UNIT IV
SAMPLING

Introduction

In digital communication system input signals should be in digital form so that DSP can be employed on the signals. Electrical signal at the output of the transducer needs to be converted into sequential of digital signals. The block performing this task is formatter. To represent digital signals into few digits as possible, a coding system can be employed. This minimizes the number of digits. This process is called source coding and the block is called source encoder. This block compresses or minimizes the number of the digits for signals transmission. To reduce the noise in the communication cannel some redundancy bits is added with the message. This is done by channel encoder.

Base band processor

The channel encoded signals are not modulated. This transmission takes place in base band. For proper detection at the receiver and to reduce noise some line coding is used. Some pulse shaping is also done. Some special filter are also used to combat noise. All these are collectively called as baseband processor. This is in case of low speed wired transmission fixed telephony. For high speed data the digital signals to be modulated.

Band pass processor

The primary purpose of this band pass modulator is to map the digital signals into high frequency analog signals. Efficiency (spectral efficiency) is calculated according to number of bits send per second. The two block are (BASEBAND/BANDPASS) are mutually exclusive blocks. In the communication channel the transmitted signals get corrupted by random noise .the noise is from various sources .Thermal noise, shot noise, electro magnetic interference .At the receiver band pass modulator block processes the transmitted (corrupted)waveform and maps them back to sequence of number. In case of baseband the task of converting back the line coded pulse waveform to transmitted data sequence is carried out by baseband decoder block.

Channel decoder

Its used to reconstruct to the original sequence from the channel encoded digital by removing the extra added redundancy bits.

Source decoder

This estimates the digital signal.. In the De formatter if the original information source was not in digital data form ,this block needed to convert back the digital data to discrete or analog form. The Output transducer Converts the estimate of digital signal to analog signal.

Sampling

A message signal may originate from a digital or analog source. If the message signal is analog in nature, then it has to be converted into digital form before it can transmit by digital means. The process by which the continuous-time signal is converted into a discrete–time signal is called Sampling. Sampling operation is performed in accordance with the sampling theorem.

Sampling Theorem For Low-Pass Signals

Statement: - "If a band –limited signal g(t) contains no frequency components for $|f| > W$, then it is completely described by instantaneous values g(kTs) uniformly spaced in time with period $T_s \leq 1/2W$. If the sampling rate, fs is equal to the Nyquist rate or greater ($f_s \geq 2W$), the signal g(t) can be exactly reconstructed.

Formatting can be done in 3 steps

Sampling-------- > discretization in time

Quantization--------- >discretization in amplitude

Encoding-------- >obtaining quantized value

Sampling discretizes an analog signal in time domain .hence instead of continuous time wave form we get continuous values of signal at discrete point of time,

Analog to Digital Conversion

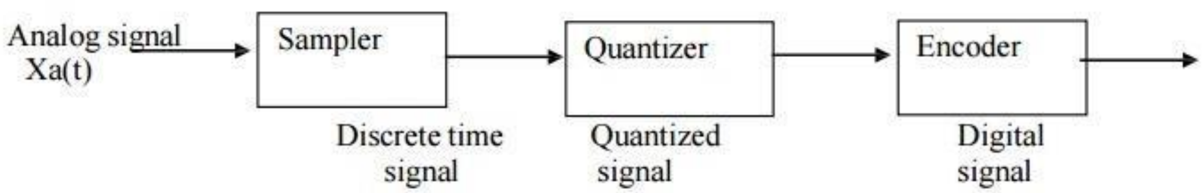

Figure 5.1.1 Analog to Digital Conversion

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Sampling theorem Definition

A bandlimited signal having no spectral components above fm Hz can be determined uniquely by values sampled at uniform intervals of $Ts \leq 1/2fm$ sec

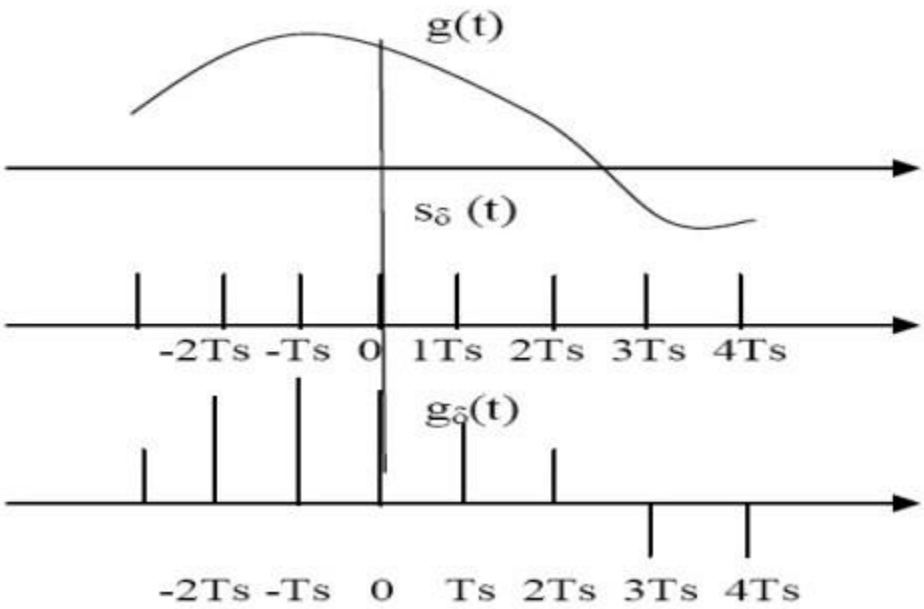

Figure 5.1.2 Sampling Process

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Nyquist rate

Minimum sampling frequency needed to reconstruct the analog signal from sampled waveform

$$fs \leq 2\,fm$$

Impulse sampling

Let as sample an analog waveform x(t) by a sequence of unit impulses ($\delta\ fn$) Assume that the spectrum of x(t) is band limited ,it is zero outside the interval -fm<f<fm

We sample x(t) at times t=nTs by aperiodic train of delta ($X\delta\,(t)\,fn$) can be represented as

$$X\delta\,(t) = \sum_{n=-\infty}^{\infty} \delta\,(t - nTs)$$

Where Ts =1/2fm-----> to satisfy Nyquist criterion

The Fourier transform of the impulse train $X\delta(f) = 1/Ts\sum_{n=-\infty}^{\infty}\delta(t-nfs)$,Fs = 1/Ts

The sampling can be modeled mathematically as a product of $X(t)$ with $X\delta(t)$

$$Xs(t) = X(t)X\delta(t)$$

$$= \sum_{n=-\infty}^{\infty} X(t)\delta(t-nTs)$$

$$== \sum_{n=-\infty}^{\infty} X(nTs)\delta(t-nTs)$$

Now the spectrum of the sampled signal $X\delta(f)$ is

$$Xs(f) = X(f)X\delta(f)$$

$$= X\delta(f)*1/Ts\sum_{n=-\infty}^{\infty}\delta(t-nfs)$$

$$= 1/Ts\sum_{n=-\infty}^{\infty} X(f)*\delta(t-nfs)$$

$$= 1/Ts\sum_{n=-\infty}^{\infty} X(t-nfs)$$

The sampled function has the same spectrum within a constant factor 1/Ts. This is within same as that of the spectrum of the input band -fm<f<fm. This spectrum repeats periodically in frequency every fs Hz. When the sampling rate just satisfies Nyquist rate just satisfies Nyquist rate fs=2fm,the upper end of each replicate touches the lower end of the higher band neighbor replicate .Here the extraction of the original waveform is possible without filtering. In practical digital system fs always>Nyquist rate for filtering rate

Natural sampling

Uses flat –top rectangular pulses of finite width to sample analog waveform. This is called natural sampling, because top of the each pulse in the sampled sequence remains the shape of the original signal during that pulse interval .

Note the flat –top pulse train of width T as $Xp(t)$

$$Xp(t) = \sum_{n=-\infty}^{\infty} 1/Ts \prod t - nTs/Ts \quad ,Ts = 1/2fm$$

The flat top pulse train is a periodic waveform .In Fourier series it is represented by

$$Xp(t) = \sum_{n=-\infty}^{\infty} Cn\,e^{j2\pi fst}$$

The Fourier coefficient Cn of the pulse train is Cn =1/Ts sinc (nTs/Ts) . The magnitude of the pulse train has the train has the character of the sinc shape.

The sampled waveform can be written as

$$Xp(t) = X(t) \sum_{n=-\infty}^{\infty} Cn\,e^{j2\pi fst}$$

The transform $Xs(f)$ of the sampled waveform is given by

$$Xp(t) = X(t) \sum_{n=-\infty}^{\infty} F\{X(t) Cn\,e^{j2\pi fst}\}$$

Using the frequency translation property of F.T

$$Xs(f) = Cn \sum_{n=-\infty}^{\infty} X(t - nfs)$$

The spectrum of natural sampled wave form is a replication of $X(f)$ periodically repeated in every frequency fs H. This is similar to impulse sampling .One difference the spectrum is weighted by Fourier series coefficient of the pulse train is compared to constant value of the impulse samples for natural sampling the pulse width should be reduced. It can be shown calculating Cn value for flat –top pulses with pulse width Ts approaching zero.

$$Cn = 1/Ts \int_{-Ts/2}^{Ts/2} X_\delta(t)\, e^{j2\pi fst} dt$$

With ih the range of integration Ts/2 to Ts/2 the only conversion of $X\delta(f)$

$$Cn = 1/Ts \int_{-Ts/2}^{Ts/2} \delta(t)\, e^{j2\pi fst} dt = 1$$

The spectrum of flat pulse approaches the spectrum of impulse samples for Ts-→0

Sampler implementation

Sampler implementation is done by sample and hold circuit In this circuit a Switch and store mechanism is used to form a sequences of samples These samples are analog waveform. They look like PAM, the amplitude of the pulses vary continuously recovery. The original analog waveform can be recovered from these PAM samples by low pass filtering technique.

If fs<fnyquist =>under sample then aliasing occurs

Aliasing =>overlapping of adjacent spectrum replicates if fs<fnyquist

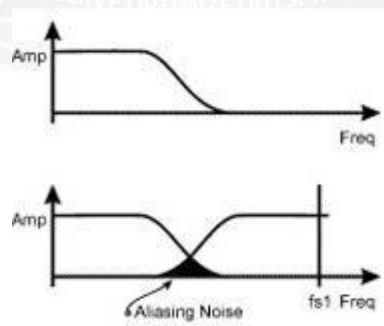

Figure 5.1.3 Aliased spectral components

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This aliased spectral components represent data appear in the frequency band fs-fm .Due to the practical difficulties in achieving Nyquist rate of sampling

,sometimes under sampling is done There are two ways to avoid aliasing both methods using antialiasing filter

The two methods are

i)pre filtering antialiasing

ii)post filtering

i)Pre filtering

Here the analog signal itself is pre filtered so that the new maximum frequency is fm

fm is reduced to fs/2 or less. Therefore sampled spectrum does not overlap

ii)Post filtering

In the method aliasing terms are eliminated after sampling wit help of low pass filter

the cut off frequency fm" needs to be less than fs-fm

Pre filtering in spectral domain

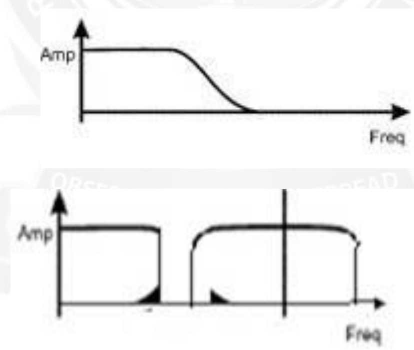

Figure 5.1.4 Pre filtering in spectral domain

Diagram Source Brain Kart

The antialiasing filter are commonly analog filters. There is alternate to them. We can oversample the signal there by removing antialiasing. The large number of samples can be filtered by digital filters instead of analog filter . This is the economic solution for sampling or the economy here A/D converter are required.

Without oversampling

In this method the signal passes through a high performance analog low pass antialiasing filter to limit its bandwidth. The antialiasing filter has a passband equal to the signal bandwidth plus the transition bandwidth. For transition bandwidth of f_T the Nyquist sampling rate $2f_m$ becomes $2f_m+f_T$. The additional spectral interval does not represent any useful signal content . It protect the signal by reserving free spectral interval between two spectral replicas. The filtered signal is sampled at Nyquist rate for the approximately Band limited signal. The samples are processed by D/A converter that's maps the continuous valued samples to a finite list of discrete output levels.

Post filtering

The disadvantages of both these antialiasing filter is same information always lost due to filtering. All realizable filters requires a nonzero bandwidth for transmission between the passband and stop band. This is known as transition bandwidth. Filter complexity and cost rise sharply with narrower transition band width.

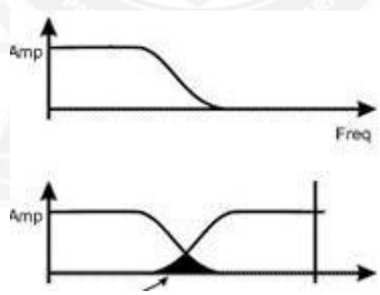

Figure 5.1.5 Post filtering for Sampling

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If we want to keep the sample rate down we should go for narrower transition bandwidth trade off is required between the cost of small transition bandwidth and cost of higher sampling rate .If we have 20%transitiuon B.W of the antialiasing filter, we have a practical Nyquist sampling rate.

$$fs \geq 2.2\ fm$$

With over sampling

Step:1

The signal is passed through a low performance analog LPF . Prefilter to limit its bandwidth. So bandwidth is reduced.

Steps:2

The prefilterd signal is over sampled at a much higher sampling rate higher than the Nyquist rate for the bandlimited signal.

Steps:3

The samples are processed by an A/D converter that maps continuous valued samples to a finite list of discrete output levels.

Steps:4

The digital samples bare then processed by a high performance a low cost digital filter to reduce the bandwidth of the digital samples

Steps:5

The sample rate at the output of the digital filter is reduced in proportion to the and width reduction obtained by the digital filter.

Cloud digital signal; processing techniques combine the filtering and resampling into a single structure .they also compensate the distortion in the first level and improve the signal quality.

Aliasing and signal reconstruction:

Nyquist's theorems as stated above and also helps to appreciate their practical implications. Let us note that while writing Eq.(1.4), we assumed that x(t) is an energy signal so that its Fourier transform exists. With this setting, if we assume that x(t) has no appreciable frequency component greater than W Hz and if fs> 2W, then

Eq.(1.4) implies that Xs(f), the Fourier Transform of the sampled signal Xs(t) consists of infinite number of replicas of X(f), centered at discrete frequencies n.fs, $-\infty < n < \infty$ and scaled by a constant fs= 1/Ts.

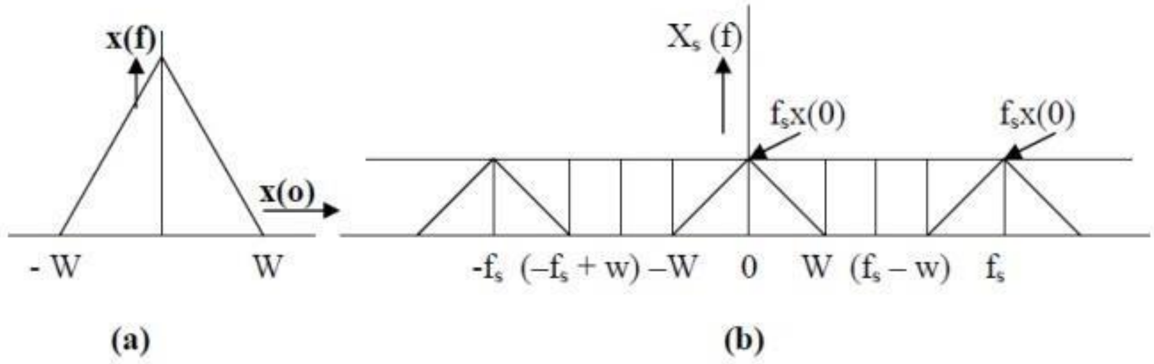

Figure 5.1.6 Spectra of Analog signal and its sampled Signal

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Fig. 5.5 indicates that the bandwidth of this instantaneously sampled wave xs(t) is infinite while the spectrum of x(t) appears in a periodic manner, centered at discrete frequency values n.fs. Part – I of the sampling theorem is about the condition fs> 2.W i.e. (fs – W) > W and (– fs + W) < – W. As seen from Fig. 1.2.1, when this condition is satisfied, the spectra of xs(t), centered at f = 0 and f = ± fs do not overlap and hence, the spectrum of x(t) is present in xs(t) without any distortion. This implies that xs(t), the appropriately sampled version of x(t), contains all information about x(t) and thus represents x(t).

The second part suggests a method of recovering x(t) from its sampled version xs(t) by using an ideal lowpass filter. As indicated by dotted lines in Fig. 1.2.1, an ideal lowpass filter (with brick-wall type response) with a bandwidth $W \leq B < (fs – W)$, when fed with xs(t), will allow the portion of Xs(f), centered at f = 0 and will reject all its replicas at f = n fs, for $n \neq 0$. This implies that the shape of the continuous time signal xs(t), will be retained at the output of the ideal filter.