## 4.3 WHEEL FRICTION

Consider a flat belt passing over a pulley or any cylindrical surface as shown in Fig.

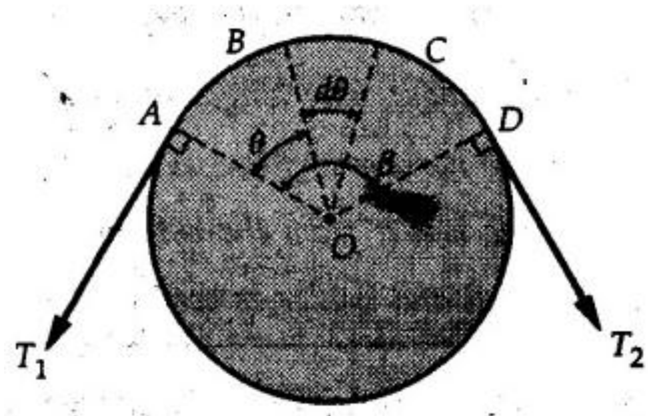



Fig.(a)

• If there is no friction between pulley and the belt, the tensions $T_1$ and $T_2$ will be equal in magnitude.

• In presence of friction, the two tensions will have different magnitudes.

We will now develop a formula relating $T_1$ and $T_2$, assuming $T_2 > T_1$.

• The belt leaves the pulley tangentially at A and D as shown in Fig. a.

• The inscribed angle over which belt is in contact with the pulley is called the **lap angle** β.

 $\beta = \angle AOD$

• Consider F.B.D. of a differential element BC of inscribed angle dθ as shown in Fig. b.

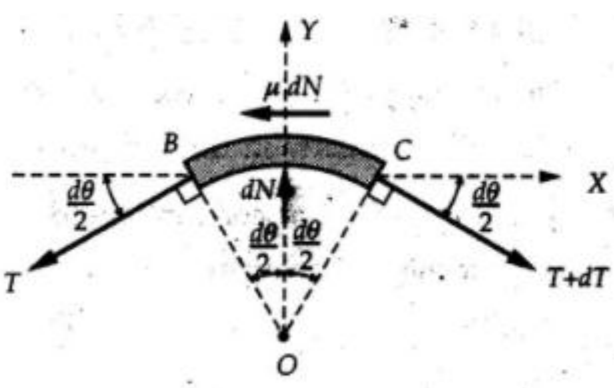


Fig. (b)

$$\sum F_x = 0$$

$$(T + dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} - \mu\, dN = 0$$

- As $d\theta$ is very small, $\cos\frac{d\theta}{2} \simeq 1$

$$\therefore \quad (T + dT) - T = \mu\, dN$$

$$\therefore \quad dT = \mu\, dN$$

$$\sum F_y = 0 \; : \; dN - T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0$$

- For small $d\theta$, $\sin\frac{d\theta}{2} \simeq \frac{d\theta}{2}$.

$$\therefore \quad dN - T\frac{d\theta}{2} - (T + dT)\frac{d\theta}{2} = 0$$

$$dN - Td\theta - dT\frac{d\theta}{2} = 0$$

a product of two differential quantities and hence is very small compared to the remaining terms.

$$\therefore \quad dN - Td\theta = 0$$

$$dN = T\, d\theta$$

• Substituting in the value of dN in dT

$$dT = \mu T\, d\theta$$

$$\frac{dT}{T} = \mu\, d\theta$$

Integrating, we get

$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu \int_0^{\beta} d\theta$$

$$\ln T_2 - \ln T_1 = \mu\beta$$

$$\therefore \quad \ln \frac{T_2}{T_1} = \mu\beta$$

$$\therefore \quad \boxed{\frac{T_2}{T_1} = e^{\mu\beta}}$$

In the above equation, value of β has to be in radians. μ can be either $\mu_k$ or $\mu_s$ depending upon whether there is relative motion between belt and pulley or not.

• If number of turns of the rope or belt are given, around the pulley, the value of β in radians can be obtained using

1 Turn = 2л radians.

• If number of turns are not given, use the concept that rope or belt leaves the pulley tangentially. Draw radii at the points where the belt leaves the pulley and find the inscribed angle for which the belt is in contact with the pulley.

• As tensions on two sides of the pulley are different, a net torque acts on the pulley given by

$$\boxed{\tau = (T_2 - T_1)\, r}$$

where *r* = Radius of the pulley

• If belt drives the pulley, the driving torque and rotation of pulley will have same sense of rotation.

• For braking mechanisms, the torque will be opposite to the rotation of pulley.

• This is useful in identifying larger tension ($T_2$) and smaller tension ($T_1$).

**Steps for Solution :**

1) Find lap angle β **in radians.**

2) Identify the larger tension ($T_2$) and smaller tension ($T_1$).

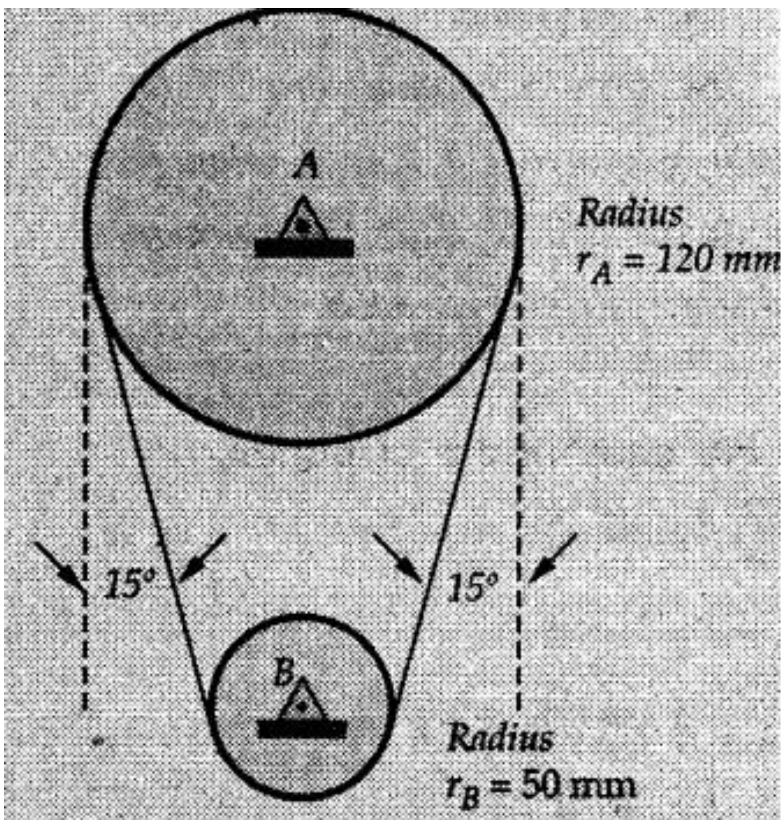
3) Use $T_2/ T_1 = e^{\mu\beta}$ for the pulley.

4) If required, use additional equations $\sum F_x = 0$; $\sum F_y = 0$ and $\sum M = 0$ for the equilibrium of pulley or any other object like a lever connected to the pulley.

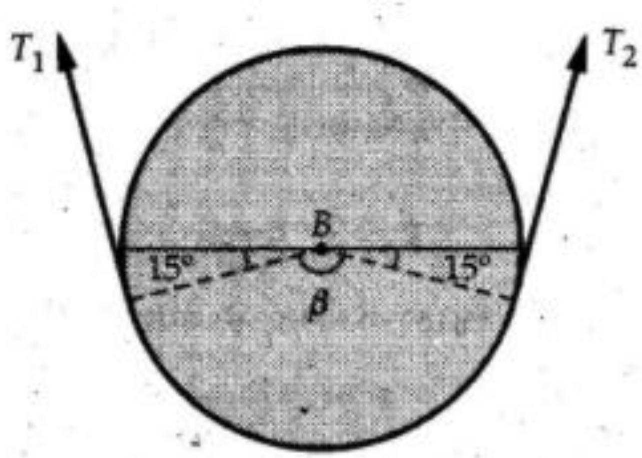
5) Solve the equations simultaneously.

**Solved Examples**

***1.A flat belt is used to transmit a torque from drum 'B' to drum 'A'. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest torque that can be exerted on drum 'A'. Refer Fig.***




**Solution:**




As torque is generated at drum $B$, we have to use $\frac{T_2}{T_1} = e^{\mu\beta}$ for $B$.

From Fig. 8.9.4 (a),

$$\beta = 180 - 15 - 15 = 150°$$

$$\therefore \quad \beta = \frac{5\pi}{6} \text{ radians.}$$

The larger tension is

$$T_2 = 450 \text{ N and } \mu = 0.4$$

$$\therefore \quad \frac{450}{T_1} = e^{(0.4)\left(\frac{5\pi}{6}\right)}$$

$$\therefore \quad T_1 = 157.914 \text{ N}$$

Torque transmitted to $A$ is $\tau_A = (T_2 - T_1)\, r_A$

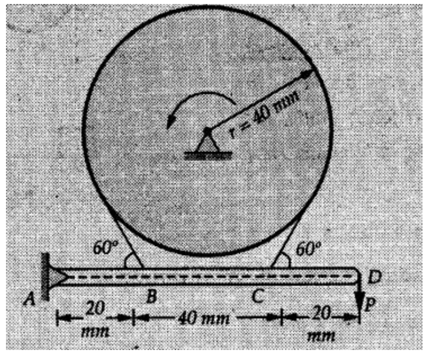
$$\tau_A = (450 - 157.914)\,(0.12)$$

$$\therefore \quad \boxed{\tau_A = 35.05 \text{ Nm}}$$

Note that the sense of rotation for the drums is not given and hence we can take tension on any one side to be the larger tension.

***2.As shown in Fig. a flexible and inextensible flat belt placed around a rotating drum of 40 mm radius, acts as a brake when the arm ABCD is pulled down. Assuming = 0.2 between drum and belt, find the force 'P' that would result in***

***braking torque of 4000 N.mm assuming that the drum is rotating counter clockwise.***




The two free body diagrams are shown in Fig. (a)

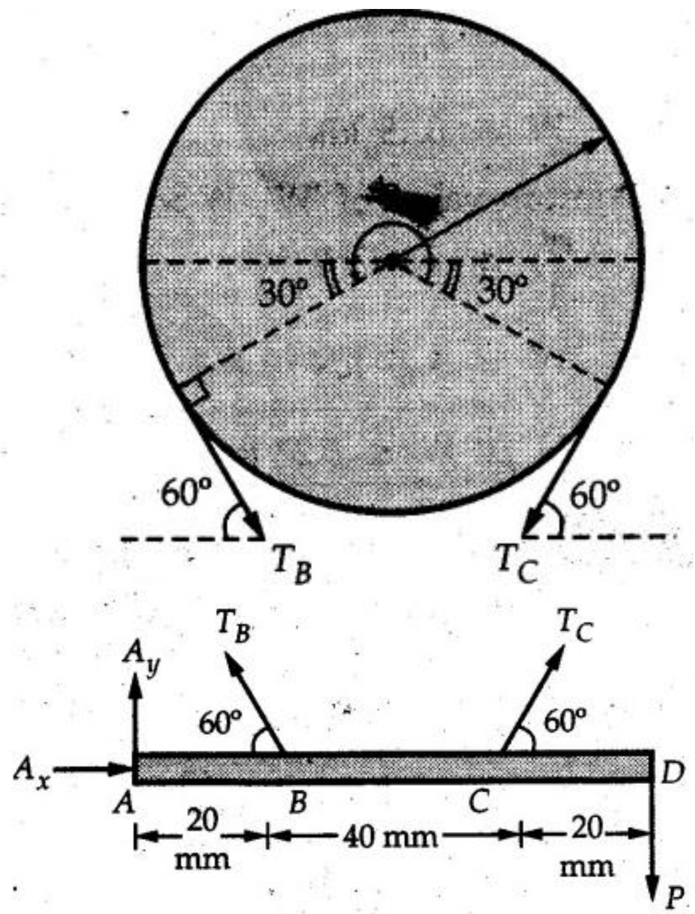



Fig. (a)

As the drum is rotating counter clockwise, the braking torque will be clockwise. Hence $T_C > T_B$ for the drum,

$$\frac{T_2}{T_1} = e^{\mu\beta}; \; T_2 > T_1$$

$$\beta = 240° = \frac{4\pi}{3} \text{ radians}$$

$$\therefore \frac{T_C}{T_B} = e^{0.2\times\frac{4\pi}{3}}$$

$$T_C = 2.3112 T_B \qquad \text{... (1)}$$

The braking torque is $\tau = 4000$ N-mm.

$$\therefore (T_C - T_B)\times 40 = 4000$$

$$\therefore \quad T_C - T_B = 100 \qquad \text{... (2)}$$

From equations (1) and (2),

$$T_B = 76.266 \text{ N}$$

$$T_C = 176.266 \text{ N}$$

For the arm ABCD, $\sum M_A = 0$

$$(T_B \sin 60)(20) + (T_C \sin 60)(60) - (P)(80) = 0$$

$$\therefore \quad \boxed{P = 131 \text{ N}}$$