The Theory of Inference

The main aim of logic is to provide rules of inference, or principles of reasoning. Here, we are concerned with the inferring of a conclusion from given premises.

We are going to check the logical validity of the conclusion, from the given set of premises by making use of Equivalence rule and implication rule, the theory associated with such things is called inference theory.

Direct Method

When a conclusion is derived from a set of premises by using the accepted rules of reasoning, then such a process of derivation is called a direct proof.

Indirect method of proof:

(i) Method of Contradiction:

In order to show that a conclusion C follows logically from the premises

$H_1, H_2, \ldots, H_m$, we assume that C is false and consider $\neg C$ as an additional premises. If the new set of premises gives contradict value, then the assumption $\neg C$ is true does not hold simultaneously with $H_1 \wedge H_2, \wedge \ldots \wedge H_m$.

Therefore, C is true whenever $H_1 \wedge H_2, \wedge \ldots \wedge H_m$ ids true. Thus C follows logically from the premises $H_1, H_2, \ldots, H_m$.

(ii) Method of contrapositive:

In order to prove $H_1 \wedge \ H_2, \wedge \ldots \wedge \ H_m \Rightarrow C$, if we prove

$\neg C \Rightarrow \neg(H_1 \wedge \ H_2, \wedge \ldots \wedge \ H_m)$ then the original problem follows. This method is called contrapositive method.

Rules of Inference

Rule P: A premise may be introduced at any point in the derivation.

Rule T: A formula S may be introduced at any point in a derivation if S is tautologically implied by any one or more of the preceding formulas.

Rule CP: If S can be derived from R and set of premises, then $R \rightarrow S$ can be derived from the set of premises alone.

Remark:

(i) Rule CP means Rule of Conditional Proof.

(ii) Rule CP is also called the deduction theorem.

Implication Rule:

$P, P \rightarrow Q \Rightarrow Q$	Modus Phones
$\neg Q, P \rightarrow Q \Rightarrow \neg P$	Modus Tollens
$\neg P, P \vee Q \Rightarrow Q$	Disjunctive syllogism
$P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$	Hypothetical syllogism (or) chain rule

$P, Q \Rightarrow P \wedge Q$	Simplification rule
$P, Q \Rightarrow P \vee Q$	Addition rule
$P \wedge \neg Q \Rightarrow \neg(P \rightarrow Q)$	Equivalence rule

Note:

In the derivation, we should use all the rules but exactly once. Also, the order is immaterial.

1.Demonstrate that R is a valid inference from the premises $\boldsymbol{P \rightarrow Q, Q \rightarrow R}$ & $\boldsymbol{P}$

Solution:

$\{1\}$	$1) P \rightarrow Q$	Rule P
$\{2\}$	$2) Q \rightarrow R$	Rule P
$\{1, 2\}$	$3) P \rightarrow R$	Rule T $(P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R)$
$\{4\}$	$4) P$	Rule P
$\{1, 2, 4\}$	$5) R$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$

2. Show that $\boldsymbol{\neg P}$ follows logically from the premises $\boldsymbol{\neg(P \wedge \neg Q), (\neg Q \vee R)}$ &$\boldsymbol{\neg R}$

Solution:

Given premises are $\neg(P \wedge \neg Q), (\neg Q \vee R), \neg R$

Conclusion: &$\neg R$

$\{1\}$	1)$\neg(P \wedge \neg Q)$	Rule P
$\{2\}$	2))$\neg P \vee Q$	Rule T (Demorgan's law)
$\{1\}$	3)$P \rightarrow Q$	Rule T ($P \rightarrow Q \Leftrightarrow \neg P \vee R$)
$\{4\}$	4)$\neg Q \vee R$	Rule P
$\{4\}$	5)$Q \rightarrow R$	Rule T ($P \rightarrow Q \Leftrightarrow \neg P \vee R$)
$\{1, 4\}$	6) $P \rightarrow R$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
$\{7\}$	7) $\neg R$	Rule P
$\{1, 4, 7\}$	8) $\neg P$	Rule T $\neg Q, P \rightarrow Q \Rightarrow \neg P$

Consistency and Inconsistency of Premises

A set of formulae $H_1, H_2, \ldots, H_m$ is said to be inconsistent if their conjunction implies contradiction.

i.e., $H_1 \wedge H_2 \wedge \ldots \wedge\ H_m \Rightarrow R \wedge \neg R$ for some formulae R.

Note:$R \wedge \neg R \Leftrightarrow F$

Consistent:

A set of formulae $H_1, H_2, \ldots, H_m$ is said to be consistent if their conjunction implies tautology.

Inconsistent:

A set of formula $H_1, H_2, \ldots, H_m$ is said to be consistent if it is not inconsistent.

1.Show that $P \to Q, P \to R, Q \to \neg R$ & P are inconsistent.

Solution:

$\{1\}$	$1)P \to Q$	Rule P
$\{2\}$	$2)Q \to \neg R$	Rule P
$\{1, 2\}$	$3)P \to \neg R$	Rule T
$\{4\}$	$4)P$	Rule P
$\{1, 2, 4\}$	$5)\neg R$	Rule T
$\{6\}$	$6)P \to R$	Rule P
$\{1, 2, 4, 6\}$	$7)\neg P$	Rule T
$\{1, 2, 4, 6\}$	$8)P \wedge \neg P$	Rule T

Which is nothing but false value.

Hence given set of premises are inconsistent.

2. Prove that $P \to Q, Q \to R, S \to \neg R$ & $P \wedge S$ are inconsistent.

Solution:

$\{1\}$	$1) P \rightarrow Q$	Rule P
$\{2\}$	$2) Q \rightarrow R$	Rule P
$\{1, 2\}$	$3) P \rightarrow R$	Rule T
$\{4\}$	$4) S \rightarrow \neg R$	Rule P
$\{ 4\}$	$5) R \rightarrow \neg S$	Rule T
$\{1, 2, 4\}$	$6) P \rightarrow \neg S$	Rule T
$\{1, 2, 4\}$	$7) \neg P \vee \neg S$	Rule T
$\{1, 2, 4\}$	$8) \neg(P \wedge S)$	Rule T
$\{ 9\}$	$9) P \wedge S$	Rule P
$\{1, 2, 4, 9\}$	$10) (P \wedge S) \wedge \neg(P \wedge S)$	Rule T

Which is nothing but false value.

Hence given set of premises are inconsistent.

3. Prove that $\boldsymbol{a \rightarrow (b \rightarrow c), d \rightarrow (b \wedge \neg c)}$, $\&\ a \wedge d$ are inconsistent.

Solution:

$\{1\}$	$1) a \wedge d$	Rule P
$\{1\}$	$2) a, d$	Rule T

$\{3\}$	3)$a \to (b \to c)$	Rule P
$\{1,3\}$	4)$b \to c$	Rule T
$\{1,3\}$	5)$\neg b \vee c$	Rule T
$\{6\}$	6)$d \to (b \wedge \neg c)$	Rule P
$\{6\}$	7)$\neg(b \wedge \neg c) \to \neg d$	Rule T
$\{6\}$	8))$\neg b \vee c \to \neg d$	Rule T
$\{1,3,6\}$	9) $\neg d$	Rule T
$\{1,3,6\}$	10) $d \wedge \neg d$	Rule T

Which is nothing but false value.

Hence given set of premises are inconsistent.