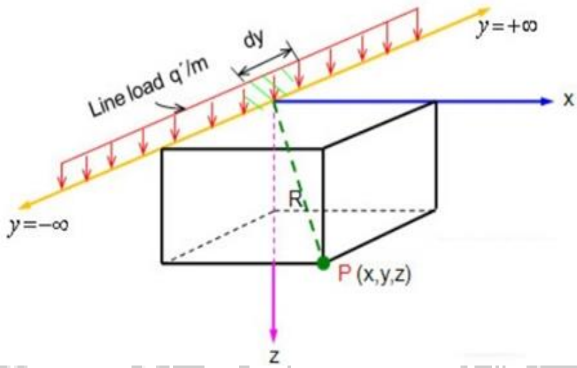
3) Stress due to line load:

The vertical stress in a soil mass due to a vertical line load can be obtained using Boussinesq solution. Let the vertical line load be of intensity q per unit length, along the y axis, acting on the surface of a semi infinite soil mass



Let us consider the load acting on a small length dy. The load can be taken as a point load of q' dy using Boussinesq solution the vertical stress at P is given by

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{(Z)^5}{(r^2+z^2)^{5/2}}\right]$$

$$\Delta\sigma_z = \frac{3q'dy}{2\pi}\left[\frac{(Z)^3}{(r^2+z^2)^{5/2}}\right] - - - -(1)$$

The vertical stress at P due to line load extending from -∞ $to + \infty$ is obtained by integration,

$$\sigma_z = \frac{3q'z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{dy}{(r^2+z^2)^{5/2}}$$

We know that $r^2 = x^2+y^2$

$$\sigma_z = \frac{3q'z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{dy}{(x^2+y^2+z^2)^{5/2}} - - - - - - - - (2)$$

Substitute $x^2+y^2=u^2$ in eqn (2)

$$\sigma_z = \frac{3q'z^3}{2\pi}\int_{-\infty}^{+\infty}\frac{dy}{(u^2+z^2)^{5/2}} \text{-------------(3)}$$

Let $y = u \tan\theta$ $\qquad$ $dy = u\sec^2\theta.\, d\theta$

When y= -∞,tan θ= -∞, θ= -π/2

y= +∞,tan θ= ∞, θ= π/2

Eqn (3) can be written as,

$$\sigma_z = \frac{3q'z^3}{2\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{dy}{(\mathrm{u}^2 + u^2\tan^2\theta)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3.2}{2\pi}\int_{0}^{\frac{\pi}{2}} \frac{dy}{(\mathrm{u}^2 + u^2\tan^2\theta)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3}{\pi}\int_{0}^{\frac{\pi}{2}} \frac{dy}{\mathrm{u}^2(1 + \tan^2\theta)^{5/2}}$$

Substitute dy=$\mathrm{u}.\sec^2\theta.\mathrm{d}\,\theta$ & $[1+\tan^2\theta = \sec^2\theta]$

$$\sigma_z = \frac{3q'z^3}{\pi}\int_{0}^{\frac{\pi}{2}} \frac{\mathrm{u}.\sec^2\theta.\mathrm{d}\,\theta}{\mathrm{u}^2(\sec^2\theta)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4}\int_{0}^{\frac{\pi}{2}} \cos^3\theta.d\theta$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4}\int_{0}^{\frac{\pi}{2}} \cos^2\theta.cos\theta.d\theta$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4}\int_{0}^{\frac{\pi}{2}} (1-\sin^2\theta).cos\theta.d\theta -----(4)$$

Let sinθ=t

Cosθ.dθ=dt

When θ=0,sinθ=0

θ=π/2, sinθ=1

Eqn (4) becomes

$$\sigma_z = \frac{3q'z^3}{\pi u^4}\int_0^1 (1-\mathrm{t}^2).\,dt$$

$$= \frac{3q'z^3}{\pi u^4}\left(t - \frac{t^3}{3}\right)_0^1$$

$$= \frac{3q'z^3}{\pi u^4}(1 - \frac{1}{3})$$

$$= \frac{3q'z^3}{\pi u^4}.\frac{2}{3}$$

$$= \frac{2q'z^3}{\pi((u^2)^2)}$$

$$\sigma_z = \frac{2q'z^3}{\pi((x^2+z^2)^2)}$$

$$\sigma_z = \frac{2q'}{\pi z\left(1+\left(\frac{x}{z}\right)^2\right)^2}$$

$$\sigma_z = \frac{2q'}{\pi z}.\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2 - - - - - - -(5)$$

$$\sigma_z = (I_B)\frac{q'}{z}$$

$$I_B = \frac{2}{\pi}.\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

I_B= Boussinesq influence factor for line load

When the point P lies vertically below the line load x=0

$$\sigma_z = \frac{2q'}{\pi z}$$

Problems

1)A line load of 100KN/m run extend to a long distance .Determine the intensity of vertical stress at a point 2m below the surface for the following two cases.

i)Directly under the line load

ii)At a distance of 2m perpendicular to the line load. Use Boussinesq theory

Solution:

q`=100 KN/m

case i)z=2m,x=0

$$\sigma_z = \frac{2q'}{\pi z}.\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$$

$$\sigma_z = \frac{2x100}{\pi x2}.\left[\frac{1}{1+\left(\frac{0}{2}\right)^2}\right]^2$$

$$\sigma_z = 31.83\ KN/m^2$$

case i)z=2m,x=2

$$\sigma_z = \frac{2x100}{\pi x2}.\left[\frac{1}{1+\left(\frac{2}{2}\right)^2}\right]^2$$

$$\sigma_z = 7.96\ KN/m^2$$