Two Branch Parallel Circuit :

Fig. 5.11(a) shows two branch parallel RLC circuit in which one branch contains resistor & inductor and the other branch contains resistor & capacitor.

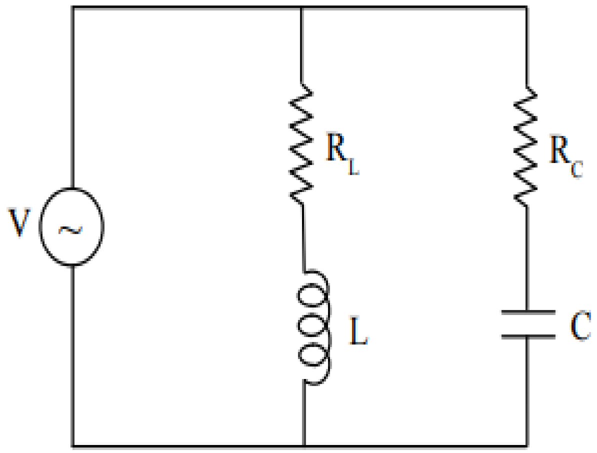

Fig. 5.11 (a) Two branch parallel circuit

The total admittance is, $Y = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$

Multiplying the numerator & denominator of each term by conjugate of the denominator we get,

$$Y = \frac{R_L - jX_L}{(R_L + jX_L)(R_L - jX_L)} + \frac{R_C + jX_C}{(R_C + jX_C)(R_C - jX_C)}$$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2} \qquad \left(\because\ (a + jb)(a - jb) = a^2 + b^2\right)$$

$$= \frac{R_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

At resonance, susceptance $B = 0$

$$\Rightarrow \frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0 \Rightarrow \frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$X_C\left(R_L^2 + X_L^2\right) = X_L\left(R_C^2 + X_C^2\right)$$

Substituting, $X_C = \frac{1}{\omega_0 C}$; $X_L = \omega_0 L$ we get

$$\frac{R_L^2 + \omega_0^2 L^2}{\omega_0 C} = \omega_0 L\left(R_C^2 + \frac{1}{\omega_0^2 C^2}\right)$$

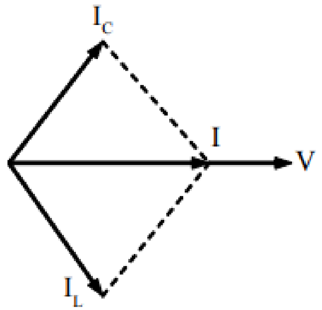

Fig. 5.11 (b) Vector diagram

$$R_L^2 + \omega_0^2 L^2 = \omega_0^2 L_C \left(\frac{\omega_0^2 R_C^2 C^2 + 1}{\omega_0^2 C^2} \right) \Rightarrow R_L^2 + \omega_0^2 L^2 = \frac{L}{C}\left(\omega_0^2 R_C^2 C^2 + 1\right)$$

$$R_L^2 + \omega_0^2 L^2 = \omega_0^2 R_C^2 \ LC + \frac{L}{C}$$

$$\omega_0^2 \ L^2 - \omega_0^2 R_C^2 LC = \frac{L}{C} - R_L^2$$

$$LC \ \omega_0^2 \left(\frac{L}{C} - R_C^2 \right) = \frac{L}{C} - R_L^2$$

$$\omega_0^2 = \frac{1}{LC}\left(\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2} \right) = \frac{1}{LC}\left(\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}} \right)$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$