ROHINI
COLLEGE OF ENGINEERING & TECHNOLOGY
Approved by AICTE and Affiliated to Anna University, (An ISO Certified Institution)
Near Anjugramam Junction, Kanyakumari Main Road, Palkulam, Variyoor P.O - 629 401

DEPARTMENT OF BIOMEDICAL ENGINEERING

III Semester- BM3301 SENSORS AND MEASUREMENTS

UNIT -1

1.7 uncertainty analysis

expression of uncertainty: accuracy and precision index, propagation of errors

Some numerical statements are exact: Mary has 3 brothers, and 2 + 2 = 4. However, all measurements have some degree of uncertainty that may come from a variety of sources. The process of evaluating the uncertainty associated with a measurement result is often called uncertainty analysis or sometimes error analysis.

The complete statement of a measured value should include an estimate of the level of confidence associated with the value.

Properly reporting an experimental result along with its uncertainty allows other people to make judgments about the quality of the experiment, and it facilitates meaningful comparisons with other similar values or a theoretical prediction.

When making a measurement, we generally assume that some exact or true value exists based on how we define what is being measured.

While we may never know this true value exactly, we attempt to find this ideal quantity to the best of our ability with the time and resources available. As we make measurements by different methods, or even when making multiple measurements using the same method, we may obtain slightly different results.

So how do we report our findings for our best estimate of this elusive true value? The most common way to show the range of values that we believe includes the true value is

measurement = (best estimate ± uncertainty) units

For example: We might say that the length of a certain stick measures 20 centimetres plus or minus 1 centimetre, at the 95 percent confidence level. This result could be written:

20 cm ±1 cm, at a level of confidence of 95%.

The statement says that we are 95 percent sure that the stick is between 19 centimetres and 21 centimetres long

how do you know that it is accurate, and how confident are you that this measurement represents the true value?

To help answer these questions, we first define the terms accuracy and precision:

Accuracy is the closeness of agreement between a measured value and a true or accepted value. Measurement error is the amount of inaccuracy.

Precision is a measure of how well a result can be determined (without reference to a theoretical or true value). It is the degree of consistency and agreement among independent measurements of the same quantity; also, the reliability or reproducibility of the result.

The accuracy and precision can be pictured as follows:

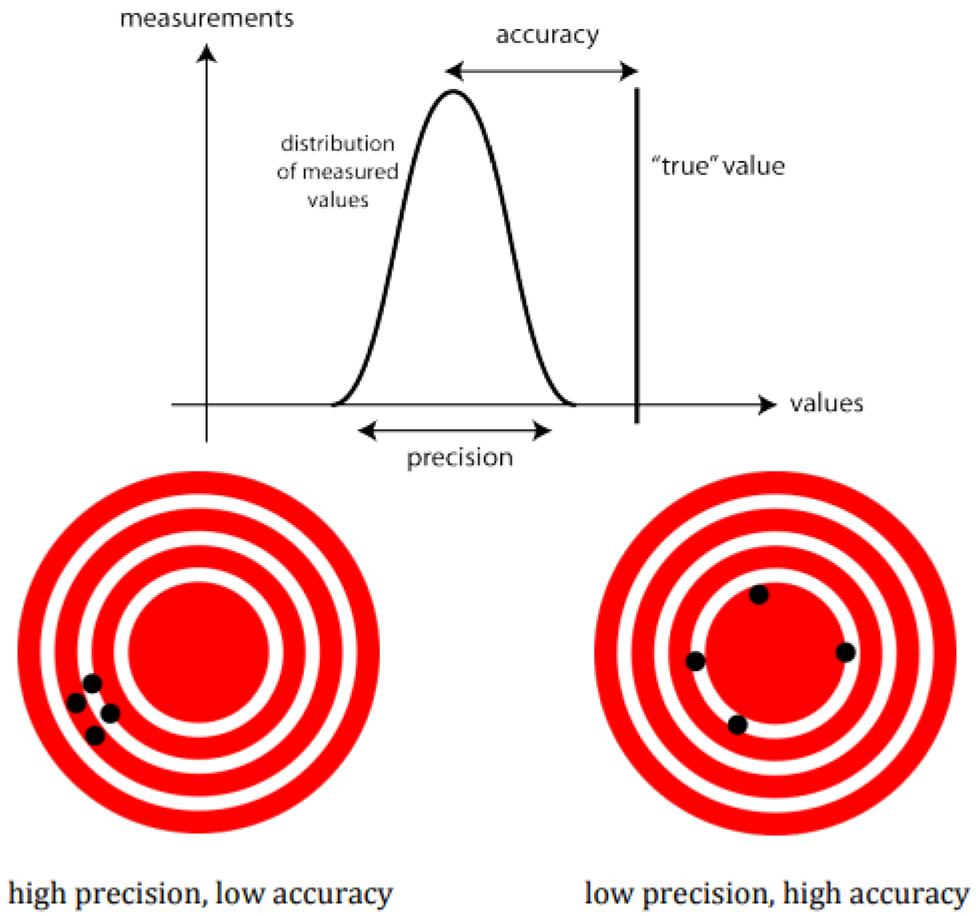

Figure 1.7.1 Accuracy vs Precision

The uncertainty estimate associated with a measurement should account for both the accuracy and precision of the measurement. Precision indicates the quality of the measurement, without any guarantee that the measurement is "correct." Accuracy, on the other hand, assumes that there is an ideal "true" value, and expresses how far your answer is from that "correct" answer. These concepts are directly related to random and systematic measurement uncertainties.

Precision is often reported quantitatively by using relative or fractional uncertainty.

$$Relative\ Uncertainty = \left|\frac{Uncertainty}{Measured\ quantity}\right|$$

For example, m = 75.5 ± 0.5 g has a fractional uncertainty of: $\frac{0.5g}{75.5g}$ $=$ 0.006 =0.7%

Accuracy is often reported quantitatively by using relative error:

$$Relative\ Error = \frac{Measured\ Value - Expected\ Value}{Expected\ Value}$$

If the expected value form is 80.0 g, then the relative error is =

$$\frac{75.5 - 80.0}{80.0} = -0.056 = -5.6\%$$

Types of Uncertainty Measurement:

Uncertainties may be classified as either random or systematic, depending on how the measurement was obtained (an instrument could cause a random uncertainty in one situation and a systematic uncertainty in another).

Random uncertainties are statistical fluctuations (in either direction) in the measured data. These uncertainties may have their origin in the measuring device, or in the fundamental physics underlying the experiment. The random uncertainties may be masked by the precision or accuracy of the measurement device. Random uncertainties can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations (see "standard error" later in this document).

Systematic uncertainties are reproducible inaccuracies that are consistently in the "same direction," and could be caused by an artifact in the measuring instrument, or a flaw in the experimental design (because of these possibilities, it is not uncommon to see the term "systematic error"). These uncertainties may be difficult to detect and cannot be analyzed statistically. If a systematic uncertainty or error is identified when calibrating against a standard, applying a correction or correction factor to compensate for the effect can reduce the bias. Unlike random uncertainties, systematic uncertainties cannot be detected or reduced by increasing the number of observations.

Estimating Uncertainty in Repeated Measurements:

If you repeat the measurement several times and examine the variation among the measured values, you can get a better idea of the uncertainty in the period. For example, here are the results of 5 measurements, in seconds: 0.46, 0.44, 0.45, 0.44, 0.41. For this situation, the best estimate of the period is the **average, or mean**:

$$Average(Mean) = \frac{x_1 + x_2 + \cdots + x_N}{N}$$

Whenever possible, repeat a measurement several times and average the results. This average is generally the best estimate of the “true” value (unless the data set is skewed by one or more outliers which should be examined to determine if they are bad data points that should be omitted from the average or valid measurements that require further investigation). Generally, the more repetitions you make of a measurement, the better this estimate will be, but be careful to avoid wasting time taking more measurements than is necessary for the precision required.

One way to express the variation among the measurements is to use the ***average deviation***. This statistic tells us on average (with 50% confidence) how much the individual measurements vary from the mean.

$$Average\ Deviation, \overline{d} = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \cdots + |x_N - \bar{x}|}{N}$$

The average deviation would seem to be a sufficient measure of uncertainty; however, it is important to understand the distribution of measurements.

Standard Deviation:

To calculate the standard deviation for a sample of N measurements:

1. Sum all the measurements and divide by N to get the average, or mean.
2. Subtract this average from each of the N measurements to obtain N “deviations.”
3. Square each of the N deviations and add them together.
4. Divide this result by (N–1) and take the square root.

To convert this into a formula, let the N measurements be called x_1, x_2, …, x_N. Let the average of the N values be called x . Then each deviation is given by

$$\delta x_i = x_i - \overline{x} \qquad for\ i = 1,2, \ldots N$$

The standard deviation is then:

$$S = \sqrt{\frac{(\delta x_1{}^2 + \delta x_2{}^2 + \cdots + \delta x_N{}^2)}{(N-1)}} = \sqrt{\frac{\sum \delta x_i{}^2}{(N-1)}}$$

Standard Deviation of the Mean (Standard Error) When reporting the average value of N measurements, the uncertainty associated with this average value is the standard deviation of the mean, often called the Standard Error (SE),

Standard Deviation of the Mean, or Standard Error (SE),

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{N}}$$

The standard error is smaller than the standard deviation by a factor of $1/\sqrt{N}$. This reflects the fact that we expect the uncertainty of the average value to get smaller when we use a larger number of measurements.

Significant Figures:

The number of significant figures in a value can be defined as all the digits between and including the first non-zero digit from the left, through the last digit. For instance, 0.44 has two significant figures, and the number 66.770 has 5 significant figures. Zeroes are significant except when used to locate the decimal point, as in the number 0.00030, which has 2 significant figures. Zeroes may or may not be significant for numbers like 1200, where it is not clear whether two, three, or four significant figures are indicated.

Propagation of Errors:

- The analysis of uncertainties (errors) in measurements and calculations is essential in the physics laboratory.
- For example, suppose you measure the length of a long rod by making three measurement $x = x_{best} \pm \Delta x$, $y = y_{best} \pm \Delta y$, and $z = z_{best} \pm \Delta z$.
- Each of these measurements has its own uncertainty Δx, Δy, and Δz respectively. What is the uncertainty in the length of the rod $L = x + y + z$? When we add the measurements do the uncertainties Δx, Δy, Δz cancel, add, or remain the same?

- Likewise, suppose we measure the dimensions $b = b_{best} \pm \Delta b$, $h = h_{best} \pm \Delta h$, and $w = w_{best} \pm \Delta w$ of a block. Again, each of these measurements has its own uncertainty Δb, Δh, and Δw respectively. What is the uncertainty in the volume of the block V = bhw? Do the uncertainties add, cancel, or remain the same when we calculate the volume? In order for us to determine what happens to the uncertainty (error) in the length of the rod or volume of the block we must analyze how the error (uncertainty) propagates when we do the calculation. In error analysis we refer to this as ***error propagation***.
- There is an error propagation formula that is used for calculating uncertainties when adding or subtracting measurements with uncertainties and a different error propagation formula for calculating uncertainties when multiplying or dividing measurements with uncertainties. Let's first look at the formula for adding or subtracting measurements with uncertainties.
- **Adding or Subtracting Measurements with Uncertainties**: Suppose you make two measurements,

$$x = x_{best} \pm \Delta x$$

$$y = y_{best} \pm \Delta y$$

What is the uncertainty in the quantity q = x + y or q = x – y? To obtain the uncertainty we will find the lowest and highest probable value of q = x + y. Note that we would like to state q in the standard form of $q = q_{best} \pm \Delta q$ where

$$q_{best} = x_{best} + y_{best}$$

(highest probable value of q = x + y):

$$(x_{best} + \Delta x) + (y_{best} + \Delta y) = (x_{best} + y_{best}) + (\Delta x + \Delta y) = q_{best} + \Delta q$$

(lowest probable value of q = x + y):

$$(x_{best} - \Delta x) + (y_{best} - \Delta y) = (x_{best} + y_{best}) + (\Delta x + \Delta y) = q_{best} - \Delta q$$

Thus, we that

$$\Delta q = \Delta x + \Delta y$$

is the uncertainty in q = x + y

Precision Index:

The precision index describes the spread or dispersion of repeated result about a central value.

(a) 105

(b) 105.0

(c) 0.00105×10^5

(a) and (c) - have three significant figure.

(b) - has four significant figure So (b) has higher precision

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