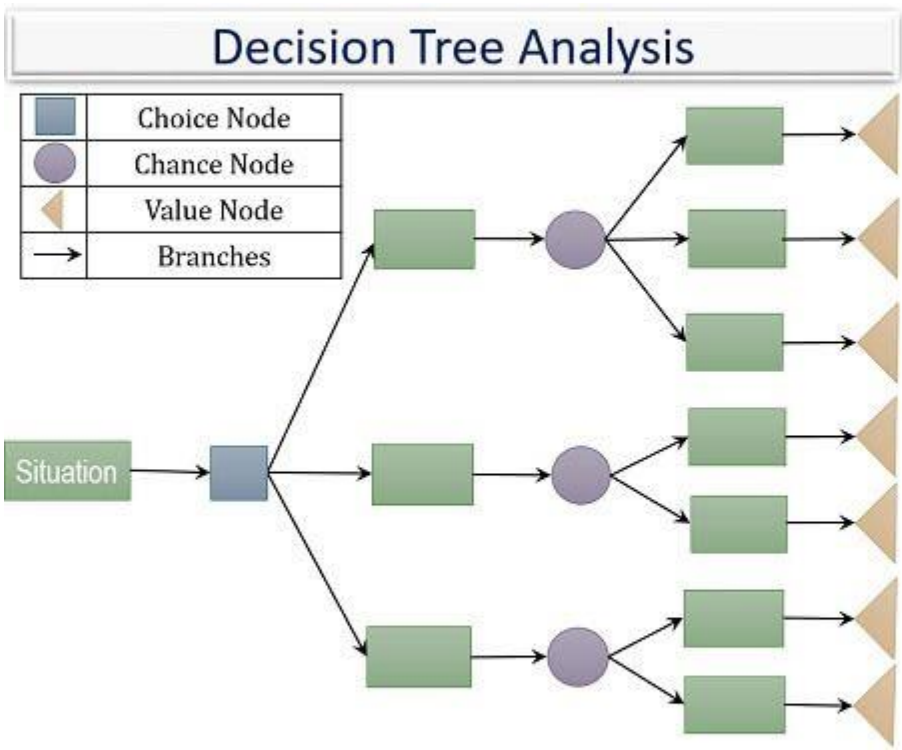
3.4 DECISION TREE

INTRODUCTION

Decision tree analysis is a powerful decision-making tool which initiates a structured nonparametric approach for problem-solving. It facilitates the evaluation and comparison of the various options and their results, as shown in a decision tree. It helps to choose the most competitive alternative. A decision tree is the graphical depiction of all the possibilities or outcomes to solve a specific issue or avail a potential opportunity. It is a useful financial tool which visually facilitates the classification of all the probable results.

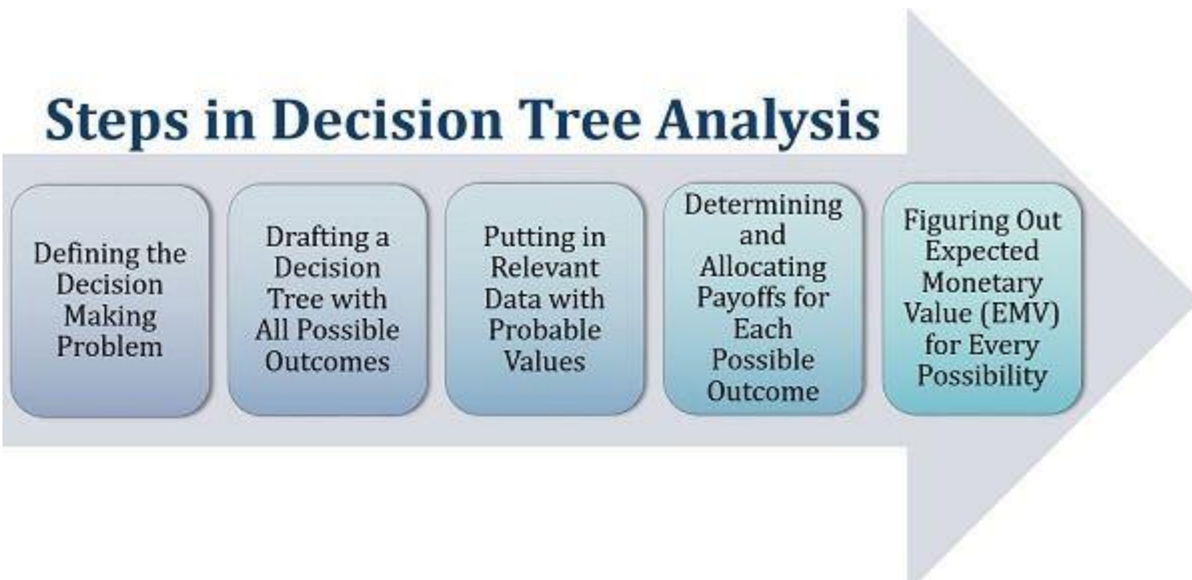


Let us understand some of the relevant concepts and terms used in the decision tree:

- **Root Node**: A root node compiles the whole sample, it is then divided into multiple sets which comprise of homogeneous variables.
- **Decision Node**: That sub-node which diverges into further possibilities, can be denoted as a decision node.
- **Terminal Node**: The final node showing the outcome which cannot be categorized any further, is termed as a value or terminal node.
- **Branch**: A branch denotes the various alternatives available with the decision tree maker.
- **Splitting**: The division of the available option (depicted by a node or sub-node) into multiple sub-nodes is termed as splitting.
- **Pruning**: It is just the reverse of splitting, where the decision tree maker can eliminate one or more sub-nodes from a particular decision node.

Steps in Decision Tree Analysis

Following steps simplify the interpretation process of a decision tree:



1. The *first step* is understanding and specifying the problem area for which decision making is required.
2. The *second step* is interpreting and chalking out all possible solutions to the particular issue as well as their consequences.

3. The *third step* is presenting the variables on a decision tree along with its respective probability values.
4. The *fourth step* is finding out the outcomes of all the variables and specifying it in the decision tree.
5. The *last step* is highly crucial and backs the overall analysis of this process. It involves calculating the EMV values for all the chance nodes or options, to figure out the solution which provides the highest expected value.

Problem : 1 : ABC Company is considering whether it should tender for two contracts (MS1 and MS2) on offer from a government department for the supply of certain components. The company has three options: tender for MS1 only; or tender for MS2 only; or tender for both MS1 and MS2. If tenders are to be submitted the company will incur additional costs. These costs will have to be entirely recouped from the contract price. The risk, of course, is that if a tender is unsuccessful the company will have made a loss. The cost of tendering for contract MS1 only is £50,000. The component supply cost if the tender is successful would be £18,000. The cost of tendering for contract MS2 only is £14,000. The component supply cost if the tender is successful would be £12,000. The cost of tendering for both contract MS1 and contract MS2 is £55,000. The component supply cost if the tender is successful would be £24,000. For each contract, possible tender prices have been determined. In addition, subjective assessments have been made of the probability of getting the contract with a particular tender price as shown below. Note here that the company can only submit one tender and cannot, for example, submit two tenders (at different prices) for the same contract.

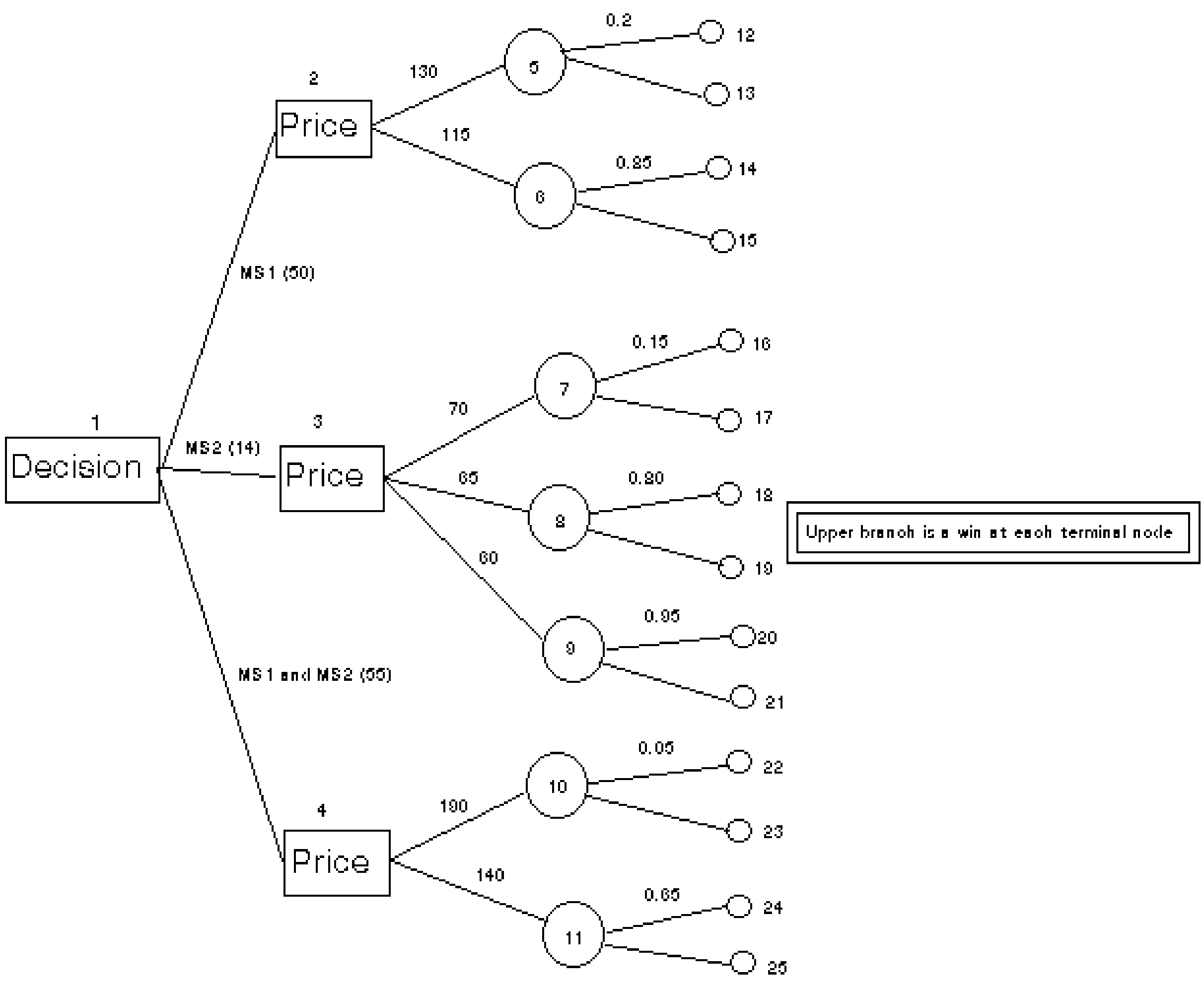
Option	Possible tender prices (£)	Probability of getting contract
MS1 only	130,000	0.20
	115,000	0.85
MS2 only	70,000	0.15
	65,000	0.80
	60,000	0.95
MS1 and MS2	190,000	0.05
	140,000	0.65

In the event that the company tenders for both MS1 and MS2 it will either win both contracts (at the price shown above) or no contract at all.

- What do you suggest the company should do and why?
- What are the downside and the upside of your suggested course of action?
- A consultant has approached your company with an offer that in return for £20,000 in cash she will ensure that if you tender £60,000 for contract MS2 only your tender is guaranteed to be successful. Should you accept her offer or not and why?

Solution

The decision tree for the problem is shown below.



Below we carry out step 1 of the decision tree solution procedure which (for this example) involves working out the total profit for each of the paths from the initial node to the terminal node (all figures in £'000).

Step 1

- path to terminal node 12, we tender for MS1 only (cost 50), at a price of 130, and win the contract, so incurring component supply costs of 18,
  total profit 130-50-18 = 62
- path to terminal node 13, we tender for MS1 only (cost 50), at a price of 130, and lose the contract, total profit -50
- path to terminal node 14, we tender for MS1 only (cost 50), at a price of 115, and win the contract, so incurring component supply costs of 18,
  total profit 115-50-18 = 47
- path to terminal node 15, we tender for MS1 only (cost 50), at a price of 115, and lose the contract, total profit -50
- path to terminal node 16, we tender for MS2 only (cost 14), at a price of 70, and win the contract, so incurring component supply costs of 12,
  total profit 70-14-12 = 44
- path to terminal node 17, we tender for MS2 only (cost 14), at a price of 70, and lose the contract, total profit -14
- path to terminal node 18, we tender for MS2 only (cost 14), at a price of 65, and win the contract, so incurring component supply costs of 12,
  total profit 65-14-12 = 39
- path to terminal node 19, we tender for MS2 only (cost 14), at a price of 65, and lose the contract, total profit -14
- path to terminal node 20, we tender for MS2 only (cost 14), at a price of 60, and win the contract, so incurring component supply costs of 12,
  total profit 60-14-12 = 34
- path to terminal node 21, we tender for MS2 only (cost 14), at a price of 60, and lose the contract, total profit -14

- path to terminal node 22, we tender for MS1 and MS2 (cost 55), at a price of 190, and win the contract, so incurring component supply costs of 24,
  total profit 190-55- 24=111
- path to terminal node 23, we tender for MS1 and MS2 (cost 55), at a price of 190, and lose the contract, total profit -55
- path to terminal node 24, we tender for MS1 and MS2 (cost 55), at a price of 140, and win the contract, so incurring component supply costs of 24,
  total profit 140-55- 24=61
- path to terminal node 25, we tender for MS1 and MS2 (cost 55), at a price of 140, and lose the contract, total profit -55

Hence we can arrive at the table below indicating for each branch the total profit involved in that branch from the initial node to the terminal node.

Terminal node	Total profit £'000
12	62
13	-50
14	47
15	-50
16	44
17	-14
18	39
19	-14
20	34
21	-14
22	111
23	-55
24	61
25	-55

We can now carry out the second step of the decision tree solution procedure where we work from the right-hand side of the diagram back to the left-hand side.

Step 2

- For chance node 5 the EMV is 0.2(62) + 0.8(-50) = -27.6
- For chance node 6 the EMV is 0.85(47) + 0.15(-50) = 32.45

Hence the best decision at decision node 2 is to tender at a price of 115 (EMV=32.45).

- For chance node 7 the EMV is 0.15(44) + 0.85(-14) = -5.3
- For chance node 8 the EMV is 0.80(39) + 0.20(-14) = 28.4
- For chance node 9 the EMV is 0.95(34) + 0.05(-14) = 31.6

Hence the best decision at decision node 3 is to tender at a price of 60 (EMV=31.6).

- For chance node 10 the EMV is 0.05(111) + 0.95(-55) = -46.7
- For chance node 11 the EMV is 0.65(61) + 0.35(-55) = 20.4

Hence the best decision at decision node 4 is to tender at a price of 140 (EMV=20.4).

Hence at decision node 1 have three alternatives:

- tender for MS1 only EMV=32.45
- tender for MS2 only EMV=31.6
- tender for both MS1 and MS2 EMV = 20.4

Hence the best decision is to tender for MS1 only (at a price of 115) as it has the highest expected monetary value of 32.45 (£'000).

The downside is a loss of 50 and the upside is a profit of 47.

With regard to the consultants offer then, ignoring ethical considerations, we could of course, tender 60 for MS2 only without her help and if we were to do that we would have a 0.95 probability of having our tender accepted. Hence there are essentially three options:

- as before, tender for MS1 only at a price of 115: EMV 32.45, downside -50 (probability 0.15), upside 47 (probability 0.85)
- tender for MS2 only at a price of 60, unaided by the consultant: EMV 31.6, downside -14 (probability 0.05), upside 34 (probability 0.95)
- tender for MS2 only at a price of 60, with the consultants help, then (assuming she can fulfil her promise of guaranteeing we will be successful), we have a certain outcome with a profit of 34 (terminal node 20) - 20
(cash paid to the consultant) = 14

On an EMV basis we would still support our original decision. Looking at the risks (probabilities) of loosing money, and considering tendering for MS2 only at 60, we would essentially be paying the consultant 20 to avoid a 0.05 chance of loosing 14, the downside of tendering unaided. Paying 20 to guarantee not incurring a loss of 14 which will occur with a probability of 0.05 (one in twenty) does not seem like an awfully good investment and so we should reject her offer

Problem : 2 : The Metal Discovery Group (MDG) is a company set up to conduct geological explorations of parcels of land in order to ascertain whether significant metal deposits (worthy of further commercial exploitation) are present or not. Current MDG has an option to purchase outright a parcel of land for £3m. If MDG purchases this parcel of land then it will conduct a geological exploration of the land. Past experience indicates that for the type of parcel of land under consideration geological explorations cost approximately £1m and yield significant metal deposits as follows:

- manganese 1% chance
- gold 0.05% chance
- silver 0.2% chance

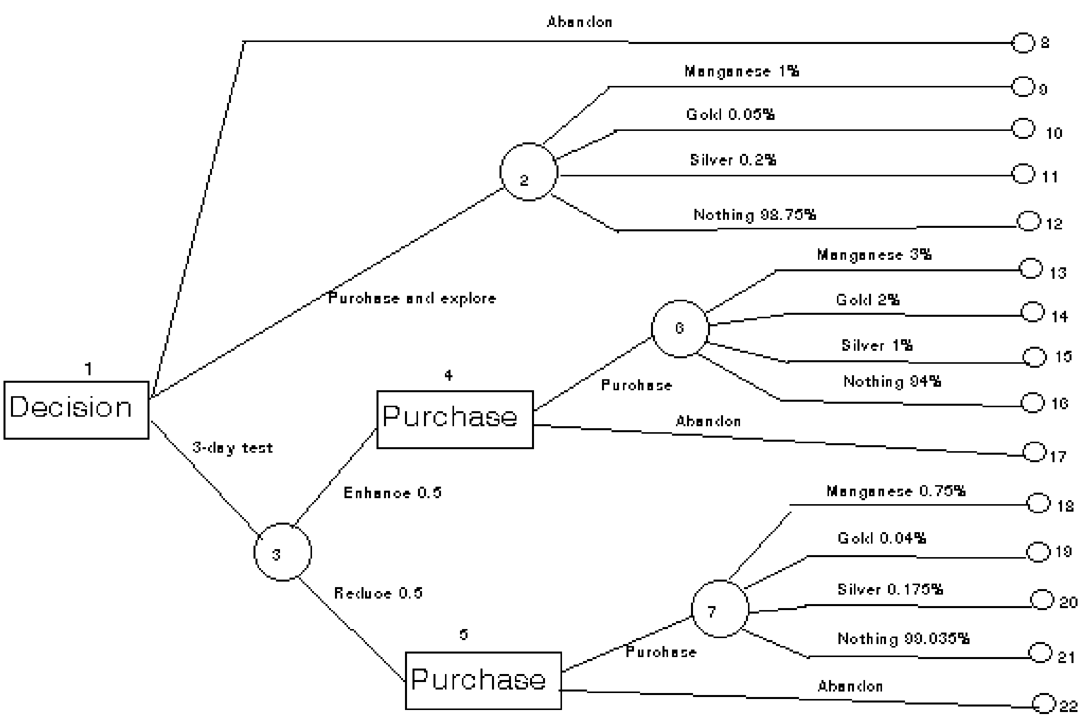
Only one of these three metals is ever found (if at all), i.e. there is no chance of finding two or more of these metals and no chance of finding any other metal. If manganese is found then the parcel of land can be sold for £30m, if gold is found then the parcel of land can be sold for £250m and if silver is found the parcel of land can be sold for £150m. MDG can, if they wish, pay £750,000 for the right to conduct a three-day test exploration before deciding whether to purchase the parcel of land or not. Such three-day test explorations can only give a preliminary indication of whether significant metal deposits are present or not and past experience indicates that three-day test explorations cost £250,000 and indicate that significant metal deposits are present 50% of the time.

If the three-day test exploration indicates significant metal deposits then the chances of finding manganese, gold and silver increase to 3%, 2% and 1% respectively. If the three-day test exploration fails to indicate significant metal deposits then the chances of finding manganese, gold and silver decrease to 0.75%, 0.04% and 0.175% respectively.

- What would you recommend MDG should do and why?
- A company working in a related field to MDG is prepared to pay half of all costs associated with this parcel of land in return for half of all revenues. Under these circumstances what would you recommend MDG should do and why?

Solution

The decision tree for the problem is shown below.



Below we carry out step 1 of the decision tree solution procedure which (for this example) involves working out the total profit for each of the paths from the initial node to the terminal node (all figures in £'000000).

Step 1

- path to terminal node 8, abandon the project - profit zero
- path to terminal node 9, we purchase (cost £3m), explore (cost £1m) and find manganese (revenue £30m), total profit 26 (£m)
- path to terminal node 10, we purchase (cost £3m), explore (cost £1m) and find gold (revenue £250m), total profit 246 (£m)
- path to terminal node 11, we purchase (cost £3m), explore (cost £1m) and find silver (revenue £150m), total profit 146 (£m)
- path to terminal node 12, we purchase (cost £3m), explore (cost £1m) and find nothing, total profit -4 (£m)
- path to terminal node 13, we conduct the three-day test (cost £0.75m + £0.25m), find we have an enhanced chance of significant metal deposits, purchase and explore (cost £4m) and find manganese (revenue £30m), total profit 25 (£m)
- path to terminal node 14, we conduct the three-day test (cost £0.75m + £0.25m), find we have an enhanced chance of significant metal deposits, purchase and explore (cost £4m) and find gold (revenue £250m), total profit 245 (£m)
- path to terminal node 15, we conduct the three-day test (cost £0.75m + £0.25m), find we have an enhanced chance of significant metal deposits, purchase and explore (cost £4m) and find silver (revenue £150m), total profit 145 (£m)
- path to terminal node 16, we conduct the three-day test (cost £0.75m + £0.25m), find we have an enhanced chance of significant metal deposits, purchase and explore (cost £4m) and find nothing, total profit -5 (£m)
- path to terminal node 17, we conduct the three-day test (cost £0.75m + £0.25m), find we have an enhanced chance of significant metal deposits, decide to abandon, total profit -1 (£m)

- path to terminal node 18, we conduct the three-day test (cost £0.75m + £0.25m), find we have an reduced chance of significant metal deposits, purchase and explore (cost £4m) and find manganese (revenue £30m), total profit 25 (£m)
- path to terminal node 19, we conduct the three-day test (cost £0.75m + £0.25m), find we have an reduced chance of significant metal deposits, purchase and explore (cost £4m) and find gold (revenue £250m), total profit 245 (£m)
- path to terminal node 20, we conduct the three-day test (cost £0.75m + £0.25m), find we have an reduced chance of significant metal deposits, purchase and explore (cost £4m) and find silver (revenue £150m), total profit 145 (£m)
- path to terminal node 21, we conduct the three-day test (cost £0.75m + £0.25m), find we have an reduced chance of significant metal deposits, purchase and explore (cost £4m) and find nothing, total profit -5 (£m)
- path to terminal node 22, we conduct the three-day test (cost £0.75m + £0.25m), find we have an reduced chance of significant metal deposits, decide to abandon, total profit -1 (£m)

Hence we can arrive at the table below indicating for each branch the total profit involved in that branch from the initial node to the terminal node.

Terminal node	Total profit £
8	0
9	26
10	246
11	146
12	-4
13	25
14	245
15	145
16	-5
17	-1

18	25
19	245
20	145
21	-5
22	-1

We can now carry out the second step of the decision tree solution procedure where we work from the right-hand side of the diagram back to the left-hand side.

Step 2

Consider chance node 7 with branches to terminal nodes 15-21 emanating from it. The expected monetary value for this chance node is given by

$0.0075(25) + 0.0004(245) + 0.00175(145) + 0.99035(-5) = -4.4125$

Hence the best decision at decision node 5 is to abandon (EMV=-1).

The EMV for chance node 6 is given by

$0.03(25) + 0.02(245) + 0.01(145) + 0.94(-5) = 2.4$

Hence the best decision at decision node 4 is to purchase (EMV=2.4).

The EMV for chance node 3 is given by $0.5(2.4) + 0.5(-1) = 0.7$

The EMV for chance node 2 is given by $0.01(26) + 0.0005(246) + 0.002(146) + 0.9875(-4) = -3.275$

Hence at decision node 1 have three alternatives:

- abandon EMV=0
- purchase and explore EMV=-3.275
- 3-day test EMV=0.7

Hence the best decision is the 3-day test as it has the highest expected monetary value of 0.7 (£m).

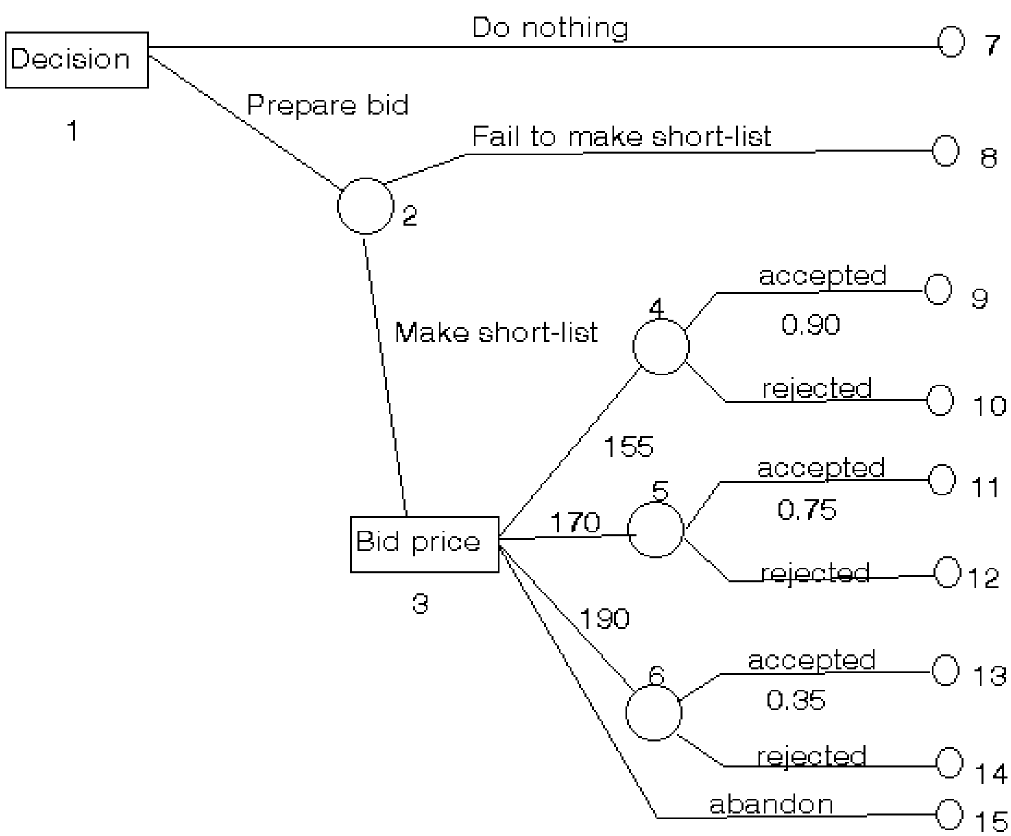
Sharing the costs and revenues on a 50:50 basis merely halves all the monetary figures in the above calculations and so the optimal EMV decision is *exactly* as before. However in a wider context by accepting to share costs and revenues the company is spreading its risk and from that point of view may well be a wise offer to accept.

Problem : 3

A company is trying to decide whether to bid for a certain contract or not. They estimate that merely preparing the bid will cost £10,000. If their company bid then they estimate that there is a 50% chance that their bid will be put on the "short-list", otherwise their bid will be rejected. Once "short-listed" the company will have to supply further detailed information (entailing costs estimated at £5,000). After this stage their bid will either be accepted or rejected. The company estimate that the labour and material costs associated with the contract are £127,000. They are considering three possible bid prices, namely £155,000, £170,000 and £190,000. They estimate that the probability of these bids being accepted (once they have been short-listed) is 0.90, 0.75 and 0.35 respectively. What should the company do and what is the expected monetary value of your suggested course of action?

Solution

The decision tree for the problem is shown below.



Below we carry out step 1 of the decision tree solution procedure which (for this example) involves working out the total profit for each of the paths from the initial node to the terminal node (all figures in £'000).

Step 1 : path to terminal node 7 - the company do nothing

Total profit = 0

path to terminal node 8 - the company prepare the bid but fail to make the short-list

Total cost = 10 Total profit = -10

- path to terminal node 9 - the company prepare the bid, make the short-list and their bid of £155K is accepted

Total cost = 10 + 5 + 127 Total revenue = 155 Total profit = 13

- path to terminal node 10 - the company prepare the bid, make the short-list but their bid of £155K is unsuccessful

Total cost = 10 + 5 Total profit = -15

- path to terminal node 11 - the company prepare the bid, make the short-list and their bid of £170K is accepted

Total cost = 10 + 5 + 127 Total revenue = 170 Total profit = 28

- path to terminal node 12 - the company prepare the bid, make the short-list but their bid of £170K is unsuccessful

Total cost = 10 + 5 Total profit = -15

- path to terminal node 13 - the company prepare the bid, make the short-list and their bid of £190K is accepted

Total cost = 10 + 5 + 127 Total revenue = 190 Total profit = 48

- path to terminal node 14 - the company prepare the bid, make the short-list but their bid of £190K is unsuccessful

Total cost = 10 + 5 Total profit = -15

- path to terminal node 15 - the company prepare the bid and make the short-list and then decide to abandon bidding (an implicit option available to the company)

Total cost = 10 + 5 Total profit = -15

Hence we can arrive at the table below indicating for each branch the total profit involved in that branch from the initial node to the terminal node.

Terminal node	Total profit £
7	0
8	-10
9	13
10	-15
11	28
11	-15
13	48
14	-15
15	-15

We can now carry out the second step of the decision tree solution procedure where we work from the right-hand side of the diagram back to the left-hand side.

Step 2

Consider chance node 4 with branches to terminal nodes 9 and 10 emanating from it. The expected monetary value for this chance node is given by 0.90(13) + 0.10(-15) = 10.2

Similarly the EMV for chance node 5 is given by 0.75(28) + 0.25(-15) = 17.25

The EMV for chance node 6 is given by 0.35(48) + 0.65(-15) = 7.05

Hence at the bid price decision node we have the four alternatives

(1) bid £155K EMV = 10.2

(2) bid £170K EMV = 17.25

(3) bid £190K EMV = 7.05

(4) abandon the bidding EMV = -15

Hence the best alternative is to bid £170K leading to an EMV of 17.25

Hence at chance node 2 the EMV is given by 0.50(17.25) + 0.50(-10) = 3.625

Hence at the initial decision node we have the two alternatives

(1) prepare bid EMV = 3.625

(2) do nothing EMV = 0

Hence the best alternative is to prepare the bid leading to an EMV of £3625. In the event that the company is short-listed then (as discussed above) it should bid £170,000.

Problem : 4

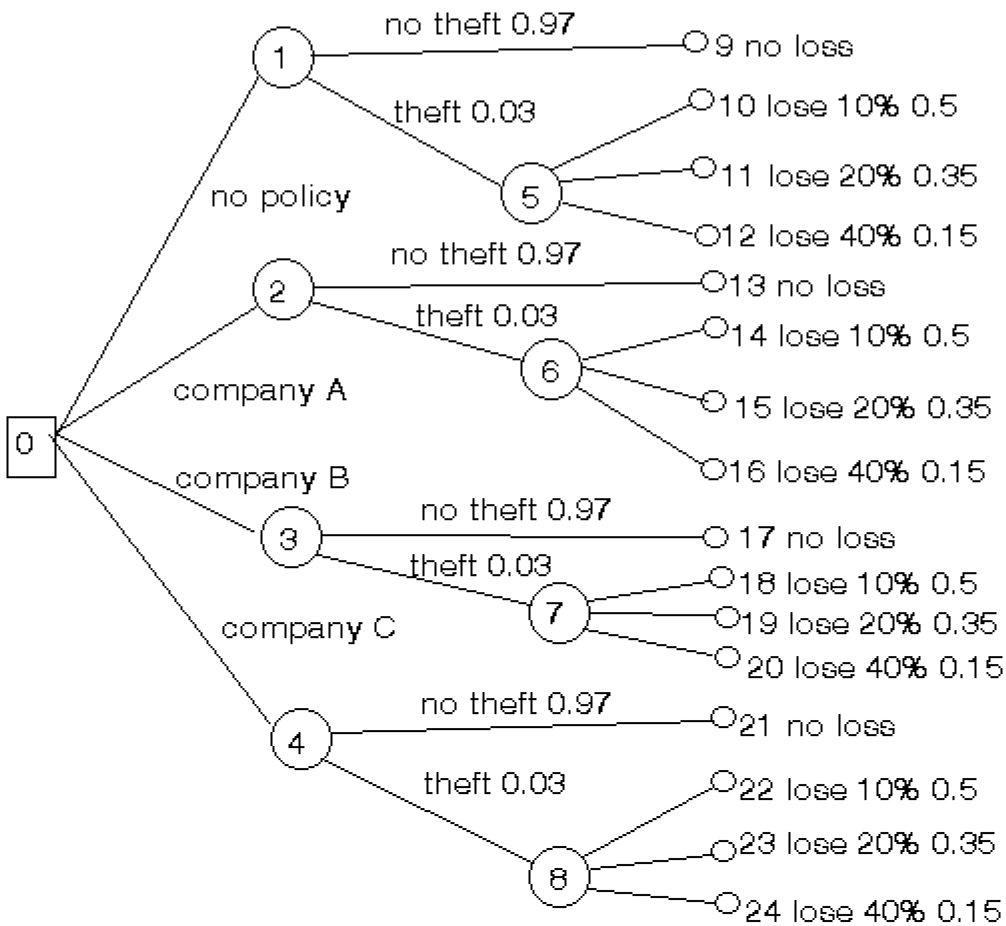
A householder is currently considering insuring the contents of his house against theft for one year. He estimates that the contents of his house would cost him £20,000 to replace. Local crime statistics indicate that there is a probability of 0.03 that his house will be broken into in the coming year. In that event his losses would be 10%, 20%, or 40% of the contents with probabilities 0.5, 0.35 and 0.15 respectively. An insurance policy from company A costs £150 a year but guarantees to replace any losses due to theft. An insurance policy from company B is cheaper at £100 a year but the householder has to pay the first £x of any loss himself. An insurance policy from company C is even cheaper at £75 a year but only replaces a fraction (y%) of any loss suffered.

Assume that there can be at most one theft a year.

- Draw the decision tree.
- What would be your advice to the householder if x = 50 and y = 40% and his objective is to maximise expected monetary value (EMV)?
- Formulate the problem of determining the maximum and minimum values of x such that the policy from company B has the highest EMV using linear programming with two variables x and y (i.e. both x and y are now variables, not known constants).

Solution

The decision tree for the problem is shown below.



Below we carry out step 1 of the decision tree solution procedure which (for this example) involves working out the total profit for each of the paths from the initial node to the terminal node.

Step 1

- path to terminal node 9 - we have no insurance policy but suffer no theft.

Total profit = 0

- path to terminal node 10 - we have no insurance policy but suffer a theft resulting in a loss of 10% of the contents.

Total cost = 0.1(20000) = 2000 Total profit = - 2000

Similarly for terminal nodes 11 and 12 total profit = -4000 and -8000 respectively.

- path to terminal node 13 - we have an insurance policy with company A costing £150 but suffer no theft.

Total cost = 150 Total profit = -150

- path to terminal node 14 - we have an insurance policy with company A costing £150 but suffer a theft resulting in a loss of 0.1(20000) = £2000 for which we are *reimbursed* in full by company A. Hence

Total revenue = 2000 Total cost = 2000 + 150 Total profit = -150

It is clear from this calculation that when the reimbursement equals the amount lost the total profit will always be just the cost of the insurance.

This will be the case for terminal nodes 15 and 16 respectively.

Continuing in a similar manner we can arrive at the table below indicating for each branch the total profit involved in that branch from the initial node to the terminal node.

Terminal node	Total profit £
9	0
10	-2000
11	-4000
12	-8000
13	-150
14	-150
15	-150
16	-150
17	-100
18	$-100-x$ ($x <= 2000$)
19	$-100-x$
20	$-100-x$
21	-75
22	$-75-2000(1-y/100)$
23	$-75-4000(1-y/100)$
24	$-75-8000(1-y/100)$

We can now carry out the second step of the decision tree solution procedure where we work from the right-hand side of the diagram back to the left-hand side.

Step 2

Consider chance node 5 with branches to terminal nodes 10, 11 and 12 emanating from it.

The expected monetary value for this chance node is given by

0.5(-2000) + 0.35(-4000) + 0.15(-8000) = -3600

Hence the EMV for chance node 1 is given by 0.97(0) + 0.03(-3600) = -108

Similarly the EMV for chance node 2 is -150.

The EMV for chance node 3 is 0.97(-100) + 0.03[0.5(-100-x) + 0.35(-100-x) + 0.15(-100-x)]

= -97 + 0.03(-100-x) = -100 - 0.03x (x <= 2000) = -101.5 since x = 50

The EMV for chance node 4 is

0.97(-75) + 0.03[0.5(-75-2000(1-y/100)) + 0.35(-75-4000(1-y/100)) + 0.15(-75- 8000(1-y/100))]

= 0.97(-75) + 0.03[-75-(1-y/100)(3600)] = -75 + 1.08y - 108 = -183 + 1.08y

= -139.8 since y = 40

Hence at the initial decision node we have the four alternatives

1. no policy EMV = -108
2. company A policy EMV = -150
3. company B policy EMV = -101.5
4. company C policy EMV = -139.8

Hence the best alternative is the policy from company B leading to an EMV of - £101.5

We know that for x = 50 policy B is best so we already have that the minimum value of x <= 2000

and so the LP for the minimum value of x is given by

minimise x

s.t. -100 - 0.03x >= -108 i.e. EMV B >= EMV no policy

-100 - 0.03x >= -150 EMV B >= EMV A

-100 - 0.03x >= -183 + 1.08y EMV B >= EMV C i.e.

minimise x

s.t.

$$x <= 266.67$$

$$x <= 1666.67$$

$$1.08y + 0.03x <= 83$$

$$x >= 0$$

$$y >= 0 \text{ and } y <= 100$$

If $x = 2000$ then EMV B becomes $-100-0.03(2000) = -160$ so if x becomes that high we would prefer no policy (EMV for chance node 1 = -108).

Problem : 5

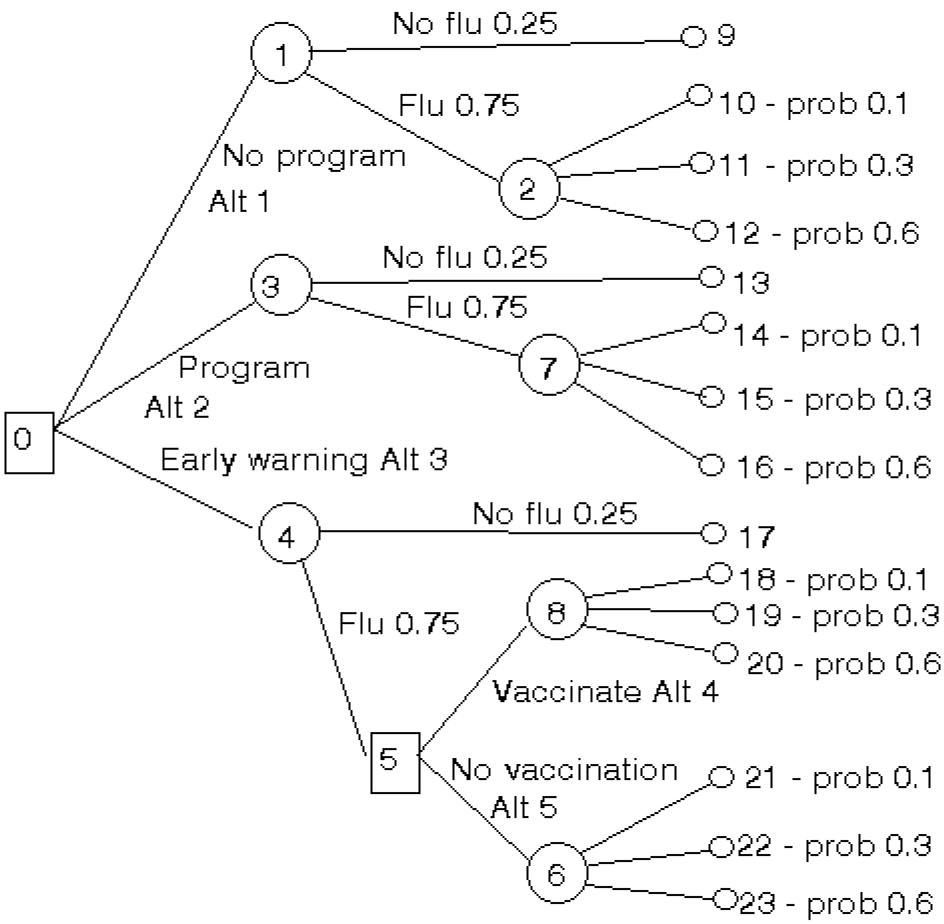
A government committee is considering the economic benefits of a program of preventative flu vaccinations. If vaccinations are not introduced then the estimated cost to the government if flu strikes in the next year is £7m with probability 0.1, £10m with probability 0.3 and £15m with probability 0.6. It is estimated that such a program will cost £7m and that the probability of flu striking in the next year is 0.75.

One alternative open to the committee is to institute an "early-warning" monitoring scheme (costing £3m) which will enable it to detect an outbreak of flu early and hence institute a rush vaccination program (costing £10m because of the need to vaccinate quickly before the outbreak spreads).

- What recommendations should the committee make to the government if their objective is to maximise expected monetary value (EMV)?
- The committee has also been informed that there are alternatives to using EMV. What are these alternatives and would they be appropriate in this case?

Solution

The decision tree for the problem is shown below.



Below we carry out step 1 of the decision tree solution procedure which (for this example) involves working out the total profit for each of the paths from the initial node to the terminal nodes.

Step 1

- path to terminal node 9 - we carry out no program and flu does not strike

Total revenue = 0

Total cost = 0

Total profit = 0

- path to terminal node 10 - we carry out no program and flu strikes costing the government £7m

Total revenue = 0

Total cost = 7

Total profit = -7 (all figures in £m)

- path to terminal nodes 11 and 12 similar to the case above giving a total profit of -10 and -15 respectively
- path to terminal node 13 - we carry out a program costing £7m and flu does not strike

Total revenue = 0

Total cost = 7

Total profit = -7

- path to terminal node 14 - we carry out a program costing £7m and flu strikes. Now we would have lost £7m with this flu outbreak but because of the program (which we assume to be 100% effective) we do not.

The key here is to regard the £7m paid for the program as "insurance" which reimburses the government for whatever losses are suffered as a result of flu striking. Hence we have

Total revenue = 7 (reimbursement)

Total cost = 7 (cost of program) + 7 (loss due to flu striking)

Total profit = -7

It is clear from the above calculation that since (in this case) the reimbursement always exactly equals the amount lost the total profit will just be the cost of the "insurance" (-£7m).

The situation with the vaccination program is very similar to household insurance where a single payment guarantees replacement of any losses suffered. Whatever happens the effect of the insurance will be "as if" nothing had occurred. Under these circumstances the only expense (in effect) is the cost of the insurance.

- path to terminal nodes 15 and 16 similar to the case above where we carry out a program costing £7m and this insures us against losses. Hence

Total profit = -7 terminal node 15

Total profit = -7 terminal node 16

- path to terminal node 17 - we carry out an early warning program costing £3m and flu does not strike giving

Total revenue = 0

Total cost = 3

Total profit = -3

- path to terminal nodes 18, 19 and 20 - we carry out an early warning program costing £3m, flu strikes and we decide to vaccinate costing £10m. Hence for a total cost of £13m we are insured against losses so that we have

Total profit = -13 terminal node 18

Total profit = -13 terminal node 19

Total profit = -13 terminal node 20

- path to terminal nodes 21, 22 and 23 - we carry out an early warning program costing £3m, flu strikes but we decide not to vaccinate, leading to costs of £7m, £10m and £15m. Hence

Total profit = -10 terminal node 21

Total profit = -13 terminal node 22

Total profit = -18 terminal node 23

Hence we can form the table below indicating for each branch the total profit involved in that branch from the initial node to the terminal node.

Terminal node	Total profit (£m)
9	0
10	-7
11	-10
12	-15
13	-7
14	-7
15	-7

16	-7
17	-3
18	-13
19	-13
20	-13
21	-10
22	-13
23	-18

We can now carry out the second step of the decision tree solution procedure where we work from the right-hand side of the diagram back to the left-hand side.

Step 2

Consider chance node 2 (with branches to terminal nodes 10, 11 and 12 emanating from it). The expected monetary value (EMV) for this chance node is given by 0.1 x (-7) + 0.3 x (-10) + 0.6 x (-15) = -12.7

Hence the EMV for chance node 1 is given by 0.25 x (0) + 0.75 x (-12.7) = -9.525

Similarly the EMV for chance node 7 is given by 0.1 x (-7) + 0.3 x (-7) + 0.6 x (-7) = -7 which leads to an EMV for chance node 3 of 0.25 x (-7) + 0.75 x (-7) = -7

The EMV for chance node 8 is 0.1 x (-13) + 0.3 x (-13) + 0.6 x (-13) = -13 and the EMV for chance node 6 is 0.1 x (-10) + 0.3 x (-13) + 0.6 x (-18) = -15.7

Hence for decision node 5 we have the two alternatives:

(4) vaccinate EMV = -13

(5) no vaccination EMV = -15.7

Hence the best alternative here is to vaccinate (alternative 4) with an EMV of -13.

The EMV for chance node 4 is therefore 0.25 x (-3) + 0.75 x (-13) = -10.5

and at the initial decision node (node 0) we have the three alternatives:

(1) no program EMV = -9.525

(2) program EMV = -7

(3) early warning EMV = -10.5

Hence the best alternative is alternative 2, institute a program costing £7m, leading to an EMV of -£7m.

Note here that it is clear that the concept of the vaccination program being an insurance against all possible losses could have enabled us to have drawn a much simpler decision tree (e.g. chance node 3 could be transformed into a "terminal" node of cost -£7m and nodes 7,13,14,15 dropped altogether (similarly for nodes 8,18,19,20)). However, for clarity, we have presented the decision tree as given above.

With respect to the last part of the question mention discounting, alternative value for a chance node (other than EMV), changing the decision node ("choose highest EMV alternative") rule and utility and briefly discuss whether appropriate/inappropriate.

Problem : 6

XYZ company is considering whether it should tender for two contracts (C1 and C2) on offer from a government department for the supply of certain components. If tenders are submitted, the company will have to provide extra facilities, the cost of which will have to be entirely recouped from the contract revenue. The risk, of course, is that if the tenders are unsuccessful then the company will have to write off the cost of these facilities. The extra facilities necessary to meet the requirements of contract C1 would cost £50,000. These facilities would, however, provide sufficient capacity for the requirements of contract C2 to be met also. In addition the production costs would be £18,000. The corresponding production costs for contract C2 would be £10,000. If a tender is made for contract C2 only, then the necessary extra facilities can be provided at a cost of only £24,000. The production costs in this case would be £12,000. It is estimated that the tender preparation costs would be £2,000 if tenders are made for contracts C1 or C2 only and £3,000 if a tender is made for both contracts C1 and C2. For each contract, possible tender prices have been determined. In addition, subjective assessments have been made of the probability of getting the contract with a particular tender price as shown below. Note here that the company can only submit one tender and cannot, for example, submit two tenders (at different prices) for the same contract.

	Possible tender prices (£)	Probability of getting contract
Tendering for C1 only	120,000	0.30

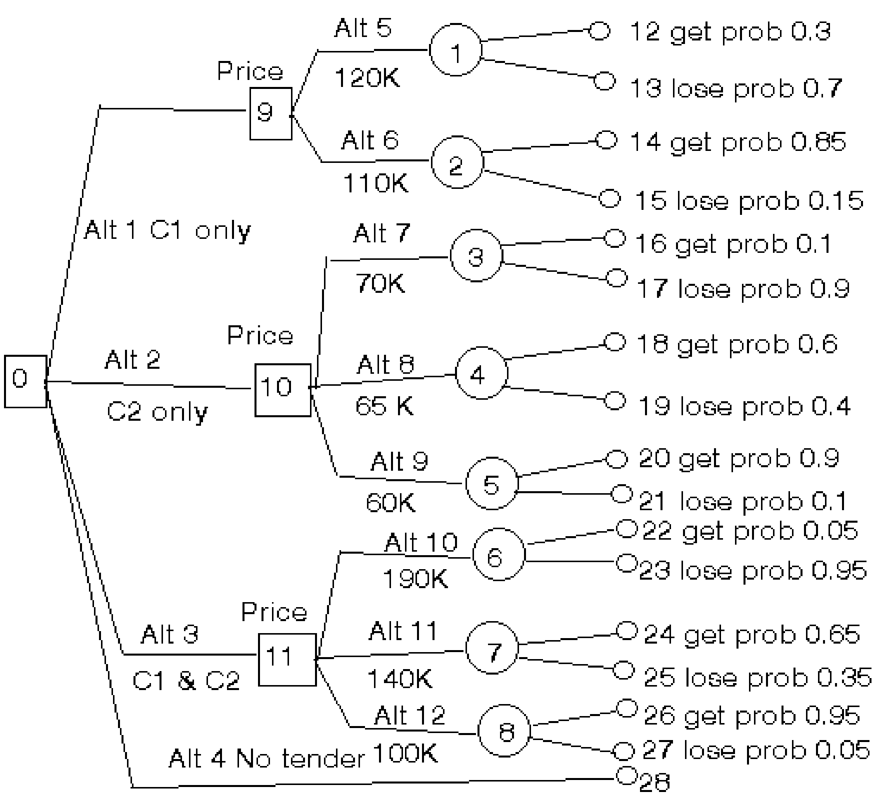
	110,000	0.85
Tendering for C2 only	70,000	0.10
	65,000	0.60
	60,000	0.90
Tendering for both C1 and C2	190,000	0.05
	140,000	0.65
	100,000	0.95

In the event that the company tenders for both C1 and C2 it will either win both contracts (at the price shown above) or no contract at all.

- What do you suggest the company should do and why?
- What is the "downside" of your suggested course of action?

Solution

The decision tree for the problem is shown below.



Below we carry out step 1 of the decision tree solution procedure which (for this example) involves calculating the total profit for each of the paths from the initial node to the terminal nodes.

Step 1

- path to terminal node 12 - we decide to tender for C1 only at a price of 120K and are successful

Total revenue = 120

Total cost = 50 + 18 + 2 = 70

Total profit = 50 (all figures in £K)

- path to terminal node 13 - we decide to tender for C1 only at a price of 120K but are unsuccessful

Total revenue = 0

Total cost = 50 + 2 = 52

Total profit = -52

- path to terminal node 14 - we decide to tender for C1 only at a price of 110K and are successful

Total revenue = 110

Total cost = 50 + 18 + 2 = 70

Total profit = 40

- path to terminal node 15 - we decide to tender for C1 only at a price of 110K but are unsuccessful

Total revenue = 0

Total cost = 50 + 2 = 52

Total profit = -52

- path to terminal node 16 - we decide to tender for C2 only at a price of 70K and are successful

Total revenue = 70

Total cost = 24 + 12 + 2 = 38

Total profit = 32

- path to terminal node 17 - we decide to tender for C2 only at a price of 70K but are unsuccessful

Total revenue = 0

Total cost = 24 + 2 = 26

Total profit = -26

- path to terminal node 18 - we decide to tender for C2 only at a price of 65K and are successful

Total revenue = 65

Total cost = 24 + 12 + 2 = 38

Total profit = 27

- path to terminal node 19 - we decide to tender for C2 only at a price of 65K but are unsuccessful

Total revenue = 0

Total cost = 24 + 2 = 26

Total profit = -26

- path to terminal node 20 - we decide to tender for C2 only at a price of 60K and are successful

Total revenue = 60

Total cost = 24 + 12 + 2 = 38

Total profit = 22

- path to terminal node 21 - we decide to tender for C2 only at a price of 60K but are unsuccessful

Total revenue = 0

Total cost = 24 + 2 = 26

Total profit = -26

- path to terminal node 22 - we decide to tender for C1/C2 at a price of 190K and are successful

Total revenue = 190

Total cost = 50 + 18 + 10 + 3 = 81

Total profit = 109

- path to terminal node 23 - we decide to tender for C1/C2 at a price of 190K but are unsuccessful

Total revenue = 0

Total cost = 50 + 3 = 53

Total profit = -53

- path to terminal node 24 - we decide to tender for C1/C2 at a price of 140K and are successful

Total revenue = 140

Total cost = 50 + 18 + 10 + 3 = 81

Total profit = 59

- path to terminal node 25 - we decide to tender for C1/C2 at a price of 140K but are unsuccessful

Total revenue = 0

Total cost = 50 + 3 = 53

Total profit = -53

- path to terminal node 26 - we decide to tender for C1/C2 at a price of 100K and are successful

Total revenue = 100

Total cost = 50 + 18 + 10 + 3 = 81

Total profit = 19

- path to terminal node 27 - we decide to tender for C1/C2 at a price of 100K but are unsuccessful

Total revenue = 0

Total cost = 50 + 3 = 53

Total profit = -53

- path to terminal node 28 - we decide not to tender at all

Total revenue = 0

Total cost = 0

Total profit = 0

Hence we can form the table below indicating for each branch the total profit involved in that branch from the initial node to the terminal node.

Terminal node	Total profit (£K)
12	50
13	-52
14	40
15	-52
16	32
17	-26
18	27
19	-26
20	22
21	-26
22	109
23	-53
24	59
25	-53
26	19
27	-53
28	0

We can now carry out the second step of the decision tree solution procedure where we work from the right-hand side of the diagram back to the left-hand side.

Step 2

Consider chance node 1 (with branches to terminal nodes 12 and 13 emanating from it). The expected monetary value (EMV) for this chance node is given by 0.3 x (50) + 0.7 x (-52) = -21.4

Consider chance node 2, the EMV for this chance node is given by 0.85 x (40) + 0.15 x (-52) = 26.2

Then for the decision node relating to the price for C1 we have the two alternatives:

(5) price 120K EMV = -21.4

(6) price 110K EMV = 26.2

It is clear that, in £ terms, alternative 6 is the most attractive alternative and so we can discard the other alternative.

Continuing the process the EMV for chance node 3 is given by 0.10 x (32) + 0.9 x (-26) = -20.2

The EMV for chance node 4 is given by 0.60 x (27) + 0.40 x (-26) = 5.8

The EMV for chance node 5 is given by 0.90 x (22) + 0.10 x (-26) = 17.2

Hence for the decision node relating to the price for C2 we have the three alternatives:

(7) price 70K EMV = -20.2

(8) price 65K EMV = 5.8

(9) price 60K EMV = 17.2

It is clear that, in £ terms, alternative 9 is the most attractive alternative and so we can discard the other two alternatives.

Continuing the process the EMV for chance node 6 is given by 0.05 x (109) + 0.95 x (-53) = -44.9

The EMV for chance node 7 is given by 0.65 x (59) + 0.35 x (-53) = 19.8

The EMV for chance node 8 is given by 0.95 x (19) + 0.05 x (-53) = 15.4

Hence for the decision node relating to the price to charge for C1 and C2 we have the three alternatives:

(10) price 190K EMV = -44.9

(11) price 140K EMV = 19.8

(12) price 100K EMV = 15.4

It is clear that, in £ terms, alternative 11 is the most attractive alternative and so we can discard the other two alternatives.

Hence for the decision node relating to the tender decision we have the four alternatives:

(1) C1 only EMV = 26.2

(2) C2 only EMV = 17.2

(3) C1 and C2 EMV = 19.8

(4) no tender EMV = 0

It is clear that, in £ terms, alternative 1 is the most attractive alternative and so we can discard the other three alternatives.

Hence we recommend that the company tenders for contract C1 only, with a tender price of £110K because this alternative has the highest EMV of £26.2K.

If the company follows this recommendation the actual outcome will be one of the terminal nodes 14 or 15 (depending upon chance events) i.e. the outcome will be one of [40, -52]. Hence the downside is that the company may lose £52K (if their tender is unsuccessful).