Jointly WSS Process:

Let {X(t)} and {Y(t)}be two random processes. Then the function of (t) and (t) is said to be JWSS process if

(i)$E[X(t)] = constant$

(ii)$E[Y(t)] = constant$

(iii)$R(t_1, t_2\ ) is\ a\ function\ of\ r$

Problems under Joint Wide Sense Stationary:

1.If $X(t) = 5\cos(10t + \theta)$ and $Y(t) = 20\sin(10t + \theta)$, where θ is uniformly distributed over $(0, 2\pi)$. Prove $\{(t)\}$ and $\{(t)\}$ are JWSS.

Solution:

Given $(t) = 5\cos(10t + \theta), Y(t) = 20\sin(10t + \theta)$, where θ is a RV uniform distributed over $(0, 2\pi)$

$$f(\theta) = \frac{1}{2\pi}, 0 < \theta < 2\pi$$

To prove $\{(t)\}$ and $\{(t)\}$ is a JWSS process.

(i)$E[X(t)] = constant$

(ii)$E[Y(t)] = constant$

(iii)$R_{XY}(t, t+r)$ *is a function of* r

(i)$[X(t)] = [5\cos(10t+\theta)]$

$$= 5\,[\cos(10t+\theta)]$$

$$= 5\int_0^{2\pi} \cos(10t+\theta)\,\frac{1}{2\pi}d\theta$$

$$= \frac{5}{2\pi}\int_0^{2\pi} \cos(10t+\theta)d\theta$$

$$= \frac{5}{2\pi}[\sin(10t+\theta)]_0^{2\pi}$$

$$= \frac{5}{2\pi}[\sin(10t+2\pi) - \sin(10t)]$$

$$= \frac{5}{2\pi}[\sin 10t - \sin 10\,t] = 0$$

$[X(t)] = 0$ *is a constant*

(ii)$[Y(t)] = [20\sin(10t+\theta)]$

$$= 20\,[\sin(10t+\theta)]$$

$$= 20\int_0^{2\pi} \sin(10t+\theta)\,\frac{1}{2\pi}\,d\theta$$

$$= \frac{20}{2\pi}\int_0^{2\pi} \sin(10t+\theta)$$

$$= \frac{10}{\pi}[-\,c(10t+\theta)]_0^{2\pi}$$

$$= \frac{10}{\pi}[-\,\cos(10t+2\pi) + \cos(10t)]$$

$$= \frac{10}{\pi}[-\cos 10t + \cos 10t] = 0$$

$[Y(t)] = 0 \; is \; a \; constant$

(iii)$R_{XY}(r) = E[X(t)Y(t+r)]$

$$= [5\cos(10t+\theta)20\sin(10(t+r)+\theta)]$$

$$= [5\cos(10t+\theta)20\sin(10t+10r+\theta)]$$

$$= \frac{100}{2}[\sin(10t+\theta+10t+10r+\theta) - \sin(10t+\theta-10t-10r-\theta)]$$

$$= 50[\sin(20t+10r+2\theta) - \sin(-10r)]$$

$$= 50[\sin(20t+10r+2\theta) + \sin(10r)]$$

$$= 50\sin 10r + \frac{50}{2\pi}\int_0^{2\pi} \sin(20t+10r+2\theta)$$

$$= 50\sin 10r + \frac{25}{\pi}\left[\frac{-\cos(20t+10c+2\theta)}{2}\right]_0^{2\pi}$$

$$= 50\sin 10r + \frac{25}{2\pi}[-\cos(20t+10r+4\pi) + \cos(20t+10r)]$$

$$= 50\sin 10r + \frac{25}{2\pi}[-\cos(20t+10r) + \cos(20t+10r)]$$

$$= 50\sin 10r$$

$R_{XX}(t, t+r\,) is \; a \; function \; of \; r$

Since the conditions (i), (ii) and (iii) for JWSS are satisfied, $\{(t)\}\ and\ \{(t)\}$ are JWSS processes.

2. Two random processes are obtained by $X(t) = A\ cos m_0 t + B sin m_0 t\ and\ Y(t) = B\ cos m_0 t - A sin m_0 t$. Show that X(t) and Y(t) are JWSS if A and B are uncorrelated random variables with zero mean and same variances and m_0 is a constant.

Solution:

Given $(t) = A\ cos\omega_0 t + B sin\omega_0 t\ and\ (t) = B\ cos\omega_0 t - A sin\omega_0 t$

Where A and B are random variables with mean zero.

∴ E (A) = E (B) = 0… .....(1)

Also given A and B have same variances

$(i.e)\ Var(A) = Var(B)$

∴ E (A^2) = E (B^2) = 0….…..(2)

Also given A and B are uncorrelated

E(AB) = E(A) E(B) = 0…...............(3)

To Prove {X(t)} and {Y(t)} is a JWSS process.

(i)$E[X(t)] = constant$

(ii)$E[Y(t)] = constant$

(iii)$R(t_1, t_2\ ) is\ a\ function\ of\ r$

(i) $[(t)] = E[A\ cos\omega_0 t + Bsin\omega_0 t]$

$= [A]\ cos\omega_0 t + [B]sin\omega_0 t]$

= 0 (by 1)

$[X(t)] = 0\ is\ a\ constant$

(ii) $[(t)] = \mathrm{E}[B\ cos\omega_0 t - Asin\omega_0 t]$

$= \mathrm{E}[B]\ cos\omega_0 t - E[A]sin\omega_0 t]$

= 0 (by 1)

$[Y(t)] = 0\ is\ a\ constant$

(iii) $R(t_1, t_2) = E[X(t_1)Y(t_2)]$

$= [(A\ cos\omega_0 t_1 + Bsin\omega_0 t_1)(B\ cos\omega_0 t_2 - Asin\omega_0 t_2)$

$= [AB\ cos\omega_0 t_1\ cos\omega_0 t_2 - A^2\ cos\omega_0 t_1 sin\omega_0 t_2 + B^2\ sin\omega_0 t_1 \quad cos\omega_0 t_2 - ABsin\omega_0 t_1\ sin\omega_0 t_2]$

$= [AB]\ cos\omega_0 t_1\ cos\omega_0 t_2 - [A^2]\ cos\omega_0 t_1 sin\omega_0 t_2 + [B^2]\ sin\omega_0 t_1 \quad cos\omega_0 t_2 - [AB]sin\omega_0 t_1\ sin\omega_0 t_2]$

$= 0 - [A^2]\ cos\omega_0 t_1 sin\omega_0 t_2 + [B^2]\ sin\omega_0 t_1 \quad cos\omega_0 t_2 - 0$

$= -[A^2]\ cos\omega_0 t_1 sin\omega_0 t_2 + E[B^2]\ sin\omega_0 t_1 \quad cos\omega_0 t_2$

$= [A^2](sin\omega_0 t_1 \quad cos\omega_0 t_2 - \ cos\omega_0 t_1 sin\omega_0 t_2)$

$= [A^2]in\omega_0(t_1 - t_2)$

$= [A^2] in\omega_0 r$, which is a function of r

$$\therefore R(t_1, t_2) is\ a\ function\ of\ r$$

$\sin A \cos B - \cos A \sin B = \sin(A - B)$

Since the conditions (1), (2) and (3) for JWSS are satisfied , $X(t) and\ Y(t) are$ is JWSS process.