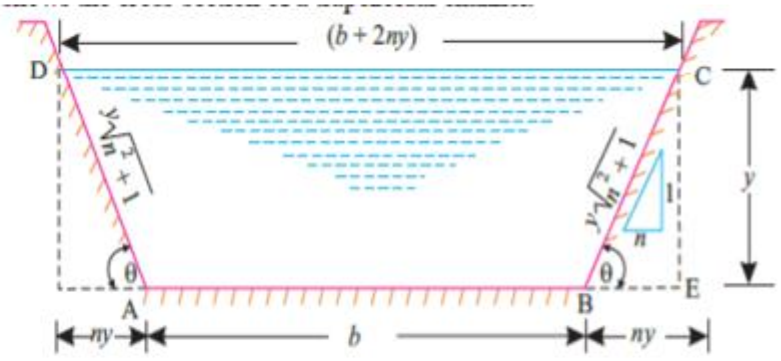
MOST ECONOMICAL TRAPEZOIDAL CHANNEL



Let b = Base width of the channel,

y = Depth of flow, and

θ = Angle made by the sides with horizontal.

Side slope = 1 vertical to n horizontal.

$$\text{Area of flow, } A = \left(\frac{AB + CD}{2}\right) \times y = \frac{b + (b + 2ny)}{2} \times y = (b + ny)\,y \qquad ...(i)$$

$$\therefore \qquad \frac{A}{y} = b + ny$$

or,
$$b = \frac{A}{y} - ny \qquad ...(ii)$$

$$\text{Wetted perimeter, } P = AD + AB + BC = AB + 2BC \qquad (\because AD = BC)$$

$$= b + 2\sqrt{BE^2 + CE^2}$$

$$= b + 2\sqrt{n^2y^2 + y^2}$$

or,
$$P = b + 2y\sqrt{n^2 + 1} \qquad ...(iii)$$

Substituting the value of b from eqn. (*ii*) in eqn. (*iii*), we get:

$$P = \frac{A}{y} - ny + 2y\sqrt{n^2 + 1} \qquad ...(iv)$$

The section of the channel will be *most economical* when its wetted perimeter (P) is *minimum*,

i.e. $\frac{dP}{dy} = 0$

or, $$\frac{d}{dy}\left[\frac{A}{y} - ny + 2y\sqrt{n^2 + 1}\right] = 0$$

or, $$-\frac{A}{y^2}-n+2\sqrt{n^2+1} = 0 \qquad (\because n \text{ is constant})$$

or, $$\frac{A}{y^2}+n = 2\sqrt{n^2+1}$$

Substituting the value of A from eqn. (i), in the above equation, we get:

$$\frac{(b+ny)\,y}{y^2}+n = 2\sqrt{n^2+1}$$

or, $$\frac{(b+ny)}{y}+n = 2\sqrt{n^2+1}$$

or, $$\frac{b+ny+ny}{y} + 2\sqrt{n^2+1} \quad \text{or} \quad \frac{b+2ny}{y} = 2\sqrt{n^2+1}$$

or, $$\frac{b+2ny}{2} = y\sqrt{n^2+1} \qquad \text{...(16.14)}$$

[*i.e. Half of top width = One of the sloping sides* ... Fig 16.9]

Hydraulic radius, *R*:

Hydraulic radius, $R = \frac{A}{P}$

$$A = (b+ny)\times y \qquad \text{[From eqn. } (i)]$$

$$P = b+2y\sqrt{n^2+1} \qquad \text{[From eqn. } (iii)]$$

But, $$2y\sqrt{n^2+1} = b+2ny \qquad \text{[From eqn. (16.14)]}$$

$\therefore$ $$P = b+(b+2ny) = 2\,(b+ny)$$

$\therefore$ $$\text{Hydraulic radius, } R = \frac{(b+ny)\,y}{2\,(b+ny)} = \frac{y}{2} \qquad \text{...(16.15)}$$

i.e., The hydraulic radius equals half the flow depth.