Convolutional codes

The operation of a convolutional encoder can be explained in several but equivalent ways such as, by a) state diagram representation. b) Tree diagram representation. c) trellis diagram representation.

Convolutional codes:

*Convolutional codes are widely used as channel codes in practical communication systems for error correction. *The encoded bits depend on the current k input bits and a few past input bits. * The main decoding strategy for convolutional codes is based on the widely used Viterbi algorithm.

*Convolutional codes are commonly described using two parameters: the code rate and the constraint length. The code rate, k/n, is expressed as a ratio of the number of bits into the convolutional encoder (k) to the number of channel symbols output by the convolutional encoder (n) in a given encoder cycle. *The constraint length parameter, K, denotes the "length" of the convolutional encoder, i.e. how many k-bit stages are available to feed the combinatorial logic that produces the output symbols. Closely related to K is the parameter m, which can be thought of as the memory length of the encoder. A simple convolutional encoder is shown below(fig.). The information bits are fed in small groups of k-bits at a time to a shift register. The output encoded bits are obtained by modulo-2 addition (EXCLUSIVE-OR operation) of the input information bits and the contents of the shift registers which are a few previous information bits.

The operation of a convolutional encoder can be explained in several but equivalent ways such as, by

a) state diagram representation.

b) Tree diagram representation.

c) trellis diagram representation.

Representing convolutional codes compactly: code trellis and state diagram:

STATE DIAGRAM:

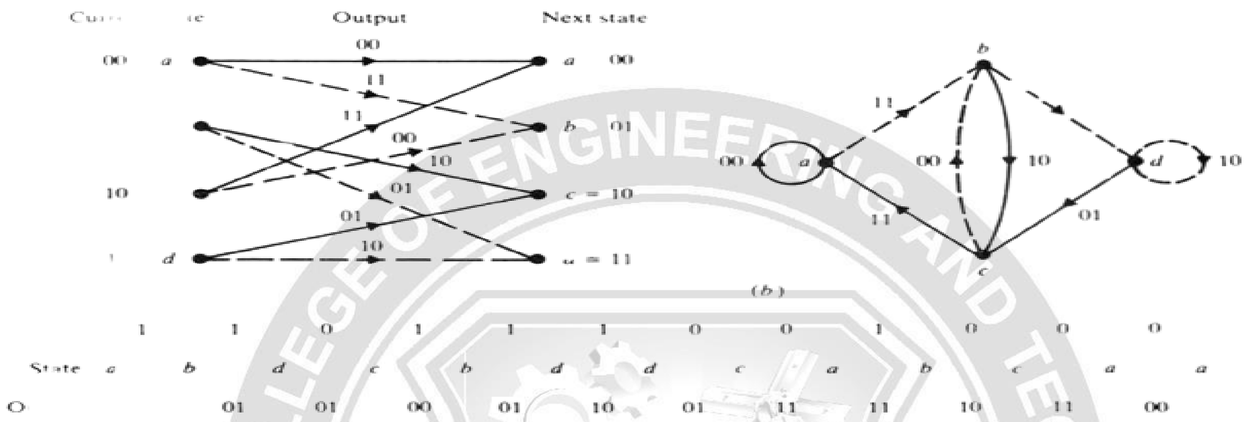
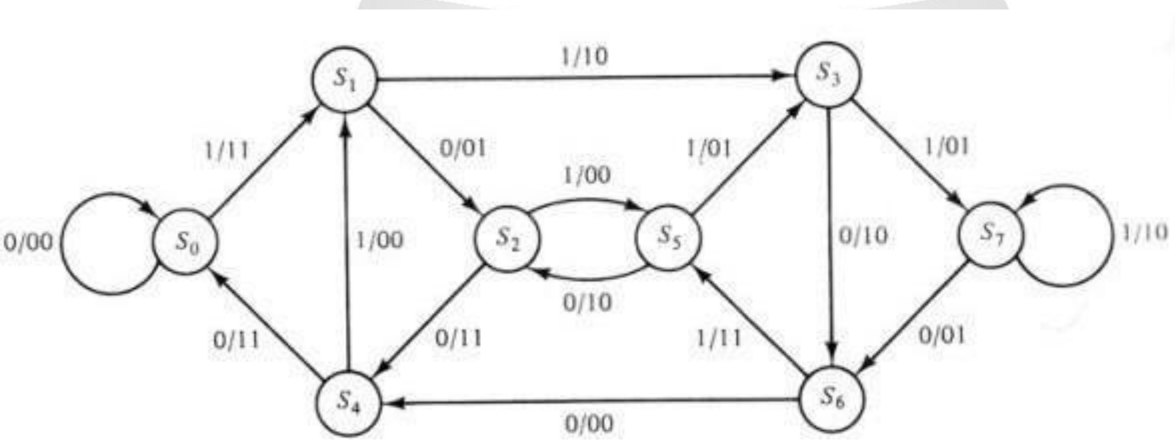


Fig 5.1 code trellis and state diagram (Source:Brainkart)

Inspecting state diagram: Structural properties of convolutional codes:

- Each new block of *k* input bits causes a transition into new state.
- Hence there are 2*k* branches leaving each state.
- Assuming encoder zero initial state, encoded word for any input of *k* bits can thus be obtained. For instance, below for **u**=(1 1 1 0 1), encoded word **v**=(1 1, 1 0, 0 1, 0 1, 1 1, 1 0, 1 1, 1 1) is produced:



(Source:Brainkart)

- Encoder state diagram for $(n,k,L)=(2,1,2)$ code
- Note that the number of states is $2L+1 = 8$.

Distance for some convolutional codes:

n	k	R_c	L	d_f	$R_c d_f/2$
4	1	1/4	3	13	1.63
3	1	1/3	3	10	1.68
2	1	1/2	3	6	1.50
			6	10	2.50
			9	12	3.00
3	2	2/3	3	7	2.33
4	3	3/4	3	8	3.00

Trellis Diagram Representation:

The trellis diagram of a convolutional code is obtained from its state diagram. All state transitions at each time step are explicitly shown in the diagram to retain the time dimension, as is present in the corresponding tree diagram. Usually, supporting descriptions on state transitions, corresponding input and output bits etc. are labelled in the trellis diagram. It is interesting to note that the trellis diagram, which describes the operation of the encoder, is very convenient for describing the behaviour of the corresponding decoder, especially when the famous Viterbi Algorithm (VA) is followed.

$$\mathbf{G} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

Hamming Code Example:

H (7,4) - Generator matrix G: first 4-by-4 identical matrix

$$\mathbf{p} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

- Message information vector p
- Transmission vector x
- Received vector r and error vector e
- Parity check matrix H

Error Correction:

$$\mathbf{Hr} = \mathbf{H}(\mathbf{x} + \mathbf{e}_i) = \mathbf{Hx} + \mathbf{He}_i = 0 + \mathbf{He}_i = \mathbf{He}_i$$

If there is no error, syndrome vector z=zeros

If there is one error at location 2 New syndrome vector z is

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathbf{Hr} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \mathbf{z}$$

$$\mathbf{Hr} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \mathbf{z}$$

1. Consider a single error correction (7,4) linear code and the corresponding decoding table.
2. Find the (7,4) linear systematic block code word corresponding to 1101.

Assume a suitable generator matrix.

Let

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

n=7 k=4 q = n-k= 3 code vector G=[Ik: P]

$$P_{4\times 3} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Check matrix C=MP C1 = m1+m2+m3 C2= m2+m3+m4 C3= m1+m2+m4 C=[010]

Complete code word can be calculated X={M:C}={1 1 0 0 0 1 0}

The parity matrix H=[pT :I] =[I: pT] =

$$\begin{bmatrix} 1 & 1 & 1 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & : & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & : & 0 & 0 & 1 \end{bmatrix}$$

Minimum weight W(X)=3

3. Determine the generator polynomial g(X) FOR A (7,4) cyclic code and fine the code vector for the following data vector 1010, 1111 and 1000

n 7

k=4

q=n- =3

To obtain the generator polynomial (p7+1) =(p+1)(p3+p2+1)(p3+p+1) Let G(p)= (p3+p+1) .

To obtain the generator matrix in systematic form determine the code vector

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix}$$

1. code vector for

M=1010 X=MG

2. code vector for M=1111

$$X = [1\,1\,1\,1]\begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix} = [1\,1\,1\,1\,1\,1\,1]$$

3. code vector for M=1000

$$X = [1\,0\,0\,0]\begin{bmatrix} 1 & 0 & 0 & 0 & : & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & : & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & : & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & : & 0 & 1 & 1 \end{bmatrix} = [1\,0\,0\,0\,1\,0\,1]$$

(Fig Source: J.G Proakis, ―Digital Communication‖, 4th Edition, Tata Mc Graw Hill Company, 2001)

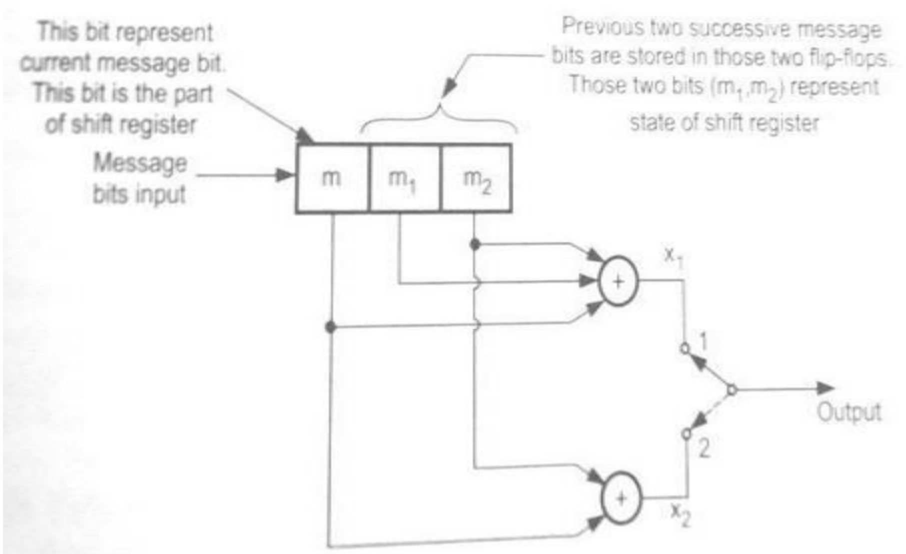


Fig 5.2 Convolutional codes(Source: J.G Proakis, ―Digital Communication‖, 4th Edition, Tata Mc Graw Hill Company, 2001)

$\cdot + \mathbf{e}_i$

$$\mathbf{Gp} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{x}$$

$$\mathbf{H} \quad \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

k = number of bits shifted into the encoder at one time

k=1 is usually used!!

n = number of encoder output bits corresponding to the *k0020*information bits

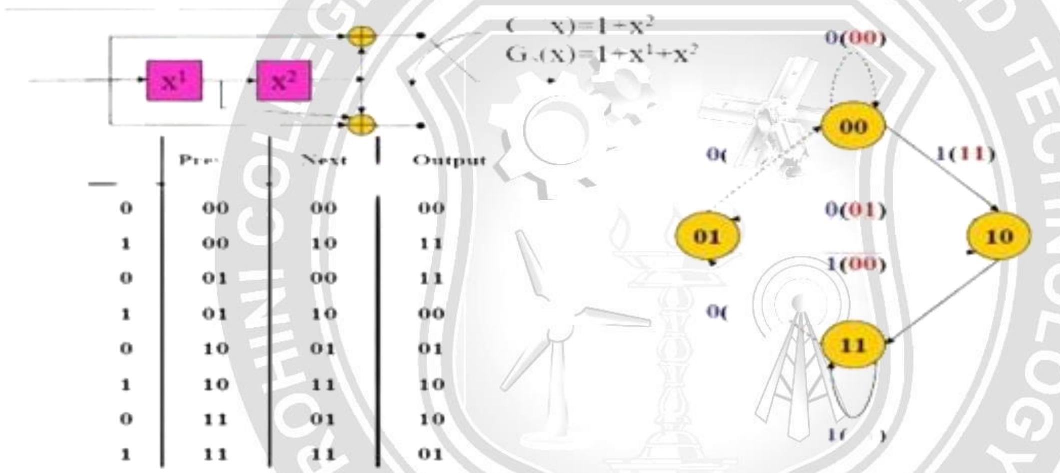
$r = k/n$ = code rate

K = constraint length, encoder memory

Each encoded bit is a function of the present input bits and their past ones.

An Example – (rate=1/2 with K=2

Trellis Diagram Representation



	Pre	Next	Output
0	00	00	00
1	00	10	11
0	01	00	11
1	01	10	00
0	10	01	01
1	10	11	10
0	11	01	10
1	11	11	01