5.1 STATE VARIABLE REPRESENTATION

The state variable approach is a powerful tool for the analysis and design of control system. The analysis and design of the following system can be carried using state space method.

STATE SPAE FORMULATION

The **state** of a dynamic system is a minimal set of variables (known as state variables) such that the knowledge of these variables at $t = t_o$ together with the knowledge of the inputs for $t > t_0$, completely determines the behavior of the system for $t > t_0$.

A set of variables which describes the system at any time instant are called **state variables**.

In the state variable formulation of a system, in general, a system consists of m-inputs, p-outputs and n-state variables.

Let, state variable = $x_1(t), x_2(t), x_3(t), \ldots\ldots x_n(t)$.

Input variables = $u_1(t), u_2(t), u_3(t), \ldots\ldots u_m(t)$.

Output variables = $y_1(t), y_2(t), y_3(t) \ldots\ldots y_p(t)$.

The different variables may be represented by the vectors as shown below

$$\text{input vector } U(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}; \qquad \text{output vector } Y(t) = \begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_p(t) \end{bmatrix}$$

$$\text{state variable vector } X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ X_n(t) \end{bmatrix}$$

STATE EQUATIONS

The state variable representation can be arranged in the form of n number of first order differential equations as shown below

$$\frac{dx_1}{dt} = \dot{x}_1 = f_1(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m)$$

$$\frac{dx_2}{dt} = \dot{x}_2 = f_2(x_1, x_2, \ldots\ldots, x_n, u_1, u_2, \ldots, u_m)$$

$$\frac{dx_n}{dt} = \dot{x}_n = f_n(x_1, x_2, \ldots\ldots, x_n, u_1, u_2, \ldots\ldots, u_m)$$

The n numbers of differential equation as may be written in vector notation as *X(t) =*

*f*X(t), U*(t)+

The set of all possible values which the input vector U(t) an have at time t forms the input space of the system.

STATE MODEL OF LINEAR SYSTEM

The state model of a system consists of the state equation and output equation. The state equation of the system is a function of state variables and inputs as defined by equation *X(t) = f{X(t), U(t)+*

The state model of a system consists of state equation and output equation. (or) the state equation and output equation together called as state model of the system.

$$X(t) = A\,X(t) + B\,U(t) \text{ ----------------- } \textit{state equation}$$

$$Y(t) = C\,X(t) + D\,U(t) \text{ ------------------ } \textit{output equation}$$

Where

X (t) = state vector of order (n x 1)

U (t) = Input vector of order (m x 1)

A = System matrix of order (n x n)

B = Input matrix of order (n x m)

Y (t) = Output vector of order (p x 1)

C = Output matrix of order (p x n)

D = Transmission matrix of order (p x m)

The matrix form of state equation is

		-0-	0-12 ■ ■■ ^ln"	-x_r-		"bn	^12 ■	■■ b_{lm}'	rUi
*2		a21	a22 ■ ■■ a2n	*2		^21	^22 ■	■■ ^2m	U_2
*3	−	a31	a32 ■■ a3n	*3	+	^31	^32 ■	■■ ^3m	U_3
-				- X					
0⁄n		-	an2 -	n-		-	^n2	- Unm	m-

The matrix form of output equation

$$\begin{bmatrix} y_1 \\ y_2 \\ y_2 \\ \vdots \\ y_p \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ c_{31} & c_{32} & \cdots & c_{3n} \\ \vdots & & & \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ d_{31} & d_{32} & \cdots & d_{3m} \\ \vdots & & & \\ d_{p1} & d_{p2} & \cdots & d_{pm} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_m \end{bmatrix}$$