## STEADY UNIFORM FLOW: CHEZY EQUATION, MANNING EQUATION

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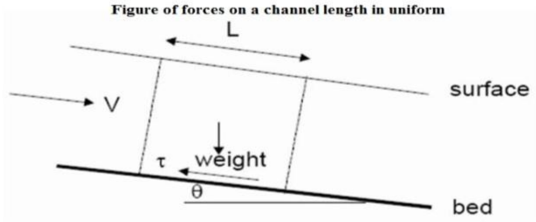
Figure of forces on a channel length in uniform




When uniform flow occurs gravitational forces exactly balance the frictional resistance forces which apply as a shear force along the boundary (channel bed and walls).

Considering the above diagram, the gravity force resolved in the direction of flow is

$$\text{Gravity force} = \rho g A L \sin\theta$$

boundary shear force resolved in the direction of flow is

$$\text{shear force} = \tau_o P L$$

In uniform flow these balance

$$\tau_o P L = \rho g A L \sin\theta$$

Considering a channel of small slope, (as channel slopes for unifor and gradually varied flow seldom exceed about 1 in 50) then

$$\sin q \sim \tan q = S_o$$

$$\tau_o = rgA\, S_o / P = rgRS_o$$

**1.The Chezy equation**

If an estimate of $\tau_o$ can be made then we can make use of Equation.

If we assume the state of rough turbulent flow then we can also make the assumption the shear force is proportional to the flow velocity squared i.e.

$$\tau_o \,\alpha\, V^2$$

$$\tau_o = KV^2$$

Substituting this into equation gives

$$V = \sqrt{\frac{\rho g}{K} R S_o}$$

Or grouping the constants together as one equal to C

$$V = C\sqrt{RS_o}$$

This is the Chezy equation and the C the 'Chezy C'

**The Manning equation**

A very many studies have been made of the evaluation of C for different naturaland manmade channels.

$$C = \frac{R^{1/6}}{n}$$

## Problem 1

Find the velocity of flow and rate of flow of water through a rectangular channel of 6m wide and 3m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take chezy's constant C = 55.

**Given:**

Width of rectangle channel, b = 6m.

Depth d = 3m

Bed Slope, i = 1 in 2000 = 1/2000

Chezy's constant C = 55

**Solution:**

Area = b × d = 6 × 3 = 18$m^2$

Perimeter P = b+2d = 6 + 2 × 3 = 12m

Hydraulic mean depth, m = A/P = 18/12 = 1.5m

V = $C\sqrt{mi}$ =55×$\sqrt{1.5 \times 1/2000}$ = 1.506 m/s

Q = V × Area = 1.506 × 18 = **27.108$m^3$/s.**

## Problem 2

Find the slope of the bed of a rectangular channel 5m when depth of water is 2m and rate of flow is given as 20 $m^3$/s. Take chezy's constant, C = 50.

**Given:**

Width of channel b = 5m.

Depth of water d = 2m

Rate of flow Q = 20 m3/s.

C = 50

Bed Slope = i

**Solution:**

$Q = AC\sqrt{mi}$

A = Area = b × d = 5×2 = 10 m²

Perimeter P = b+2d = 5+ 2×2 = 9m

Hydraulic mean depth, m = A/P

= 10/9 m

$$20.0 = 10 \times 50 \times \sqrt{10/9 \times i}$$

Therefore Bed slope is 1 in 694.44

**Problem 3**

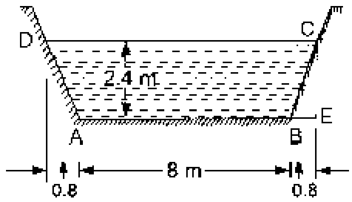
Find the discharge through a trapezoidal channel of width 8m and side slope of 1 horizontal to 3 vertical. The depth of flow of water is 2.4m and value of Chezy's constant, C = 50. The slope of the bed of the channel is given 1 in 4000.

**Given:**

Width b = 8m

Side Slope = 1 horizontal to 3 vertical

Depth d = 2.4m




Chezy's constant C = 50, Bed Slope I = 1/4000

**Solution**

Horizontal distance BE = 2.4 × 1/3 = 0.8m

Therefore Top Width of the channel,

CD = AB + 2 × BE = 8.0 + 2×0.8 = 9.6m

Therefore Area of trapezoidal channel, ABCD is given as,

A = (AB + CD) × CE/2 = (8+9.6) × 2.4/2 = 17.6 × 1.2 = 21.12m²

Wetted Perimeter, P = AB + BC + AD = AB = 2BC

$$BC = \sqrt{BE^2 + CE^2}$$

$$= \sqrt{(0.8)^2 + (2.4)^2} = 2.529\text{m}$$

P = 8 + 2 × 2.529 = 13.058m

Hydraulic mean depth m = A/P

= 12.12/13.058 = 1.617m

$$Q = AC\sqrt{mi}$$

$$= 21.12 \times 50\sqrt{1.617 \times 1/4000}$$

= **21.23 m³/s.**

## Problem 4

Find the bed slope of trapezoidal channel of bed width 6m, depth of water 3m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is 30 $m^3/s$. Take Chezy's Constant, C= 70

**Given:**

Bed width, b = 6.0m

depth of flow, d = 3.0m

side slope = 3 horizontal to 4 vertical

discharge q = 30 $m^3/s$

Chezy s Constant = 70

**Solution**

Depth of flow CE = 3m

BE = 3 × 3/4 = 2.25m

Therefore Top Width, CD = AB + 2 × BE

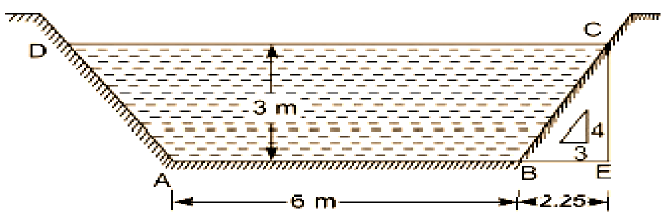
= 6.0 + 2 × 2.25 = 10.50m

Wetted Primeter, P = AD + AB + BC

= AB + 2ABC ($\because$BC = AD)

= AB + 2 $\sqrt{BE^2 + CE^2}$

=6.0 + 2$\sqrt{2.25^2 + 3^2}$= 13.5m




A = Area of trapezoidal ABCD

$= (AB+CD) \times CE/2$

$= (6+10.50)/2 \times 3 = 24.75m^2$

Hydraulic mean depth, $m = A/P = 24.75/13.50 = 1.833$

$$Q = AC\sqrt{mi}$$

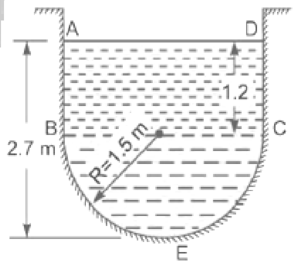
$$30.0 = 24.75 \times 70\ \sqrt{1.833 \times i} = 2345.6\ \sqrt{i}$$

$$i = (30/2345.6)^2 = 1/(2345.6/30)^2 = 1/6133$$

**i = 1/6133**

## problem 5

Find the discharge of water through the channel shown in the fig. Take thevalue of Chezy's constant = 60 and slope of the bed as 1 in 2000




**Given:**

Chezy s Constant $C = 60$

Bed Slope, $i = 1/2000$

**Solution:**

A = Area ABCD + Area BEC

$= (1.2 \times 3.0) + \pi R^2/2$ S

$= 3.6 + (1.5)^2\pi/2 = 7.134m^2$

Wetted Perimeter, P = AB + BEC + CD

$= 1.2 + \pi R + 1.2 = 1.2 + \pi 1.5 + 1.2$

$= 7.1124m$

Hydraulic mean depth, $m = A/P$

$= 7.134/7.1124 = 1.003$

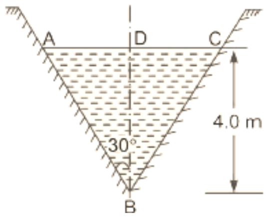
$$Q = AC\sqrt{mi}$$

$$= 7.134 \times 60 \times \sqrt{\left(1.003 \times \frac{1}{2000}\right)}$$

$$= \mathbf{9.585\ m^3/s}$$

**problem 6**

Find the rate of flow of water through a V- Shaped channel as shown in the fig. Take the value of C = 55 and slope of the bed 1 in 2000




**Given:**

C = 55

Bed Slope i = 1/1000

Depth of flow, d = 4.0m

Angle made by each side with vertical i.e <ABD = <CBD = 30°

**Solution:**

Area A = Area of ABC

= 2×Area of ABCD = (2×AD×BD)/2 = AD×BD

= BD tan 3°0 ×BD

= 4 tan 30° ×4 = 9.2376m$^2$

Wetted Perimeter, P = AB + BC = 2AB

$$= 2\sqrt{BD^2 + AD^2}$$

$$=2\sqrt{4^2 + (4\tan 30)^2}$$

$$= 2\sqrt{(16.0 + 5.333)} = 9.2375\text{m}$$

Hydraulic mean depth, m = A/P

$$= 9.2376/9.2375 = 1.0\text{m}$$

$$Q = AC\sqrt{mi}$$

$$= 9.2376 \times 55\sqrt{(1 * 1/1000)}$$

$$\mathbf{= 16.066 m^3/s}$$

## HYDRAULICALLY EFFICIENT CHANNEL SECTIONS

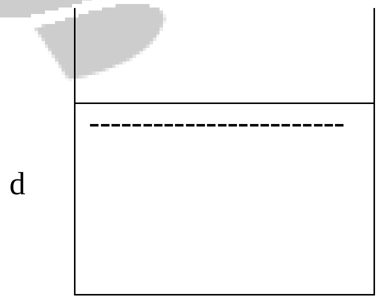
**Most Economical Section of Channels:**

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction of a channel depends on excavation and the lining. To keep the cost down or minimum, the wetted perimeter, for a given discharge, should be minimum. This condition is utilized for determining the dimensions of economical sections of different forms of channels.

Most economical section is also called the best section or most efficient section as the discharge, passing through a most economical section of channel for a given cross- sectional area A, slope of the bed S and a resistance coefficient, is maximum.

**Most Economical Rectangular Channel:**

Consider a rectangular section of channel as shown



Let B = width of channel,

D = depth of flow.

$\therefore$ area of flow A = b× $d$--------------(1) b

wetted perimeter, P = b + 2d------------(2)

=Ad+2d=Ad+2d - - - - (3)

for most economical section, P should be minimum for a given area.

$\frac{\partial p}{\partial d}$=0

difference the equation (3) with respect to 'd' and equating the same to zero, we get,

$$\frac{d}{d(d)}\left[\frac{A}{d} - 2d\right] = 0$$

$$-\frac{A}{d^2} - 2 = 0$$

$$A = 2d^2$$

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But A = b x d

$\therefore$ b×d=$2d^2$

b = 2d

Now hydraulic mean depth,m=A/P

$=\frac{b\times d}{b+2d}$

$=\frac{2d\times d}{2d+2d}$

$=\frac{2d}{4d^2}$

m = $\frac{d}{2}$

**problem 1**

A rectangular channel of width, 4m is having a bed slope of 1 in 1500. Find the maximum discharge through the channel. Take value of C = 50

**Given:**

b = 4 m

$i = \frac{1}{1500}$

C = 50

**Solution**

b = 2d

$d = \frac{b}{2}$

$d = \frac{4}{2} = 2m$

$m = \frac{d}{2} = \frac{2}{2} = 1.0m$

Area of economical rectangular channel,

$A = b \times d = 4 \times 2 = 8m^2$

$Q = AC\sqrt{mi}$

$Q = 4 \times 2 \times 50 \times \sqrt{1 \times \frac{1}{1500}}$

**Q= 10.328 m3 /s.**

## problem 2

A rectangular channel carries water at the rate of 400 lt is when bed slope is 1 in 2000. Find the most economical dimension of the channel of C = 50

**Given:**

Q = 400 lts/s = 0.4 $m^3$/s

$i = \frac{1}{2000}$

C = 50

**Solution**

For the rectangular channel to be most economical,

i. Width b = 2d.

ii. Hydraulic mean depth $m = \frac{d}{2}$

Area = b×d = 2d × d = $2d^2$

$Q = AC\sqrt{mi}$

$0.4 = 2d^2 \times 50\sqrt{\frac{d}{2} \times \frac{1}{2000}}$

$= 2 \times 50\sqrt{5/4000} \times d^{5/2}$

$d^{5/2} = 0.253$

**d=0.577m**

$b = 2 \times d = 2 \times 0.577 = \mathbf{1.154m}$