

### 3.3 ANALYSIS OF DT SYSTEMS

#### IMPULSE RESPONSE

#### LTI SYSTEM WITH AND WITHOUT MEMORY

Convolution is given by  $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau$

Convolution is commutative  $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$

A CT system is memoryless if present output depends on present input. Above condition is true only for  $h(\tau) = k\delta(\tau)$  and such a memory system has the form

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(t - \tau)k\delta(\tau)d\tau$$

$$= kx(t) * \delta(t) \text{-----(1)}$$

We know that,  $x(t) * \delta(t) = x(t)$

$$(1) \rightarrow y(t) = kx(t)$$

K is the constant

The system is memory less or static if

$$h(t) = \delta(t) \text{-----(2)}$$

If (2) is not satisfied system is dynamic system

Hence the output is equal to the input, this system becomes an identity system.

## INVERTIBILITY OF LTI SYSTEM

Consider a CT LTI system with impulse response  $h(t)$ . This system is invertible, if it has an LTI inverse system.

We know, for an identity system

$$x(t) * \delta(t) = x(t)$$

The impulse response of an inverse system should satisfy the condition

$$h(t) * h_1(t) = \delta(t)$$

## CAUSALITY FOR LTI SYSTEM

For a system to be causal, the output of the system must depend on present and past inputs only.

$$\therefore h(t) = 0 \text{ for } t < 0$$

The convolution integral becomes

$$y(t) = \int_{-\infty}^t x(\tau)h(t - \tau)d\tau$$

$$y(t) = \int_0^{\infty} h(\tau)x(t - \tau)d\tau$$

## STABILITY OF LTI SYSTEM

For a LTI system to be stable

$$\int_{-\infty}^{\infty} |h(\tau)|d\tau < \infty$$

**FOURIER METHOD FOR ANALYSIS****FREQUENCY RESPONSE OF LTI SYSTEM**

Consider a LTI system

$$y(t) = x(t) * h(t)$$

Take Fourier transform on both sides

$$Y(j\omega) = X(j\omega).H(j\omega) \text{ --- (1)}$$

Convolution in time domain gives multiplication in frequency domain.

Where,  $H(j\omega)$  is the frequency response of the system

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Magnitude of equation (1) is

$$|Y(j\omega)| = |X(j\omega)|. |H(j\omega)|$$

$|H(j\omega)|$  is the gain of the system

phase of equation (1) is

$$\angle Y(j\omega) = \angle X(j\omega). \angle H(j\omega)$$

$\angle H(j\omega)$  is the phase shift of the system

**ANALYSIS AND CHARACTERIZATION OF LTI SYSTEM USING LAPLACE TRANSFORM**

Output of the system is given by

$$y(t) = x(t) * h(t)$$

Taking Laplace Transform on both sides.

$$Y(S) = X(S).H(S)$$

Where,  $H(S)$  is the system function or transfer function

If  $S = j\omega$ , then  $H(S)$  is frequency response of LTI system.

### **CAUSALITY:**

For causal LTI system

$$h(t) = 0 \text{ for } t < 0$$

1. ROC for a causal system is in the right half plane.
2. ROC for causal system with rational system is right half plane to right of right most pole.

### **STABILITY:**

An LTI system is stable if and only if ROC of its system function  $H(S)$  includes  $j\omega$  axis .ie,  $Re(S) = 0$

A causal system with rational system function is stable if and only if all poles(X) of  $H(S)$  lie in left half of S-plane.