4.7.1 A Model of Neuromuscular Reflex Motion

Examination of the dynamics of neuromuscular reflex motion can yield valuable insight into the status of patients who have neurological disorders. The model that we will consider assumes the following test. The patient is seated comfortably and his shoulder and elbow are held by adjustable supports so that the upper arm remains in a fixed horizontal position throughout the test. The subject's forearm is allowed to move only in the vertical plane. At the start of the experiment, he is made to flex his arm by pulling on a cord that has been attached to a cuff on his wrist. The cord runs around a pulley system and supports a sizeable weight. The initial angle between the forearm and upper arm is 135°. The subject is not given any specific instructions about maintaining this angle, except to relax his arm as much as possible while supporting the weight. Then, at time $t = 0$, an electromagnetic catch is switched off so that an additional weight is abruptly added to the original load. Changes in angular motion, $\theta(t)$, of the forearm about the elbow are recorded during and after the quick release of the weight. The mathematical model used to interpret the results of this test is based on the work of Soechting et al. (1971).

4.7.1.1. Limb Dynamics. Figure 4.10a shows a schematic diagram of the forearm, with the black filled circle representing the elbow joint. M_x represents the change in external moment acting on the limb about the elbow joint; in this experiment, M_x would be a step. M represents the net muscular torque exerted in response to the external disturbance. Neglecting the weight of the forearm itself, application of Newton's Second Law yields the following equation of motion:

$$M_x(t) - M(t) = J\ddot{\theta} \qquad (4.91)$$

where J is the moment of inertia of the forearm about the elbow joint.

4.7.1.2. Muscle Model. Although this reflex involves both the biceps and triceps muscles, we will assume for simplicity that the net muscular torque in response to M_x is generated by a single equivalent muscle model, illustrated in Figure 4.10b. Note that in this mechanical analog, M is treated as if it were a "force," although it is actually a torque. Accordingly, the "displacements" that result are in fact angular changes, θ and θ_1. As such, the muscle stiffness parameter, k, and the viscous damping parameter, B, have units consistent with this representation. The equations of motion for the muscle model are:

$$M(t) = k(\theta - \theta_1) \tag{4.92}$$

and

$$M(t) = M_0(t) + B\dot{\theta}_1 \tag{4.93}$$

where $M_0(t)$ is the torque exerted by the muscle under isometric conditions. $M_0(t)$ is represented as a function of time, since it is dependent on the pattern of firing of the alpha motorneurons.

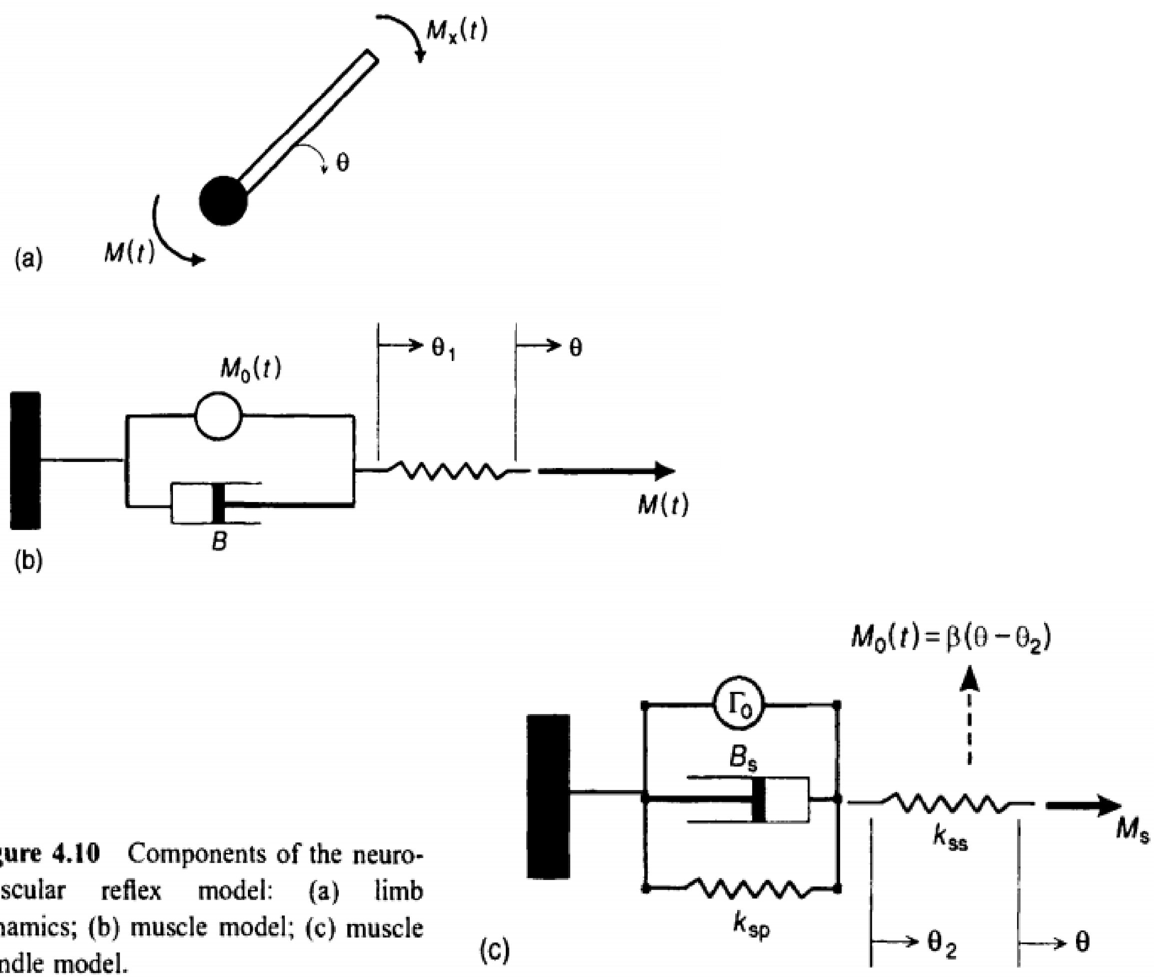


Figure 4.10 Components of the neuromuscular reflex model: (a) limb dynamics; (b) muscle model; (c) muscle spindle model.

4.7.1.3. Plant Equations. By combining Equations (4.91) through (4.93), we obtain an equation of motion that characterizes the dynamics of the plant, i.e., describing how θ would change due to the torque exerted by the external disturbance M_x and the resulting muscular response:

$$\frac{BJ}{k}\dddot{\theta} + J\ddot{\theta} + B\dot{\theta} = M_x(t) - M_0(t) \tag{4.94}$$

4.7.1.4. Muscle Spindle Model. This model describes the dynamics by which changes in θ are transduced at the level of the muscle spindles into afferent neural signals. The latter travel to the spinal cord, which sends out efferent signals to the contractile machinery of the muscle to generate $M_0(t)$. We assume that the neural output of the spindle is proportional to the amount by which its nuclear bag region is stretched, so that ultimately

$$M_0(t) = \beta(\theta - \theta_2) \tag{4.95}$$

Figure 4.10c shows the mechanical analog of the muscle spindle model. k_{sp} and B_s are parameters that represent the elastic stiffness and viscous damping properties, respectively, of the pole region of the spindle, while k_{ss} represents the elastic stiffness of the nuclear bag region. Γ_0 represents the contractile part of the pole region, which allows the operating length of the spindle to be reset at different levels, using the gamma motorneuronal pathways. We will assume Γ_0 to be constant at the equilibrium length of the spindle, so that this parameter does not play a role in the dynamics of changes about this equilibrium length. With this consideration in mind, the dynamics of the muscle spindle model may be characterized by the following equations:

$$M_s = K_{ss}(\theta - \theta_2) \tag{4.96}$$

and

$$M_s = B_s\dot{\theta}_2 + k_{sp}\theta_2 \tag{4.97}$$

Another important factor that must be taken into account is the fact that, although θ is sensed virtually instantaneously by the spindle organs, there is a finite delay before this feedback information is finally converted into corrective action at the level of the muscle. This total delay, T_d, includes all lags involved in neural transmission along the afferent and efferent pathways as well as the delay taken for muscle potentials to be converted into muscular force. Eliminating the intermediate variables, M_s and θ_2, from Equations (4.95) through (4.97), we obtain the following equation for the feedback portion of the stretch reflex model:

$$M_0 + \frac{M_0}{\tau} = \beta\left(\dot{\theta}(t - T_d) + \frac{\theta(t - T_d)}{\eta\tau}\right) \tag{4.98}$$

where

$$\tau = \frac{B_s}{k_{ss} + k_{sp}} \tag{4.99}$$

and

$$\eta = \frac{k_{ss} + k_{sp}}{k_{sp}} \tag{4.100}$$

4.7.1.5. Block Diagram of Neuromuscular Reflex Model. Taking the Laplace transforms of Equations (4.94) and (4.98), we obtain the following equations that are represented schematically by the block diagram shown in Figure 4.11:

$$\theta(s) = \frac{M_x(s) - M_0(s)}{s\left(\frac{BJ}{k}\, s^2 + Js + B\right)} \tag{4.101}$$

and

$$M_0(s) = \beta \frac{\tau s + 1/\eta}{\tau s + 1}\, e^{-sT_d}\theta(s) \tag{4.102}$$

4.7.2 SIMULINK Implementation

The SIMULINK implementation of the neuromuscular reflex model is depicted in Figure 4.12. This program has been saved as the file "`nmreflex.mdl`." Note that the model parameters appear in the program as variables and not as fixed constants. This gives us the flexibility of changing the parameter values by entering them in the MATLAB command window or running a MATLAB m-file immediately prior to running the SIMULINK program. In this case, we have chosen the latter path and created an m-file called

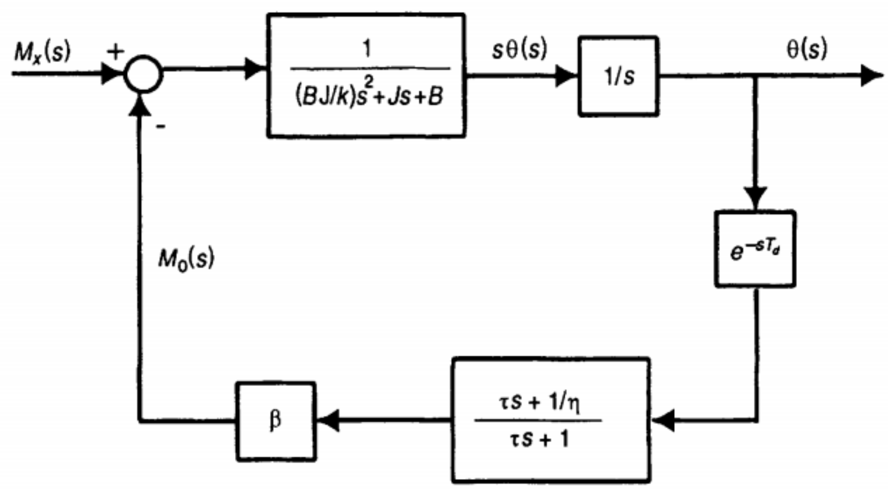


Figure 4.11 Block diagram of neuromuscular reflex model.

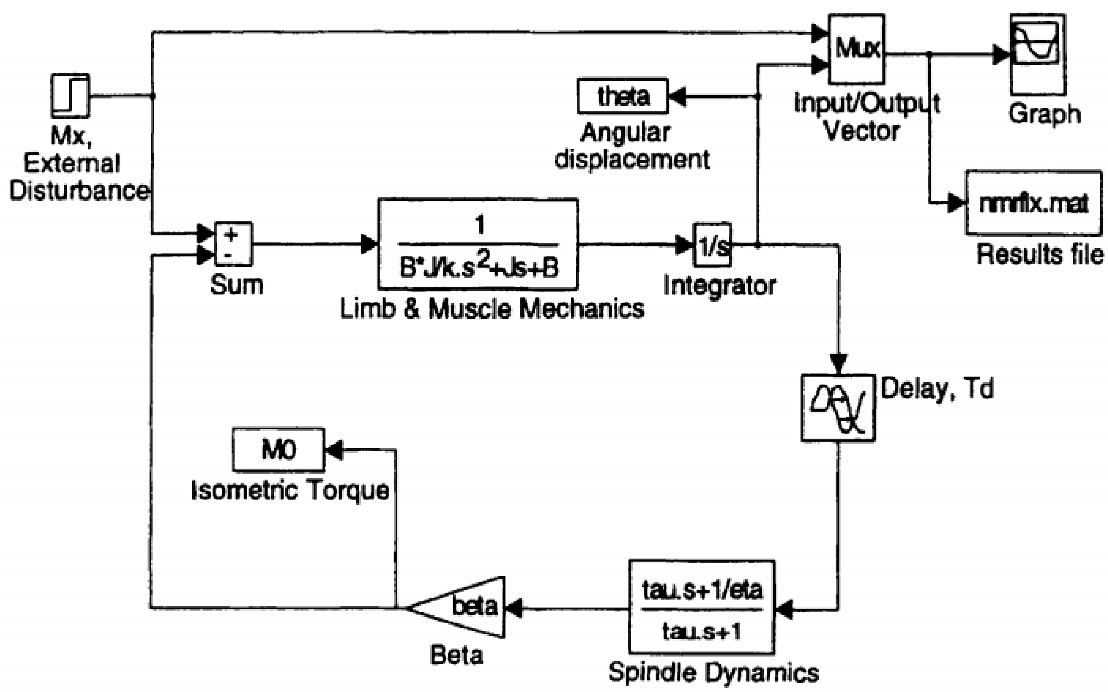
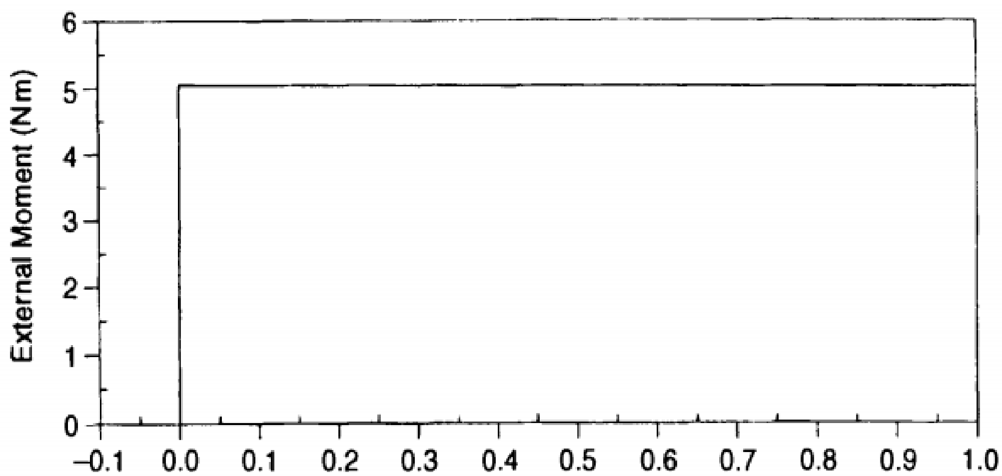


Figure 4.12 SIMULINK implementation of neuromuscular reflex model.

"nmr_var.m" that specifies the parameter values. The nominal parameter values used in the simulation are as follows: $J = 0.1\,\mathrm{kg\,m^2}$, $k = 50\,\mathrm{N\,m}$, $B = 2\,\mathrm{N\,m\,s}$, $T_d = 0.02\,\mathrm{s}$, τ ("tau" in Figure 4.12) = 1/300 s, η ("eta" in Figure 4.12) = 5, and β ("beta" in Figure 4.12) = 100. These values are consistent with the average physiological equivalents found in normal adult humans.

Figure 4.13 displays the results of three simulation runs with "nmreflex.mdl" using the nominal parameter values mentioned above. The upper panel shows the time-course of the external disturbance, M_x, which is a step increase of 5 N m in the moment applied to the forearm. The solid tracing in the lower panel represents the corresponding response in θ, the angular displacement of the forearm, when β was set equal to 100. Note that positive values of θ correspond to increases in the angle of flexion between the forearm and the upper arm. There is a slight overshoot in θ, followed by an almost undetectable oscillation before the



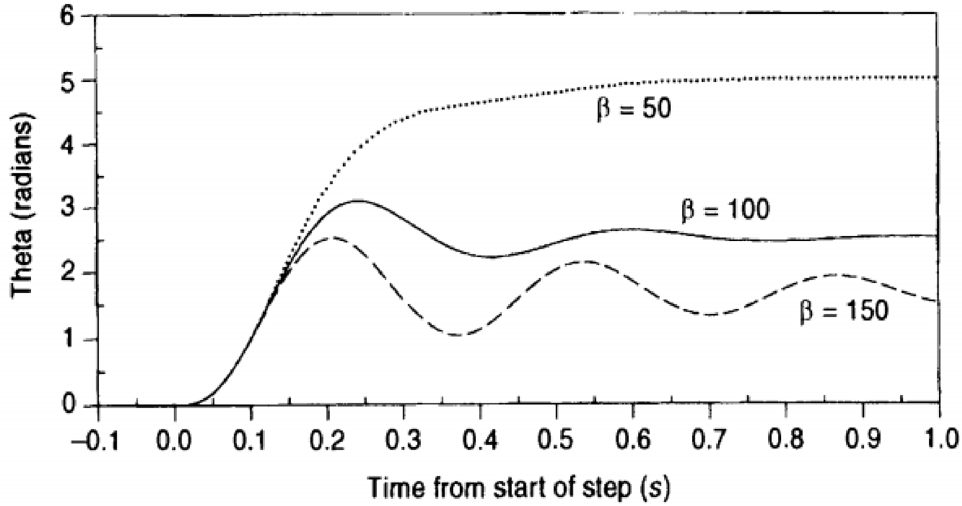


Figure 4.13 Sample results of simulations using the SIMULINK implementation of the neuromuscular reflex model.

steady-state value of approximately 0.25 radian is attained. Note that β represents the overall gain of the reflex arc. When β was increased to 150, the response was a damped oscillation, but the steady-state value achieved by θ became smaller than that obtained with the nominal value of β. In the third simulation, β was decreased to half the nominal value (i.e., 50). This produced an overdamped response and also resulted in a larger end-value for θ. These results reiterate the point that increased feedback gain leads to better attenuation of the effects of imposed disturbances—higher values of β produced smaller ending values for θ. On the other hand, the responses also become more oscillatory. This issue of instability will be discussed further in Chapter 6.