UNIT III

ORBITS AND PALTFORMS

1. MOTIONS OF PLANETS AND SATELLITES

- **Planets and satellites** move in an elliptical path around the body that they orbit.
- **Orbit** is the path of a body as it moves under the influence of a second body.
- An example is the path of a planet or comet as it moves around the Sun.
- Planets and satellites that orbit other bodies trace out a path called an **ellipse**.
- Planetary motion, as well as satellite motion, is governed by **Kepler's laws and Newton's laws of Gravitation.**

1.1 KEPLER'S LAW OF PLANETARY MOTION:

Johannes Kepler formulated three laws of planetary motion which describes the orbits of planets around the sun. the three laws are described as follows,

(i) First Law (The Law of Ellipses) states that the path of each planet around the sun is an ellipse with the sun at one focus.

(ii) Second Law (The law of Equal Areas) states that a line that connects a planet to the sun sweeps out equal areas in equal time, i.e., the aerial velocity of the planet is always constant.

(iii) Third Law (The Law of Periods) states that the square of the period of revolution of any planet around the sum is proportional to the cube of the semi-major axis of the orbit.

➢ **The Law of Ellipses:**

- Kepler's first law explains that planets are orbiting the sun in a path described as an ellipse.
- An ellipse is a special curve in which the sum of the distances from every point on the curve to two other points is a constant.
- The two other points are known as the foci of the ellipse.

➢ **The Law of Equal Areas:**

- Kepler's second law describes the speed at which any given planet will move while orbiting the sun.
- The speed at which any planet moves through space is constantly changing.
- A planet moves fastest when it is closest to the sun and slowest when it is furthest from the sun.

➢ **The Law of Periods:**

- Kepler's third law compares the orbital period and radius of orbit of a planet to those of other planets.
- The comparison being made is that the ratio of the squares of the periods to the cubes of their average distances from the sun is the same for every one of the planets.
- The law accurately describes the T2/R3 ratio for any satellite about any planet.

1.2 NEWTON'S LAW OF GRAVITATION:

- Newton's law of gravitation, also known as the law of universal gravitation, is a fundamental principle that describes how two objects with mass attract each other due to the force of gravity.
- This law was formulated by Sir Isaac Newton and can be stated as follows:

"Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres."

Mathematically, this can be expressed as:

$$F = G * (m1 * m2) / r^2$$

Where:

F is the gravitational force between two objects.

G is the gravitational constant, a fundamental constant of nature that determines the strength of the gravitational force (approximately 6.674×10^{-11} $N \cdot m^2/kg^2$).

m1 and m2 and are the masses of the two objects.

r is the distance between the centres of the two objects.

Key points about Newton's law of gravitation:

- **Inverse Square Law:** The force of gravity decreases with the square of the distance between the objects. This means that if you double the distance between two objects, the gravitational force between them becomes one-fourth as strong, and if you triple the distance, it becomes one-ninth as strong, and so on.

- **Proportional to Mass:** The force of gravity is directly proportional to the product of the masses of the two objects. This means that if one of the masses is doubled, the gravitational force between them is also doubled.

- **Universal:** Newton's law of gravitation applies to all objects with mass in the universe.

- **Applicability:** For most practical purposes, Newton's law of gravitation remains highly accurate and is still used in many calculations.

1.3 GRAVITATIONAL FIELD AND POTENTIAL:

- **Gravitational Field:**
  - The gravitational field is a concept that describes the influence of a massive object exerts on the space around it due to its gravitational force.

- It is a vector field, meaning it has both magnitude and direction at every point in space.
- The gravitational field at a point in space is defined as the force per unit mass experienced by a test mass placed at that point.
- The gravitational field points radially inward toward the centre of the massive object creating it.
- The SI unit of the gravitational field is Nkg^{-1}.
- Mathematically, it is represented by the vector g and is given by the equation:

$$g = -GM/r^2$$

Where:

g is the gravitational field vector.
G is the universal gravitational constant.
M is the mass of the massive object creating the field.
r is the distance from the centre of the massive object to the point where the field is being measured.

➢ **Gravitational Potential:**

- Gravitational potential is another important concept related to gravity.
- It describes the amount of potential energy per unit mass that a test mass would have at a particular point in a gravitational field.
- The gravitational potential is a scalar quantity.
- The gravitational potential (V) at a point in space is defined as the negative of the work done per unit mass in bringing a test mass from infinity to that point in the gravitational field.
- The unit of gravitational potential is the joule per kilogram (J/kg).
- Mathematically, it is given by the equation:

$$V = -GM/r$$

Where:

V is the gravitational potential.
G is the universal gravitational constant.
M is the mass of the massive object creating the field.
r is the distance from the centre of the massive object to the point where the potential is being measured.

➢ **Relationship between Gravitational Field and Potential:**

- The gravitational field (g) at a point is related to the gravitational potential (V) at that point through the equation:

$$g = -\nabla V$$

Where:

∇V is the gradient of the gravitational potential, which gives the direction and magnitude of the gravitational field.

1.4 ESCAPE VELOCITY

- Escape velocity is the minimum velocity an object must reach in order to break free from the gravitational influence of a massive body, such as a planet or a moon, without any additional propulsion.
- It is the speed required for an object to overcome the gravitational pull of the celestial body and travel indefinitely into space.
- The formula for calculating the escape velocity (Ve) from the surface of a massive body is given by:

$$Ve = \sqrt{(2 * G * M / R)}$$

Where:

Ve is the escape velocity.
G is the universal gravitational constant.
M is the mass of the celestial body (e.g., a planet).
R is the distance from the centre of the celestial body to the point where the escape velocity is being calculated. Typically, this is the radius of the celestial body (e.g., the radius of the planet).

Key points about escape velocity:

- **Dependence on Mass and Radius:** Escape velocity depends on both the mass and the radius of the celestial body. A massive body with a smaller radius will have a higher escape velocity.

- **Direction of Motion:** Escape velocity only considers the speed required to escape the gravitational pull of the celestial body and the direction of motion doesn't matter for it.

- **Zero Escape Velocity:** If an object's initial velocity is equal to or greater than the escape velocity, it can escape the gravitational field of the celestial body. If its velocity is less than the escape velocity, it will bring it back to the celestial body.

- **Applications:** Escape velocity is a critical concept in space exploration and rocketry. It helps to determine the minimum speed required for spacecraft to leave the Earth's or another celestial body's orbit.
- **No Atmosphere Assumption:** The escape velocity formula assumes there is no atmospheric resistance. In real, atmospheric drag can affect the actual velocity required for launch from a planet with an atmosphere.

- **Orbital Velocity:** The escape velocity is related to the orbital velocity of an object. Orbital velocity is the speed required for an object to maintain a stable orbit around a celestial body. If an object reaches orbital velocity, it will not escape but will continue to orbit the body.

2. ORBIT ELEMENTS AND TYPES:

The path of satellite revolving around the earth is known as **orbit**. This path can be represented with mathematical notations. Orbital mechanics is the study of the motion of the satellites that are present in orbits.

2.1 Orbital Elements

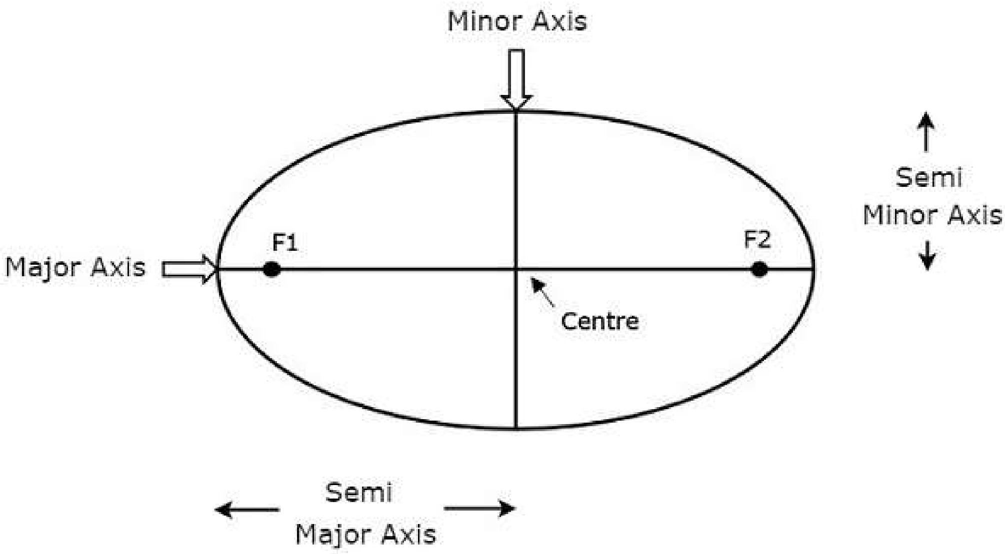
Orbital elements are the parameters, which are helpful for describing the orbital motion of satellites. Following are the **orbital elements**.

- Semi major axis
- Eccentricity
- Mean anomaly
- Argument of perigee
- Inclination
- Right ascension of ascending node

The above six orbital elements define the orbit of earth satellites. Therefore, it is easy to discriminate one satellite from other satellites based on the values of orbital elements.

➢ Semi major axis

- The length of **Semi-major axis (a)** defines the size of satellite’s orbit.
- It is half of the major axis.
- So, it is the radius of an orbit at the orbit's two most distant points.



- Length of semi **major axis (a)** not only determines the size of satellite’s orbit, but also the period of revolution.

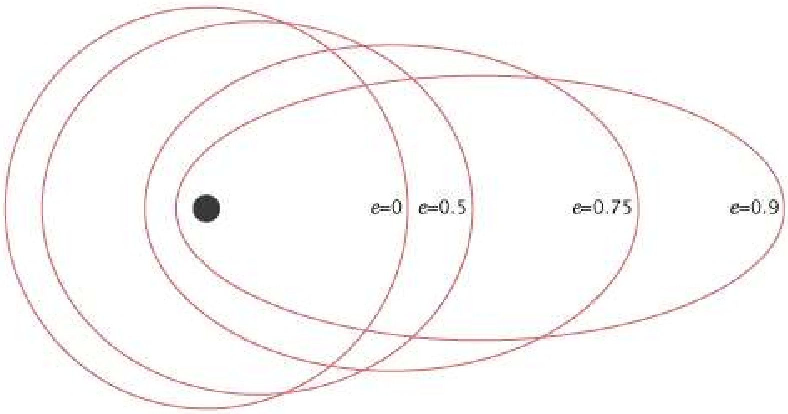
- If circular orbit is considered as a special case, then the length of semi-major axis will be equal to **radius** of that circular orbit.

- **Eccentricity**

- The value of **Eccentricity (e)** fixes the shape of satellite's orbit.
- This parameter indicates the deviation of the orbit's shape from a perfect circle.
- If the lengths of semi major axis and semi minor axis of an elliptical orbit are a & b, then the mathematical expression for **eccentricity (e)** will be,

$$e=a^2-b^2$$

- The value of eccentricity of a circular orbit is **zero**, since both a & b are equal.
- Whereas the value of eccentricity of an elliptical orbit lies between zero and one.



- The satellite orbit corresponding to eccentricity (e) value of zero is a circular orbit.
- And the remaining three satellite orbits are of elliptical corresponding to the eccentricity (e) values 0.5, 0.75 and 0.9.

- **Mean Anomaly**

- For a satellite, the point which is closest from the Earth is known as Perigee.
- **Mean anomaly** (M) gives the average value of the angular position of the satellite with reference to perigee.
- If the orbit is circular, then Mean anomaly gives the angular position of the satellite in the orbit.
- But, if the orbit is elliptical, then calculation of exact position is very difficult.

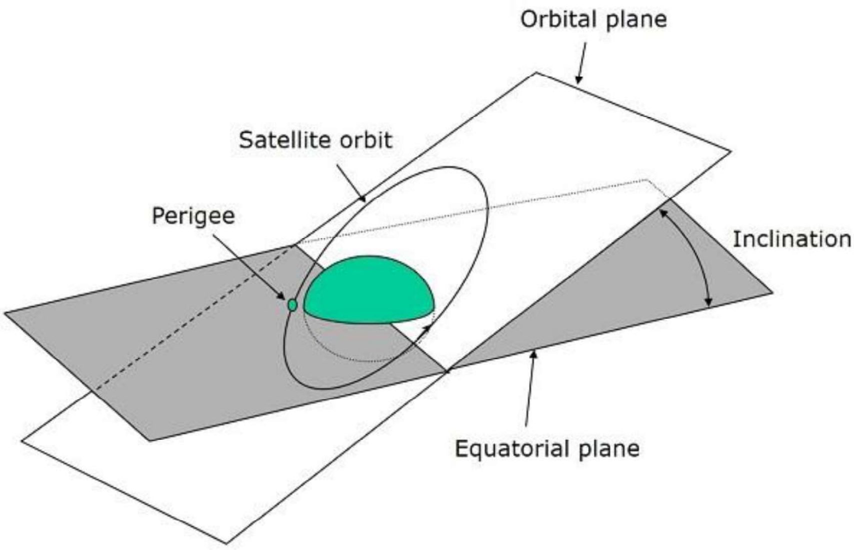
- **Argument of Perigee**

- Satellite orbit cuts the equatorial plane at two points.
- First point is called as **descending node**, where the satellite passes from the northern hemisphere to the southern hemisphere.

- Second point is called as **ascending node**, where the satellite passes from the southern hemisphere to the northern hemisphere.
- **Argument of perigee (ω)** is the angle between ascending node and perigee.
- If both perigee and ascending node are existing at same point, then the argument of perigee will be zero degrees.
- Argument of perigee is measured in the orbital plane at earth's center in the direction of satellite motion.

➢ **Inclination**

- The angle between orbital plane and earth's equatorial plane is known as **inclination (i)**.
- It is measured at the ascending node with direction being east to north.
- So, inclination defines the orientation of the orbit by considering the equator of earth as reference.



- There are four types of orbits based on the angle of inclination.

  ✓ **Equatorial orbit** − Angle of inclination is either zero degrees or 180 degrees.
  ✓ **Polar orbit** − Angle of inclination is 90 degrees.
  ✓ **Prograde orbit** − Angle of inclination lies between zero and 90 degrees.
  ✓ **Retrograde orbit** − Angle of inclination lies between 90 and 180 degrees.

➢ **Right Ascension of Ascending node**

- Right Ascension of ascending node **(Ω)** is the angle between line of Aries and ascending node towards east direction in equatorial plane. Aries is also called as vernal and equinox.
- Satellite's **ground track** is the path on the surface of the Earth, which lies exactly below its orbit.
- The ground track of a satellite can takes a number of different forms depending on the values of the orbital elements.