MATRIX OF LINEAR TRANSFORMATION WITH STANDARD BASES

1. Find the matrix of the linear transformation $T: R^2 \rightarrow R^2$ given by $T(a,b) = (2a - 3b, a + b)$ relative to the basis (i)$\{(1,0),(0,1)\}$ (ii) $\{(2,3),(1,2)\}$

Solution

Given, $T(a,b) = (2a - 3b, a + b)$

(i)The standard bases of R^2 is $\beta = \gamma = \{(1,0),(0,1)\}$

Given, $T(a,b) = (2a - 3b, a + b)$

∴the matrix of the linear transmission is $[T]_\beta^\gamma = \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$

(ii)the basis is $\beta = \{(2,3),(1,2)\}$

$$v_1 = (2,3), v_2 = (1,2)$$

$$T(a,b) = (2a - 3b, a + b)$$

$$T(v_1) = T(2,3)$$

$$= (2(2) - 3(3), 2 + 3)$$

$$= (-5,5)$$

The first column of the matrix of T is $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$

$$T(v_2) = T(1,2)$$

$$= (2(1) - 3(2), 1 + 2)$$

$$= (-4,3)$$

The second column of the matrix of T is $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

Matrix of T is $\begin{bmatrix} -5 & -4 \\ 5 & 3 \end{bmatrix}$

2. Let $T: V_2(R) \to V_3(R)$ and $U: V_2(R) \to V_3(R)$ be the linear transformations respectively defined by $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$ and $U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$. Let β and γ be the standard bases of $V_2(R)$ and $V_3(R)$ respectively. Verify $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$

Solution:

Given, $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$

Since β and γ be the standard bases of $V_2(R)$ and $V_3(R)$

the matrix corresponds to $\beta = [T]_\beta^\gamma = \begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 2 & -4 \end{bmatrix}$ ..... (1)

Given, $U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$.

Since β and γ be the standard bases of $V_2(R)$ and $V_3(R)$

the matrix corresponds to $\beta = [U]_\beta^\gamma = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 2 \end{bmatrix}$ ..... (2)

$$(1)+(2) \Rightarrow [T]_{\beta}^{\gamma}+[U]_{\beta}^{\gamma}=\begin{bmatrix}2 & 2\\2 & 0\\5 & -2\end{bmatrix} \ldots\ldots(3)$$

$$(T+U)(a_1,a_2)=(a_1+3a_2,0,2a_1-4a_2)+(a_1-3a_2,2a_1,3a_1+2a_2)$$

$$=(a_1+3a_2+a_1-a_2,0+2a_1,2a_1-4a_2+\ 3a_1+2a_2)$$

$$=(2a_1+2a_2,2a_1,5a_1-2a_2)$$

Since β and γ be the standard bases of $V_2(R)$ and $V_3(R)$

the matrix corresponds to $\beta=[T+U]_{\beta}^{\gamma}=\begin{bmatrix}2 & 2\\2 & 0\\5 & -2\end{bmatrix} \ldots\ldots(4)$

$$From\ (3)\ and\ (4) \Rightarrow [T+U]_{\beta}^{\gamma}=[T]_{\beta}^{\gamma}+[U]_{\beta}^{\gamma}$$

3. Let $T: V_2(R) \to V_3(R)$ be the linear transformations defined by $T(a_1,a_2)=(a_1+3a_2,0,2a_1-4a_2)$. Let β and γ be the standard bases of $V_2(R)$ and $V_3(R)$ respectively. Verify $[\alpha T]_{\beta}^{\gamma}=\alpha[T]_{\beta}^{\gamma}$

Solution

Given, $T(a_1,a_2)=(a_1+3a_2,0,2a_1-4a_2)$

Since β and γ be the standard bases of $V_2(R)$ and $V_3(R)$

the matrix corresponds to $\beta=[T]_{\beta}^{\gamma}=\begin{bmatrix}1 & 3\\0 & 0\\2 & -4\end{bmatrix}$

the matrix corresponds to $\beta = \alpha[T]_\beta^\gamma = \alpha \begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 2 & -4 \end{bmatrix}$

$$= \begin{bmatrix} \alpha & 3\alpha \\ 0 & 0 \\ 2\alpha & -4\alpha \end{bmatrix} \dots (1)$$

We have, $T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$

$$\therefore \alpha T(a_1, a_2) = \alpha(a_1 + 3a_2, 0, 2a_1 - 4a_2)$$

$$(\alpha T)(a_1, a_2) = (\alpha a_1 + 3\alpha a_2, 0, 2\alpha a_1 - 4\alpha a_2)$$

Since β and γ be the standard bases of $V_2(R)$ and $V_3(R)$

the matrix corresponds to $\beta = \alpha[T]_\beta^\gamma = \begin{bmatrix} \alpha & 3\alpha \\ 0 & 0 \\ 2\alpha & -4\alpha \end{bmatrix} \dots (1)$

$$From\ (1)\ and\ (2) \Rightarrow\ [\alpha T]_\beta^\gamma = \alpha[T]_\beta^\gamma$$

4. **Let $T: P_3(R) \to P_2(R)$ be the linear transformations defined by $T(f(x)) = f'(x)$. Let β and γ be the standard bases of $P_3(R)$ and $P_2(R)$ respectively. Then find $[T]_\beta^\gamma$**

Solution :

Let, $\beta = \{1, x, x^2, x^3\}$ be the standard bases of $P_3(R)$

Let, $\gamma = \{1, x, x^2\}$ be the standard bases of $P_2(R)$

Let, $w_1 = 1, w_2 = x, w_3 = x^2$

Given, $T(f(x)) = f'(x)$.

Let, $(f(x)) = 1$. Then $f'(x) = 0$

$$T(1) = T\big(f(x)\big) = f'(x) = 0 = 0.1 + 0.x + 0.x^2$$

$$= 0.w_1 + 0.w_2 + 0.w_3$$

The first column of $[T]_\beta^\gamma = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Let, $(f(x)) = x$. Then $f'(x) = 1$

$$T(x) = T\big(f(x)\big) = f'(x) = 1 = 1.1 + 0.x + 0.x^2$$

$$= 1.w_1 + 0.w_2 + 0.w_3$$

The second column of $[T]_\beta^\gamma = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Let, $(f(x)) = x^2$. Then $f'(x) = 2x$

$$T(x^2) = T\big(f(x)\big) = f'(x) = 2x = 0.1 + 2.x + 0.x^2$$

$$= 0.w_1 + 2.w_2 + 0.w_3$$

The third column of $[T]_\beta^\gamma = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

Let, $(f(x)) = x^3$. Then $f'(x) = 3x^2$

$$T(x^3) = T\big(f(x)\big) = f'(x) = 3x^2 = 0.1 + 0.x + 3.x^2$$

$$= 0.w_1 + 0.w_2 + 3.w_3$$

The fourth column of $[T]_\beta^\gamma = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

So, $[T]_{\beta}^{\gamma} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$