mm

Figure shows a pinion with centre O1 , in mesh with internal gear of centre O2. It may be noted that the internal gears have the addendum circle and the tooth faces inside the pitch circle. We know that the length of path of contact is the length of the common tangent to the two base circles cut by the addendum circles. From Figure, we see that the addendum circles cut the common tangents at points K and L. Therefore the length of path of contact is KL which is equal to the sum of KP (i.e. path of approach) and PL (i.e. path of recess)



We know that pitch circle radius of the pinion,

$$r = O_1P = m.t/2 = 4 \times 18/2 = 36 \text{ mm}$$

and pitch circle radius of the gear,

$$R = O_2P = m.T/2 = 4 \times 72/2 = 144 \text{ mm}$$

$\therefore$ Radius of addendum circle of the pinion,

$$r_A = O_1L = O_1P + \text{ Addendum on pinion} = 36 + 8.5 = 44.5 \text{ mm}$$

and radius of addendum circle of the gear,

$$R_A = O_2K = O_2P - \text{Addendum on wheel} = 144 - 3.5 = 140.5 \text{ mm}$$

From Fig. 12.12, radius of the base circle of the pinion,

$$O_1M = O_1P \cos\phi = r \cos\phi = 36 \cos 20° = 33.83 \text{ mm}$$

and radius of the base circle of the gear,

$$O_2N = O_2P \cos\phi = R \cos\phi = 144 \cos 20° = 135.32 \text{ mm}$$

We know that length of the path of approach,

We know that length of the path of approach

and length of the path of recess,

$$PL = ML - MP = \sqrt{(O_1L)^2 - (O_1M)^2} - O_1P \sin 20°$$

$$= \sqrt{(44.5)^2 - (33.83)^2} - 36 \times 0.342 = 28.9 - 12.3 = 16.6 \text{ mm}$$

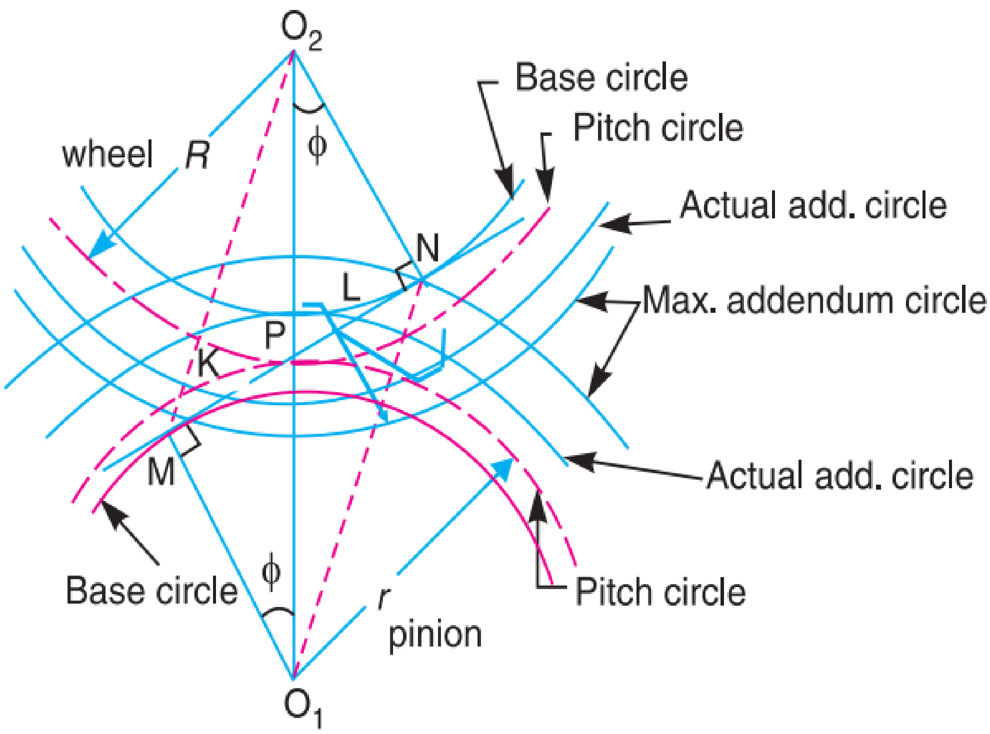
$\therefore$ Length of the path of contact,

$$KL = KP + PL = 11.45 + 16.6 = 28.05 \text{ mm}$$

2.4 Interference in Involute Gears

If the radius of the addendum circle of pinion is increased to O1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as interference, and occurs when the teeth are being cut. In brief, the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

Similarly, if the radius of the addendum circle of the wheel increases beyond O2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called interference points. Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is *O1 N and of the wheel is O2M.



Problem

A pair of spur gears with involute teeth is to give a gear ratio of 4 : 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch ?

Least number of teeth on each wheel

Let t = Least number of teeth on the smaller wheel i.e. pinion,

T = Least number of teeth on the larger wheel i.e. gear, and r = Pitch circle radius of the smaller wheel i.e. pinion.

We know that the maximum length of the arc of approach and circular pitch, $p_c = \pi m = \dfrac{2\pi r}{t}$ $\quad ...\left(\because m = \dfrac{2r}{t}\right)$

Since the arc of approach is not to be less than the circular pitch, therefore

$$r \tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25$$

and $T = G.t = 4 \times 25 = 100$. $\quad ...(\because G = T/t)$

We know that addendum of the wheel

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{m \times 100}{2}\left[\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5°} - 1\right]$$

$$= 50\,m \times 0.017 = 0.85\ m = 0.85 \times p_c/\pi = 0.27\ p_c$$