

### **5.3 STABILITY CONSIDERATIONS:**

#### **STABILITY:**

Is the ability of an amplifier to maintain effectiveness in its nominal operating characteristics such as physical temperature, signal frequency, source & load coefficient etc.

#### **STABILITY REQUIREMENT:**

Stability then implies that the magnitudes of the reflection coefficients are less than unity.

$$|\Gamma_L| < 1, |\Gamma_S| < 1$$

$$|\Gamma_{in}| = S_{11} + \left| \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right| < 1$$

$$|\Gamma_{out}| = S_{22} + \left| \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} \right| < 1$$

#### **TYPES OF STABILITY:**

##### **i) UNCONDITIONAL STABILITY:**

A network is said to be “unconditionally stable” in a frequency range if:

$$|\Gamma_{in}| < 1 \text{ \& } |\Gamma_{out}| < 1 \star$$

$$|\Gamma_L| < 1 \text{ \& } |\Gamma_S| < 1$$

##### **ii) CONDITIONAL STABILITY:**

A network is said to be “conditionally stable” or “potentially unstable” in a frequency range if:

$$|\Gamma_{in}| < 1 \text{ \& } |\Gamma_{out}| < 1$$

#### **STABILITY CIRCLES:**

- i. Input stability circle
- ii. Output stability circle

#### **OUTPUT STABILITY CIRCLE:**

In Fig 5.31, For output stability circle  $|\Gamma_{in}| = 1$

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right| = 1$$

$$|S_{11} (1 - S_{22} \Gamma_L) + S_{12} S_{21} \Gamma_L| = |1 - S_{22} \Gamma_L|$$

$$|S_{11} - S_{11} S_{22} \Gamma_L + S_{12} S_{21} \Gamma_L| = |1 - S_{22} \Gamma_L|$$

$$|S_{11} - (S_{11} S_{22} - S_{12} S_{21}) \Gamma_L| = |1 - S_{22} \Gamma_L|$$

$$S_{11} S_{22} - S_{12} S_{21} = \Delta$$

$$|S_{11} - \Delta \Gamma_L| = |1 - S_{22} \Gamma_L|$$

$$|S_{11}| - \Delta |\Gamma_L| = 1 - |S_{22} \Gamma_L|$$

$$|S_{22} \Gamma_L| - \Delta |\Gamma_L| = 1 - |S_{11}|$$

$$(S_{22} - \Delta) |\Gamma_L| = 1 - |S_{11}|$$

Multiply  $(S_{22}^* + \Delta^*)$  both sides

$$|\Gamma_L| \{(S_{22} - \Delta)\} \{(S_{22}^* + \Delta^*)\} = \{1 - |S_{11}|\} \{(S_{22}^* + \Delta^*)\}$$

$$|\Gamma_L| \{|S_{22}|^2 - |\Delta|^2\} = |S_{22}^*| + |\Delta^*| - |S_{11} S_{22}^*| - |S_{11} \Delta^*|$$

$$|\Gamma_L| \{|S_{22}|^2 - |\Delta|^2\} = |S_{22}^*| + |S_{11}^* S_{22}^*| - |S_{12}^* S_{21}^*| - |S_{11} S_{22}^*| - |S_{11} \Delta^*|$$

$$|\Gamma_L| = \frac{|S_{22}^*| + |S_{11}^* S_{22}^*| - |S_{12}^* S_{21}^*| - |S_{11} S_{22}^*| - |S_{11} \Delta^*|}{|S_{22}|^2 - |\Delta|^2}$$

$$|\Gamma_L| = \frac{|S_{22}^*| - |S_{12}^* S_{21}^*| - |S_{11} \Delta^*|}{|S_{22}|^2 - |\Delta|^2}$$

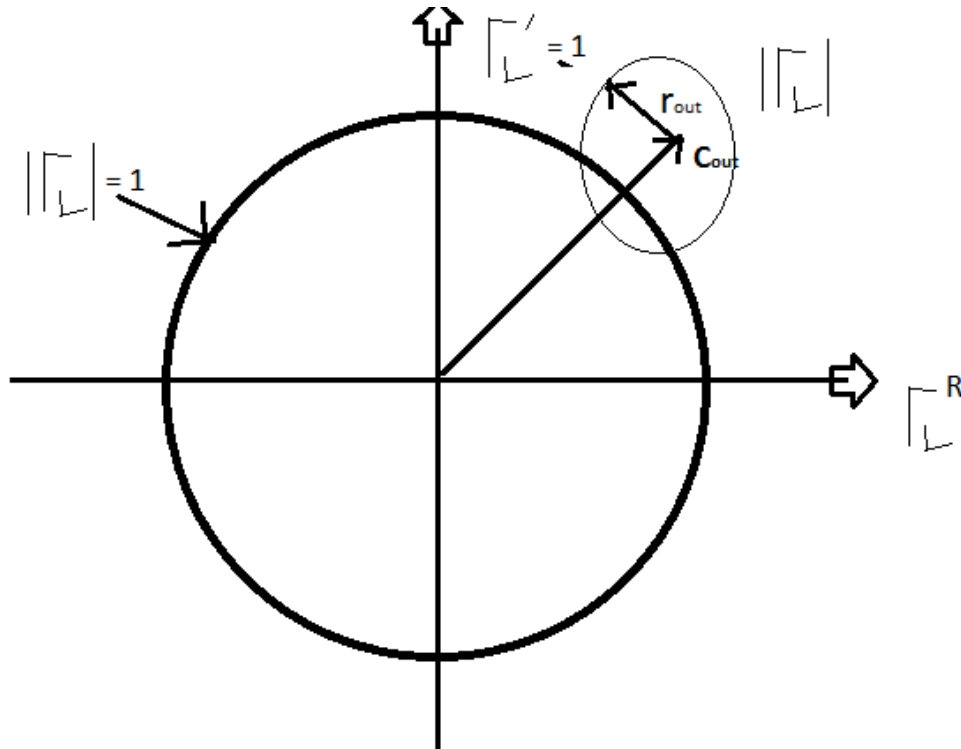
$$\left| \Gamma_L - \frac{|S_{22}^* - S_{11} \Delta^*|}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

The equation for the output stability circle ( $|\Gamma_{in}| = 1$ ) drawn in the  $\Gamma_L$  plane can be written as,

$$|\Gamma_L - C_L| = R_L$$

$$C_L = C_{out} = \frac{(S_{22} - S_{11}^* \Delta)^*}{|S_{22}|^2 - |\Delta|^2}$$

$$R_L = r_{out} = \left| \frac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$



**Fig: 5.3.1 Output stability circle**

**INPUT STABILITY CIRCLE:**

In Fig 5.3.2, The input stability circle ( $|\Gamma_{out}| = 1$ ) drawn in the  $\Gamma_S$  plane can be written as,

$$|\Gamma_S - C_S| = R_S$$

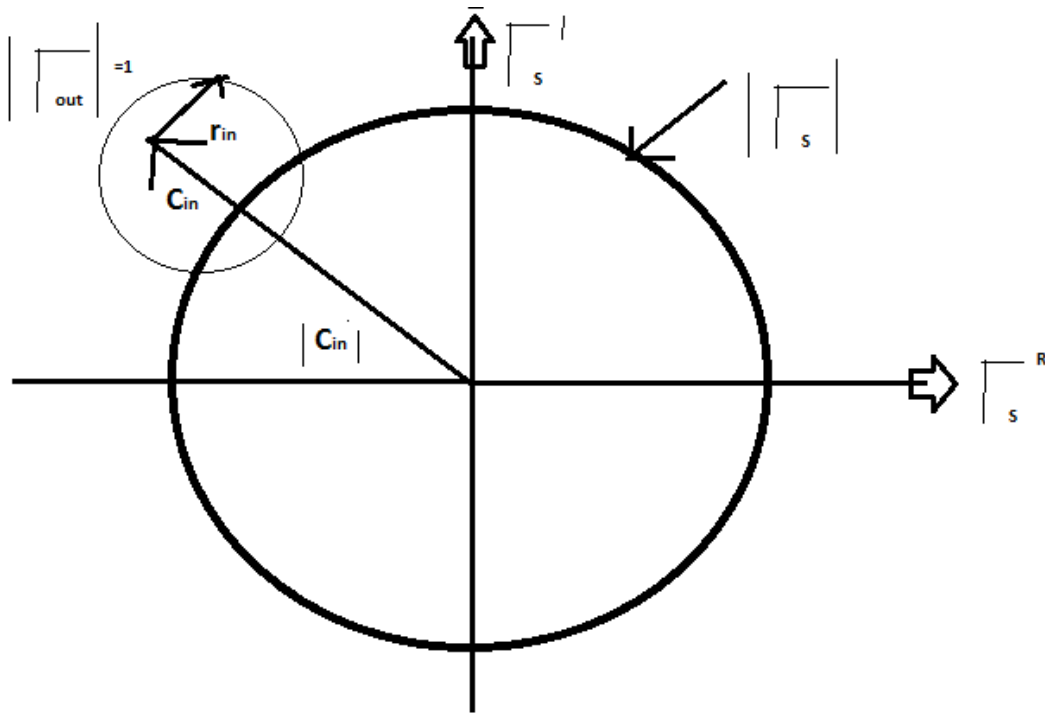
$$C_S = C_{in} = \frac{(S_{11} - S_{11}^* \Delta)^*}{|S_{11}|^2 - |\Delta|^2}$$

$$R_S = r_{in} = \left| \frac{S_{12} S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

**Special case: unconditional stability:**

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2}{2|S_{12}| |S_{21}|} > 1$$

$$|\Delta| = |S_{11} S_{22} - S_{12} S_{21}| < 1$$



**Fig: 5.3.2 Input stability circle**

