

ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING



ME3391 ENGINEERING THERMODYNAMICS

UNIT III AVAILABILITY AND IRREVERSIBILITY

Notes on Availability and Irreversibility

You are already familiar with the basic second law concepts of a heat engine. In particular, the heat engine draws heat from a source at T_H , converts some of this heat into useful work, and discards the rest to an environment at T_C . The second law placed a limit on the thermal efficiency of the heat engine; the maximum efficiency corresponding to a reversible cycle for which

$$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

and the maximum work that could be obtained from the heat source would be

$$\dot{W}_{revHE} = \dot{Q}_H \eta_{rev} = \dot{Q}_H \left(1 - \frac{T_L}{T_H}\right)$$

The maximum work \dot{W}_{revHE} can be viewed as the *available work* or, alternatively, the *useful work potential*, of the source \dot{Q}_H . It represents the theoretical maximum work that could be derived from the source, and would be obtained if all processes in the heat engine were completely reversible. The available work (or *availability*), in this case, depends both on the temperature of the heat source and the temperature of the environment.

What we want to do here is extend (or generalize) the concept of availability beyond an application to heat engines. As in the past, we will deal separately with closed and open systems in developing availability concepts. With regard to closed systems, an availability analysis seeks to quantify the work potential (i.e., availability) of system at a specified initial state. To derive work from this system, the system would need to undergo a process to a final state. The final state, in our availability analysis, will always correspond to the environment conditions (often called the *dead state*) and characterized by properties T_0, P_0, s_0, \dots

An availability analysis applied to an open system typically attempts to derive the maximum work potential of a work producing/consuming device such as a turbine or compressor. As is the case with the closed system, the availability will depend on the environment conditions.

Reversible Work and Irreversibility

Consider a system which undergoes a process from state 1 to 2. Let the actual amount of work transferred during this process be denoted as W_{1-2} . Say now the system undergoes a reversible process between the same two states, which yields a work of W_{1-2}^{rev} . Since the reversible process corresponds to the maximum value of useful work for any process between states 1 and 2, it follows that

$$W_{1-2}^{rev} \geq W_{1-2}$$

If the system expands (and produces work), the reversible work will be larger than the actual work. And if the system is compressed (requiring a work input), the reversible work will be smaller in magnitude than the actual. If the final state happens to be the dead state (the environment state), the reversible work would correspond to the availability of the system at the initial state.

The *irreversibility* of the process between 1 and 2, denoted as I_{1-2} , is simply the difference between the reversible and actual works;

$$I_{1-2} = W_{1-2}^{rev} - W_{1-2} \geq 0 \tag{1}$$

The irreversibility, which obviously has units of energy, represents the lost potential of the process.

Often the irreversibility associated with a process is not obvious. Here are a couple of examples which illustrate:

Example:

A 500 kg iron block is initially at 200°C and is allowed to cool to 27°C by transferring heat to the surrounding air at 27°C. Determine the reversible work and the irreversibility.

Solution: There is no actual work involved in this process; the block simply cools to the ambient temperature. Because of Eq. (1), the irreversibility will simply be the reversible work.

You may recall a problem similar to this from the homework on heat engines. As the block cools, the heat from the block could (in theory) be transferred into a reversible HE operating between the current temperature of the block and the environment temperature. The amount of work produced by the heat would be

$$\delta W_{rev} = \delta Q \eta_{rev} = \delta Q \left(1 - \frac{T_0}{T}\right)$$

where T_0 and T are the environment and current block temperatures. The δW and δQ quantities represent ‘path differentials’; you can view them simply as ‘small’ amounts of work and heat that are transferred during some point in the process.

From the first law,

$$\delta Q = -mC dT$$

Why the minus sign? You should figure this out yourself. The total reversible work is then

$$\begin{aligned} W_{rev} &= \int_{T_1}^{T_0} \delta W_{rev} = -mC \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T}\right) dT \\ &= mC \left((T_1 - T_0) + T_0 \ln \left(\frac{T_0}{T_1}\right) \right) \end{aligned}$$

Using $C = 0.45$ kJ/kg K for iron gives $W_{rev} = 8191$ kJ. Note that the first term in the result is the total heat transfer from the block to the engine, i.e.,

$$Q_B = mC(T_1 - T_0) = 38,925 \text{ kJ} \quad (2)$$

Only 21% of this heat (i.e., 8191/38,925) could have been theoretically converted into useful work. If the specified ambient temperature of 27°C is the lowest available environment temperature, the reversible work would also represent the availability.

The previous example used a *mechanical* analysis procedure to determine the reversible work. That is, the solution was obtained by incorporating a device (a perfect heat engine) to transform the heat flow from the block into useful work.

There is another procedure available to determine the reversible work; a procedure referred to as an *availability* method. This approach recognizes that the maximum work potential of the process would be obtained from a perfectly reversible process. And a reversible process would have no *net* change in entropy. This approach will yield the same result as the mechanical method – and it is somewhat more methodical – yet it does not explicitly account for the devices (heat engines, heat pumps) which may be required to obtain the theoretical best performance.

To illustrate this approach in the same problem, we first have to recognize the relevant energy transfers. Heat Q_B will flow from the block, and some of this will be converted into work W_{rev} . From the first law, the heat that flows into the environment is simply the amount of Q_B that is not converted, i.e.,

$$Q_0 = Q_B - W_{rev} \quad (3)$$

Observe that we are taking all energy transfers to be positive, i.e., we are not applying the usual first law sign convention.

The total entropy change is zero for the reversible process, and the total entropy change is that of the block and that of the environment:

$$\Delta S_B + \Delta S_0 = 0 \quad (4)$$

We can now apply formulas for the entropy changes. The block is a solid with a constant specific heat, and its temperature changes from $T_1 = 200^\circ\text{C} = 473\text{ K}$ to $T_0 = 27^\circ\text{C} = 300\text{ K}$. So

$$\Delta S_B = m C \ln\left(\frac{T_0}{T_1}\right) = -102.4\text{ kJ/K}$$

The environment entropy change will be

$$\Delta S_0 = \frac{Q_0}{T_0}$$

and putting this into Eq. (4) gives

$$Q_0 = -T_0 \Delta S_B = 30,734\text{ kJ}$$

The block heat transfer was computed in Eq. (2), and from Eq. (3), the reversible work is

$$W_{rev} = Q_B - Q_0 = 8191\text{ kJ}$$

which is the same value as obtained from the mechanical analysis – it has to be, as there is only one answer to the problem.

Here is another example which illustrates the two ways to approaching an analysis of an ‘ideal’ process.

Example:

The iron block discussed above is now used to maintain a house at 27°C when the outside temperature is $T_L = 5^\circ\text{C}$. Determine the maximum amount of heat which can be supplied to the house as the iron cools to 27°C .

Mechanical solution: You will probably think that the maximum heat will simply be the heat given off from the block as it cools, i.e., $Q_B = 38,925\text{ kJ}$. This is likely the practical answer, but it is not the theoretical maximum. Recall that the heat *could have* been used to power a reversible heat engine, and that this engine would have produced $W_{rev} = 8191\text{ kJ}$ of work. This work, in turn, could be used to power a reversible heat pump operating between T_L and T_0 . The schematic of this setup is shown in Fig. 1. The COP of the heat pump would be

$$COP_{rev} = \frac{1}{1 - T_L/T_0} = \frac{1}{1 - 278/300} = 13.64$$

and the heat supplied (to the house) by the heat pump would be $Q_{H,HP} = W_{rev} COP = 111,700\text{ kJ}$. The heat engine (which operates the heat pump) would also reject $Q_{L,HE} = Q_B - W_{rev} = 38,925 - 8191 = 30,734\text{ kJ}$ to the house. The total heat provided by the process would therefore be

$$Q_{net} = 30,734 + 111,700 = 142,434\text{ kJ}$$

which is quite a bit larger than the heat given off by the block! Again, this represents the theoretical best performance.

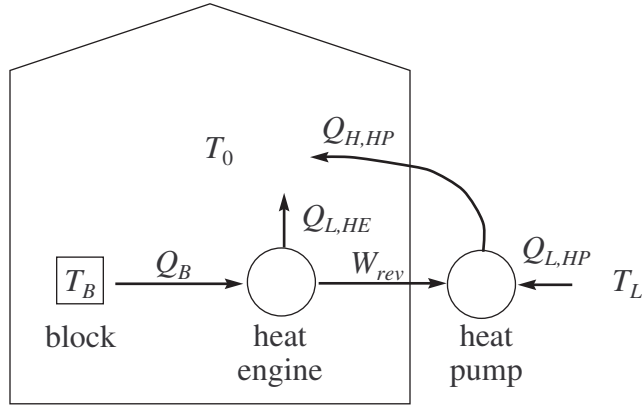


Figure 1: Reversible heating of the house with the block

Availability solution: The point here is to recognize where and how the entropies of the various components in the problem are changing. There are three components: the block, the house, and the outside. If the process is reversible, then the total entropy change is zero:

$$\Delta S_B + \Delta S_0 + \Delta S_L = 0 \quad (5)$$

Note that S_0 refers to the house entropy and S_L is the outside. The outside is typically taken to be the environment and would be given the 0 subscript; I'm trying to keep the notation consistent with the previous solutions.

The first two terms in Eq. (5) account for the total entropy change had the block simply cooled to the house temperature with no interaction with the outside. This number will be positive, as such a process is irreversible. To make the total entropy change zero, it follows that the outside entropy must decrease in an amount equal to the block + house entropy increase. The way to do this is to transfer an amount of heat Q_L from the outside and into the house. This is illustrated in Fig. 2.

The total heat transfer to the house will be $Q_B + Q_L$, and the resulting entropy increase of the house will be $(Q_B + Q_L)/T_0$. The block entropy change is given in the previous example, and from Eq. (5) we have

$$mC \ln \left(\frac{T_0}{T_1} \right) + \frac{Q_B + Q_L}{T_0} - \frac{Q_L}{T_L} = 0 \quad (6)$$

The last term is the entropy change of the outside. A minus sign is needed because the outside entropy will decrease (heat flows *from* the outside) and Q_L is taken to be positive. Solve the above equation for Q_L , to get

$$Q_L = \frac{\frac{Q_B}{T_0} + mC \ln \left(\frac{T_0}{T_1} \right)}{\frac{1}{T_L} - \frac{1}{T_0}} = 103,509 \text{ kJ}$$

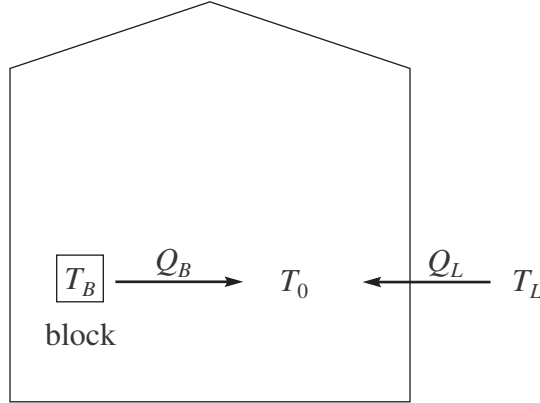


Figure 2: Relevant heat transfers for the availability analysis

and the theoretical maximum heat transfer to the house is $Q_B + Q_L = 142,434$ kJ: which is the same result, albeit without all the hardware.

The key issue here is that the best performance corresponds to the completely reversible process. The simple transfer of heat from the block to the house is an irreversible process (because it is heat transfer over a finite temperature difference). A combined HE/HP system offers a way to make the cooling of the block reversible. That is, if we reverse the process, the heat pump will become a heat engine operating between the house temperature ($T_0 = 27^\circ\text{C}$) and the outside temperature ($T_L = 5^\circ\text{C}$). And this heat engine will operate a heat pump (the reversed heat engine from the original case) which transfers heat from the house into the block to bring the block back to 200°C . Alternatively, we can dispense with the hardware, and view the process simply as the addition of heat from the outside (from some unknown means) in an amount necessary to cancel the entropy change of the block and the house. We know that this outside heat transfer will not happen ‘naturally’ (as this would be heat flow from cold to hot) and that some sort of heat engine/heat pump system would be required to accomplish the task. The availability solution merely identifies the best possible case without having to go through all the design.

The following sections will formalize the availability analysis to closed and open system. As the name implies, the objective of the analysis is to determine the maximum work potential of a process or a stream.

Closed system analysis

Say we have a system, initially at state 1 (T_1, P_1, V_1, \dots), which undergoes a process to state 2. The system is surrounded by an environment at T_0, P_0 , and exchanges heat only with this environment.

The first law has

$$Q_0 = U_2 - U_1 + W_{1-2} \quad (7)$$

in which Q_0 denotes the heat transfer exchanged with the environment during the process. The second law is

$$\Delta S_{net} = S_2 - S_1 + \Delta S_{surr} = S_2 - S_1 - \frac{Q_0}{T_0} \geq 0 \quad (8)$$

Observe that the change in entropy of the environment is the heat transfer to the environment ($= -Q_0$) divided by the environment temperature. The net change in entropy ΔS_{net} must be greater than or equal to zero. Eliminate the heat transfer from the previous two equations, which gives

$$W_{1-2} = T_0(S_2 - S_1) - (U_2 - U_1) - T_0\Delta S_{net} \quad (9)$$

This is the actual work transferred during the process.

If the process were *totally* reversible, the net change in entropy would be zero. The reversible work corresponding to this process would then be

$$W_{1-2}^{rev} = T_0(S_2 - S_1) - (U_2 - U_1) \quad (10)$$

This represents the maximum work potential of a process going from state 1 to 2 and exchanging heat only with the environment at T_0 . The heat transfer exchanged with the environment would also be different for the reversible process. By setting $\Delta S_{net} = 0$ in Eq. (8), we would have

$$Q_0^{rev} = T_0(S_2 - S_1) \quad (11)$$

which could have been obtained by applying the first law to Eq. (10). The point is that when we consider a reversible process between states 1 and 2, the heat and work transfers would be expected to change because heat and work are functions of the process path between states 1 and 2.

If, during the process, the system could also receive an amount of heat Q_H from a reservoir at T_H (this heat is denoted ${}_1Q_2$ on p. 350), the reversible work would be increased by an amount corresponding to the work for a reversible heat engine operating between T_H and T_0 . The more general form of the equation is therefore

$$W_{1-2}^{rev} = T_0(S_2 - S_1) - (U_2 - U_1) + Q_H \left(1 - \frac{T_0}{T_H}\right) \quad (12)$$

This is Eq. (10.17) in the text.

The *available* work is the maximum (reversible) work minus the work done against the surroundings by the system. For example, if the system expands it will do work against the surroundings by pushing the surroundings out. This work would not be available to us for other purposes, i.e., lifting a weight. Since the surroundings are at a constant pressure P_0 , it follows that

$$W_{1-2}^{surr} = P_0(V_2 - V_1) \quad (13)$$

and

$$W_{1-2}^{avail} = W_{1-2}^{rev} - W_{1-2}^{surr} = T_0(S_2 - S_1) - (U_2 - U_1) - P_0(V_2 - V_1) + Q_H \left(1 - \frac{T_0}{T_H}\right) \quad (14)$$

Note that the available work is not precisely the same as the system availability. The latter quantity refers to the available work if the system follows a reversible process from a given state to the dead state (the environment state), whereas the former refers to the available work obtained from an initial state to an arbitrary final state. Confusing, isn't it?

The irreversibility is the difference between the reversible and actual work transfer. The book shows that the irreversibility is

$$I_{1-2} = T_0 \left[S_2 - S_1 - \frac{Q_0}{T_0} - \frac{Q_H}{T_H} \right] = T_0 \Delta S_{net} \geq 0$$

Open system (CV) analysis

Consider a SSSF device with one inlet and one outlet and surrounded by an environment at T_0 . As was the case with the closed system, we want to distinguish between heat (per unit mass) exchanged with the environment (denoted as q_0) and heat exchanged with a reservoir at T_H (denoted as q_H). Both q_0 and q_H can be positive or negative depending on the direction of heat flow; they may also be zero. Neglecting KE and PE changes (these could be thrown in if needed), the first law gives the actual work per unit mass as

$$w = h_1 - h_2 + q_0 + q_H \quad (15)$$

where 1 and 2 are the inlet and exit states. The second law has the entropy generation per unit mass as

$$s_{gen} = (s_2 - s_1) - \frac{q_0}{T_0} - \frac{q_H}{T_H} \geq 0 \quad (16)$$

The three terms in the right-hand-side of the above equation account for the entropy change of the flow, the entropy change of the environment, and the entropy change of the heat reservoir. If q_0 is negative (which it typically is, indicating a heat loss from the CV to the environment) the environment entropy will increase as a result of the heat loss, and likewise for q_H .

A reversible process has a net entropy generation of zero. By eliminating q_0 from Eq. (16) and using the result in Eq. (15), the reversible work for the CV, per unit mass, would be

$$w^{rev} = T_0(s_2 - s_1) - (h_2 - h_1) + q_H \left(1 - \frac{T_0}{T_H}\right) \quad (17)$$

where 1 and 2 denote the inlet and exit states. This is Eq. 10.9 in the text. Note that heat which is transferred between the system and the environment, q_0 , will not contribute to Eq. (17); you could view q_0 as heat exchanged with a reservoir at T_0 , for which the contribution in Eq. (17) will be zero (i.e., $1 - T_0/T_0 = 0$). This issue comes up in example 10.2 in the text.

The *irreversibility* of a SSSF process, denoted as i (per unit mass), is the difference between the reversible and actual works. Subtracting Eq. (15) from Eq. (17) and using Eq. (16) results in

$$i = w^{rev} - w = T_0(s_2 - s_1) - q_H \frac{T_0}{T_H} - q_0 = T_0 s_{gen} \quad (18)$$

The irreversibility is a measure of the lost work potential in a process. A reversible process will have an irreversibility of zero (duh!).

The form of Eq. (17) suggests the definition of a new property of the flow. In particular, we could write Eq. (17) as

$$w^{rev} = \psi_1 - \psi_2 + q_H \left(1 - \frac{T_0}{T_H}\right) \quad (19)$$

where (in the most general sense – including KE and PE) the *flow availability* ψ is defined by

$$\psi = h - h_0 - T_0(s - s_0) + \frac{\mathcal{V}^2}{2} + g(Z - Z_0) \quad (20)$$

The flow availability is a property and is thus subject to the usual rules and behaviors involving properties (as distinct from path functions such as work and heat). The dead state properties h_0 , $T_0 s_0$, and gZ_0 (the dead state velocity is zero) have been included in the definition of ψ simply to make $\psi = 0$ when evaluated at the dead state – observe that these terms would cancel in $\psi_1 - \psi_2$ in Eq. (19). As the name implies, ψ is a measure of the available work potential of a flow, and it represents the maximum work that could be derived from the flow if it is sent through a process leading to the environment state.

Example: the gas turbine

A gas turbine receives air at $T_1 = 1200$ K, $P_1 = 8$ atm and exhausts to $T_2 = 700$ K, $P_2 = 1$ atm. The turbine is adiabatic, and kinetic energy changes are negligible. Calculate the specific work output of the turbine, the irreversibility, and the maximum available (i.e., reversible) work assuming an environment temperature of $T_0 = 298$ K.

We will assume that the air is an ideal gas with constant specific heats, with $C_P = 1.005$ kJ/kg K and $R = 0.287$ kJ/kg K. From the first law,

$$w = h_1 - h_2 = C_P(T_1 - T_2) = 502.5 \text{ kJ/kg}$$

This is the actual work provided by the turbine. The irreversibility is obtained from Eq. (18), with $q_H = 0$ in this case. For an ideal gas with constant specific heats,

$$i = T_0(s_2 - s_1) = T_0 \left(C_P \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right) \right) = 16.4 \text{ kJ/kg}$$

and, from Eq. (17),

$$w_{rev} = w + i = 518.9 \text{ kJ/kg}$$

The above results allow us to define an efficiency of the turbine, that being the ratio of the actual to reversible work. We will refer to this efficiency as the second law efficiency, $\eta_{t,II}$. For this example,

$$\eta_{t,II} = \frac{w}{w_{rev}} = 1 - \frac{i}{w_{rev}} = 0.968$$

This efficiency is different than the isentropic turbine efficiency that was discussed in Ch. 9. The second law efficiency compares the actual work to the maximum work that could be obtained *between the same inlet and exit states*. The isentropic efficiency, on the other hand, compares the actual work to the work produced by an isentropic turbine operating with the same inlet state and the same exit pressure. For the example at hand, the isentropic exit temperature would be

$$T_{2s} = T_1 \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = 662.5 \text{ K}$$

and the isentropic work and isentropic efficiency are

$$w_s = h_1 - h_{2s} \approx C_P(T_1 - T_{2s}) = 540.2 \text{ kJ/kg}$$

$$\eta_{t,s} = \frac{w}{w_s} = 0.930$$

The book refers to the isentropic efficiency as ‘the first law efficiency’, a reference which makes absolutely no sense to me.

As can be seen, the two definitions of the efficiencies give two different measures of performance. The isentropic efficiency deals more with the device (i.e., turbine, compressor) than the states leading into and out of it, and visa-versa for the second-law efficiency. For example, the isentropic efficiency tells us how much work we could have obtained if we replaced our actual turbine with a perfect (reversible) turbine that exhausts to the same exit pressure. The exit temperature will be different (lower) for the isentropic turbine compared to the actual one. On the other hand, the second-law efficiency tells us how much work could be

theoretically derived from a system of reversible devices operating across the same inlet and exit streams as our actual device. That is, we don't alter the states, rather, we alter the devices to obtain the theoretical maximum work. In many respects, the second law efficiency (and the associated availability and irreversibility analysis) is more general; we can apply it to devices with heat transfer (recall that the isentropic efficiency assumes an adiabatic device) as well as devices which produce no work. The next example provides such an illustration.

Example: the condenser in a refrigerator

The condenser on a refrigerator is a heat exchanger which receives the high-pressure vapor from the exit of the compressor and condenses it to a liquid by heat transfer to the environment. Condensers are typically installed on the back or underneath the refrigerator and consist of fin-and-tube heat exchangers.

A particular condenser receives R-12 at 1000 kPa, 50°C and the fluid exits as a saturated liquid at 1000 kPa. The environment temperature is $T_0 = 25^\circ\text{C} = 298\text{ K}$. Compute the irreversibility per unit mass of R-12.

Properties are

$$T_1 = 50^\circ\text{C}, P_1 = 1000\text{ kPa} : h_1 = 210.3\text{ kJ/kg}, s_1 = 0.7026\text{ kJ/kg K}$$

$$x_2 = 0, P_2 = 1000\text{ kPa} : T_2 = 41.6^\circ\text{C}, h_2 = 76.3\text{ kJ/kg}, s_2 = 0.277\text{ kJ/kg K}$$

This is a single inlet, single outlet, SSSF device. There is no work transfer, and KE is neglected. The heat transfer, per unit mass, is

$$q = h_2 - h_1 = -134\text{ kJ/kg}$$

The heat is exchanged with the environment at T_0 . From Eq. (18), the irreversibility is

$$i = T_0(s_2 - s_1) - q = 7.171\text{ kJ/kg}$$

Since there is no actual work transfer in the process, the irreversibility is the same as the reversible work. That is, it represents the lost work potential for the process. Theoretically, it would be possible to use the heat transfer from the condenser to power a reversible heat engine which would provide 7.171 kJ of work per kg of R-12.

The source of irreversibility, in this case, comes from the fact that the heat is transferred over a finite temperature difference. The R-12 enters at 50°C and leaves at 41.6°C, whereas the environment is at 25°C. We could lower the irreversibility by raising the environment temperature – but this would not make much sense from a practical point of view since the environment is usually at a fixed temperature. A more logical approach would be to lower the pressure in the condenser, which would lower the saturation temperature. At 25°C the saturation pressure of R-12 is 651 kPa; as the condenser pressure becomes closer to this value the irreversibility would decrease. A lower condenser pressure would also translate into a lower compressor work input, and the overall coefficient of performance of the refrigerator would increase.

On the other hand, a lower temperature difference between the condenser and the environment would decrease the *rate* of heat transfer from the condenser, and to compensate the condenser would have to have a larger surface area and/or a forced convection cooling system (i.e., cooling fans). These sorts of trade-offs often appear in power and refrigeration system design and analysis. An availability/irreversibility analysis may identify a point of improvement (i.e., a source of high irreversibility), yet actual improvement of the process may be impractical or prohibitively expensive. The next example illustrates such a situation.

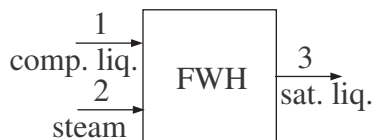


Figure 3: Open feedwater heater

Example: the open feedwater heater

An open feedwater heater (FWH) is a device in which a stream of compressed liquid, at a moderate temperature, is mixed with a stream of saturated or superheated vapor, at a higher temperature. The streams are mixed in proportions so that the exit of the FWH is a saturated liquid, i.e., a liquid at the boiling point. Typically the devices operate at constant pressure, in that the pressures of the two entering streams and the exiting stream are the same. And typically the devices are adiabatic. The FWH is frequently used in vapor power cycles, the point being to ‘pre-heat’ the liquid water leaving the pump prior to entering the boiler. The steam is obtained from a ‘bleed’ on a turbine, and even though this results in less steam flowing through the turbine and less power output, the overall effect of the FWH is to improve the thermal efficiency of the cycle. You will learn more about these devices if/when you take Thermo II.

A diagram of an open FWH is given in Fig. 3. For this particular example the FWH operates at $P = 1$ MPa. Stream 1 is at $T_1 = 40^\circ\text{C}$, stream 2 is at $T_2 = 200^\circ\text{C}$, and the exit (3) is a saturated liquid at $P = 1$ MPa. The mass flow rate of the compressed liquid is $\dot{m}_1 = 10$ kg/s. We want to

1. Calculate the mass flow rate of the steam, \dot{m}_2 ,
2. Calculate the rate of irreversibility, and
3. Calculate the theoretical minimum steam mass flow rate needed to produce a saturated liquid at 3.

First obtain the required properties:

$$\begin{aligned} h_1 &= 168.5 \text{ kJ/kg}, & s_1 &= 0.572 \text{ kJ/kg K} \\ h_2 &= 2827.9 \text{ kJ/kg}, & s_2 &= 6.694 \text{ kJ/kg K} \\ h_3 &= 762.9 \text{ kJ/kg}, & s_3 &= 2.139 \text{ kJ/kg K} \end{aligned}$$

The first question involves a simple application of the first law. The device is adiabatic, there is no work transfer, and KE is negligible:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$$

or

$$\dot{m}_2 = \dot{m}_1 \frac{h_1 - h_3}{h_3 - h_2} = 2.88 \text{ kg/s}$$

This is the actual steam mass flow rate required for the FWH.

The irreversibility of the FWH is obtained from Eq. (18), generalized to multiple inlets/outlets. We will assume an environment temperature of $T_0 = 298$ K, so

$$\dot{I} = T_0 ((\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2) = 762.5 \text{ kW}$$

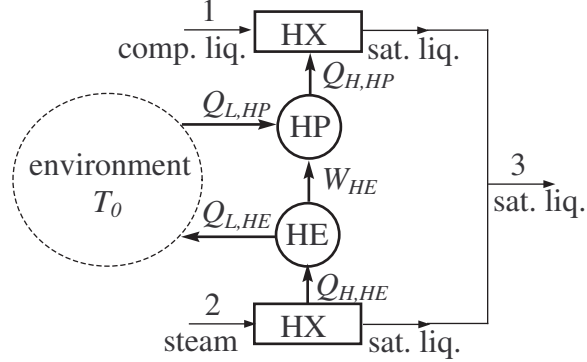


Figure 4: Reversible feedwater heater system

and since there is no actual work for the process, the reversible work would be

$$\dot{W}_{rev} = \dot{W} + \dot{I} = \dot{I} = 762.5 \text{ kW}$$

This is the power that, theoretically, could have been produced from the same inlet/exit streams.

The FWH is not intended to produce work, so a calculation of the reversible work is not especially useful except to show that the overall process is irreversible. A more relevant calculation would be to compute the minimum required mass flow rate of steam, $\dot{m}_{2,rev}$, needed to produce a saturated liquid. This would correspond to a perfectly reversible process.

For this ideal case the FWH would be a bit more complicated than that shown in Fig. 3. We would not simply dump the hot steam into a tank with the cold liquid to produce the saturated liquid (an irreversible process); rather, the system would be replaced by a system of components as illustrated in Fig. 4, all designed to produce a reversible process. The steam would first flow through a heat exchanger. Heat would be transferred from the steam and the steam would condense and exit as a saturated liquid. The heat from the heat exchanger would be transferred into a reversible heat engine. The work produced by the HE would drive a reversible heat pump, which would transfer heat from the environment to another heat exchanger which would heat the compressed liquid to a saturated liquid. The exits from both heat exchangers, both being saturated liquids at the same pressure, would then be mixed (mixing of two streams at the same state is a reversible process).

In principle, each component in Fig. 4 could be made reversible, and the total process, as a whole, would be reversible.

The mechanical description of the reversible process, in Fig. 4, would be difficult to analyze on a component-by-component basis. On the other hand, we can quickly calculate the mass flow rate of the steam, $\dot{m}_{2,rev}$, by applying the availability analysis. No net work is produced by the system (the heat engine work is entirely consumed by the heat pump), and if the overall process is reversible, then the reversible work must be zero. Alternatively, the irreversibility of the process (which is the reversible work, since no actual work is produced) is also zero. So we can go to Eq. (17) (generalized to multiple inlets/outlets) and set it to zero:

$$\dot{W}_{rev} = 0 = \dot{m}_1(h_1 - T_0 s_1) + \dot{m}_{2,rev}(h_2 - T_0 s_2) - (\dot{m}_1 + \dot{m}_{2,rev})(h_3 - T_0 s_3)$$

Now solve for $\dot{m}_{2,rev}$:

$$\dot{m}_{2,rev} = \dot{m}_1 \frac{h_1 - h_3 - T_0(s_1 - s_3)}{h_3 - h_2 - T_0(s_3 - s_2)} = 1.8 \text{ kg/s}$$

The reversible mass flow rate is smaller than the actual (or irreversible) flow rate. With regard to the power cycle application of a FWH, the use of a reversible FWH would mean that less steam would have to be bled off of a turbine to produce the saturated liquid at the boiler inlet. Less steam from the turbine equates to more steam through the turbine, and the overall efficiency and power output of the power cycle would improve. On a practical point of view, however, the improved efficiency might not justify the increased expense of the device – it would be a whole lot cheaper, up front, to build a simple open FWH than the complicated device shown in Fig. 4. The analysis, however, is useful in identifying where improvements in efficiency could be made.

The second law of thermodynamics:

1. *German physicist Rudolph Clausius states the second law of thermodynamics, "Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time".*
2. *Scottish physicist Lord Kelvin states, "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature".*
3. *Simply put, transferring heat from lower to higher temperatures is impossible without external force.*

Applications:

1. *The law states that heat always moves from a body that is warmer to a colder body. All heat engine cycles, including Otto, Diesel, etc., as well as all working fluids employed in the engines, are covered by this rule. Modern automobiles have advanced as a result of this law.*
2. *Another illustration of how this idea is used is in reverse-cycle refrigerators and heat pumps. If we want to move heat from a body with a lower temperature to a body with a higher temperature, we must perform external work. The original Carnot cycle uses heat to create work, as opposed to the reversed Carnot cycle, which transfers heat from a lower temperature reservoir to a higher temperature reservoir using work.*

Availability

The maximum useful work that can be obtained from the system such that the system comes to a dead state, while exchanging heat only with the surroundings, is known as availability of the system. Here the term dead state means a state where the system is in thermal and mechanical equilibrium with the surroundings.

Therefore for a closed system availability can be expressed as

$$\phi = (U - U_o) + p_o (V - V_o) - T_o (S - S_o)$$

similarly for an open system

$$\psi = (H - H_o) - T_o (S - S_o)$$

n steady flow systems the exit conditions are assumed to be in equilibrium with the surroundings. The change in availability of a system when it moves from one state to another can be given as:

for a closed system

$$f_1 - f_2 = (U_1 - U_2) + p_0 (V_1 - V_2) - T_0 (S_1 - S_2)$$

Availability Change Involving Heat Exchange with Reservoirs

Consider a system undergoing a change of state while interacting with a reservoir kept at T_R and atmosphere at pressure p_0 and temperature T_0 . Net heat transfer to the system

Irreversibility

Work obtained in an irreversible process will always be less than that of a reversible process. This difference is termed as irreversibility (i.e) the difference between the reversible work and the actual work for a given change of state of a system is called irreversibility.

$$I = W_{rev} - W_{act}$$

Let a stationary closed system receiving Q kJ of heat is giving out W_{act} kJ of work. From first law of thermodynamics.

$$Q - W_{act} = U_2 - U_1$$

$$W_{act} = U_1 - U_2 + Q$$

$$W_{rev} = (U_1 - U_2) - T_0 (S_1 - S_2)$$

$$= (U_1 - U_2) + T_0 (\Delta S)_{system}$$

$$\therefore I = W_{rev} - W_{act}$$

$$= (U_1 - U_2) + T_0 (\Delta S)_{system} - (U_1 - U_2) - Q$$

$$= T_0 (\Delta S)_{system} - Q \quad \text{Where } Q = -Q_{surroundings} = T_0 (\Delta S)_{surroundings}$$

$$= T_0 (\Delta S)_{system} + T_0 \Delta S_{surroundings}$$

$$= T_0 (\Delta S)_{universe}$$

1. A reversible heat engine receives 3000 KJ of heat from a constant temperature source at 650 K. If the surroundings is at 295 K,

determine

i) the availability of heat energy

ii) Unavailable heat.

Given Data:

$$Q_1 = 3000 \text{ KJ}$$

$$T_1 = 650 \text{ K}$$

$$T_0 = 295 \text{ K}$$

To find:

1) The availability of heat energy (A)

2) Unavailable heat (U.A)

Solution:

$$\begin{aligned} \text{Change in entropy } (\Delta S) &= \int \frac{dQ}{T} \\ &= \frac{Q_1}{(T_1 + T_0)} \\ &= \frac{3000}{(650 + 295)} \\ &= 3.17 \text{ KJ/K} \end{aligned}$$

The availability of heat energy,

$$\begin{aligned} A &= Q_1 - T_0 (\Delta S) \\ &= 3000 - 295 (3.17) \\ &= 2064.85 \text{ KJ.} \end{aligned}$$

Unavailable heat (U.A) = $T_0 (\Delta S)$

$$\begin{aligned} &= 295 (3.17) \\ &= 935.15 \text{ KJ.} \end{aligned}$$

Result:

1) *The availability of heat energy (A) = 2064.85 KJ*

2) *Unavailable heat (U.A) = 935.15 KJ.*

2. Air in a closed vessel of fixed volume 0.15 m^3 exerts pressure of 12 bar at 250°C . If the vessel is cooled so that the pressure falls to 3.5 bar , determine the final pressure, heat transfer and change of entropy.

Given Data:

$$V_1 = 0.15 \text{ m}^3$$

$$p_1 = 12 \text{ bar} = 1200 \text{ KN/m}^2 \quad p_2 = 3.5 \text{ bar} = 350 \text{ KN/m}^2$$

$$T_1 = 250^\circ\text{C} = 273 + 250 = 523 \text{ K}$$

To find:

- 1) The final pressure,
- 2) Heat transfer
- 3) Change of entropy

Solution:

From ideal gas equation

$$\begin{aligned} m &= \frac{p_1 V_1}{R T_1} \\ &= \frac{1200 \times 0.15}{0.287 \times 523} \\ &= 1.2 \text{ kg.} \end{aligned}$$

1) The final pressure:

For constant volume process,

$$\begin{aligned} T_2 &= T_1 \times \frac{p_2}{p_1} \\ &= 523 \times \frac{3.5}{12} \\ &= 152.54 \text{ K.} \end{aligned}$$

2) Heat transfer:

$$\begin{aligned} \text{Heat transfer, } Q &= m C_p (T_2 - T_1) \\ &= 1.2 \times 1.005 (152.54 - 523) \\ &= -446.78 \text{ KJ.} \end{aligned}$$

(- sign indicates that the heat is rejected from the system)

3) Change of entropy:

$$\begin{aligned} \text{Entropy change, } \Delta S &= m \left[C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right] \\ &= 1.2 \left[1.005 \ln \frac{152.54}{523} - 0.287 \ln \frac{3.5}{12} \right] \\ &= -1.06 \text{ KJ/K.} \end{aligned}$$

3. A domestic food freezer maintains a temperature of -15°C . The ambient air is at 30°C . If the heat leaks into the freezer at a continuous rate of 1.75 KJ/s , what is the least power necessary to pump the heat out continuously?

Given Data:

$$T_L = -15^{\circ}\text{C} = 273 - 15 = 258\text{ K}$$

$$T_H = 30^{\circ}\text{C} = 273 + 30 = 303\text{ K}$$

$$Q_S = 1.75\text{ KW}$$

To find:

Least power, (W)

Solution:

Solution:

$$\begin{aligned}\text{Carnot COP} &= \frac{T_L}{T_H - T_L} \\ &= \frac{258}{303 - 258} \\ &= 5.733\end{aligned}$$

$$\text{Actual COP of refrigerator} = \frac{Q_S}{W}$$

For minimum power required to pump the heat,

$$\text{Carnot COP} = \text{Actual COP}$$

$$5.733 = \frac{1.75}{W}$$

$$W = 0.305\text{ KW.}$$

