Module –II

Derive thermal efficiency expression for a Diesel cycle.

This cycle, proposed by a German engineer, Dr. Rudolph Diesel to describe the processes of his engine, is also called the constant pressure cycle. This is believed to be the equivalent air cycle for the reciprocating slow speed compression ignition engine. The P-V and T-s diagrams are shown in Figs 4 and 5 respectively.

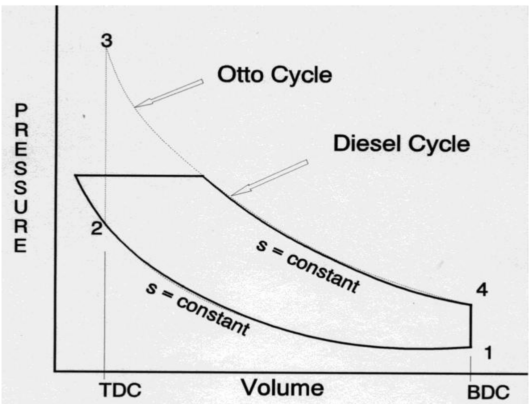

Fig.4: P-V Diagram of Diesel Cycle.

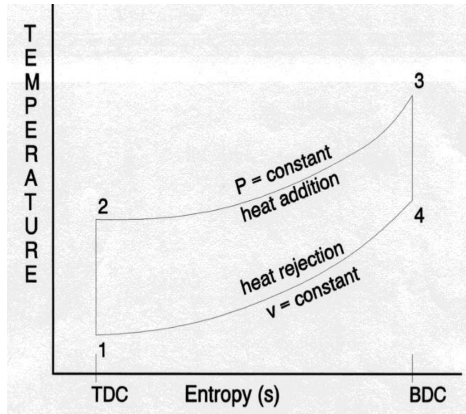

Fig.5: T-S Diagram of Diesel Cycle.

The cycle has processes which are the same as that of the Otto cycle except that the heat is added at constant pressure

The heat supplied, Qs is given by cp(T3 – T2) (22)

Whereas the heat rejected, Qr is given by cv (T4 – T1) (23) and

The thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)}\right\} \qquad (24)$$

From the T-s diagram, Fig. 5, the difference in enthalpy between points 2 and 3 is the same as that between 4 and 1, thus

$$\Delta s_{2-3} = \Delta s_{4-1}$$

$$\therefore c_v \ln\left(\frac{T_4}{T_1}\right) = c_p \ln\left(\frac{T_3}{T_2}\right)$$

$$\therefore \ln\left(\frac{T_4}{T_1}\right) = \gamma \ln\left(\frac{T_3}{T_2}\right)$$

$$\therefore \frac{T_4}{T_1} = \left(\frac{T_3}{T_2}\right)^{\gamma} \text{ and } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{1}{r^{\gamma-1}}$$

Substituting in eq. 24, we get

$$\eta_{th} = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\left(\frac{T_3}{T_2}\right)^{\gamma} - 1}{\frac{T_3}{T_2} - 1}\right] \qquad (25)$$

$$\text{Now } \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c = cut-off\ ratio$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right] \qquad (26)$$

When Eq. 26 is compared with Eq. 8, it is seen that the expressions are similar except for the term in the parentheses for the Diesel cycle. It can be shown that this term is always greater than unity.

Now $r_c = \frac{V_3}{V_2} = \frac{V_3}{V_4}\Big/\frac{V_2}{V_1} = \frac{r}{r_e}$ **where r is the compression ratio and re is the expansion ratio**

Thus, the thermal efficiency of the Diesel cycle can be written as

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\left(\frac{r}{r_e}\right)^{\gamma} - 1}{\gamma\left(\frac{r}{r_e} - 1\right)}\right] \qquad (27)$$

Let $r_e = r - \Delta$ since r is greater than r_e. Here, Δ is a small quantity. We therefore have

$$\frac{r}{r_e} = \frac{r}{r - \Delta} = \frac{r}{r\left(1 - \frac{\Delta}{r}\right)} = \left(1 - \frac{\Delta}{r}\right)^{-1}$$

We can expand the last term binomially so that

$$\left(1 - \frac{\Delta}{r}\right)^{-1} = 1 + \frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \cdots$$

$$\text{Also } \left(\frac{r}{r_e}\right)^{\gamma} = \frac{r^{\gamma}}{(r - \Delta)^{\gamma}} = \frac{r^{\gamma}}{r^{\gamma}\left(1 - \frac{\Delta}{r}\right)^{\gamma}} = \left(1 - \frac{\Delta}{r}\right)^{-\gamma}$$

We can expand the last term binomially so that

$$\left(1 - \frac{\Delta}{r}\right)^{-\gamma} = 1 + \gamma\frac{\Delta}{r} + \frac{\gamma(\gamma+1)}{2!}\frac{\Delta^2}{r^2} + \frac{\gamma(\gamma+1)(\gamma+2)}{3!}\frac{\Delta^3}{r^3} + \cdots$$

Substituting in Eq. 27, we get

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\frac{\Delta}{r} + \frac{(\gamma+1)}{2!}\frac{\Delta^2}{r^2} + \frac{(\gamma+1)(\gamma+2)}{3!}\frac{\Delta^3}{r^3} + \cdots}{\frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \cdots}\right] \qquad (28)$$

6. Derive the mean effective pressure expression for a diesel cycle

$$mep = \frac{1}{V_s}\left[P_2(V_3 - V_2) + \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1}\right] \qquad (29)$$

The pressure ratio P3/P2 is known as explosion ratio rp

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\nu} = r^{\nu} \Rightarrow P_2 = P_1 r^{\nu},$$

$$P_3 = P_2 = P_1 r^{\nu}$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\nu} = P_1 r^{\nu}\left(\frac{V_2}{V_1}\right)^{\nu} = P_1 r_c^{\ \nu}$$

$$V_4 = V_1, V_2 = V_c,$$

$$\frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = r$$

$$\therefore V_s = V_c(r-1)$$

Substituting the above values in Eq 29 to get Eq (29A)

In terms of the cut-off ratio, we can obtain another expression for mep/p1 as follows

$$mep = P_1 \frac{\gamma r^{\gamma}(r_c - 1) - r(r_c^{\gamma} - 1)}{(r-1)(\gamma-1)} \qquad (29A)$$

We can obtain a value of rc for a Diesel cycle in terms of Q" as follows:

$$r_c = \frac{Q'}{c_p T_1 r^{\gamma-1}} + 1 \qquad (30)$$

We can substitute the value of η from Eq. 38 in Eq. 26, reproduced below and obtain the value of mep/p1 for the Diesel cycle.

$$\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_1} \frac{1}{\left[1 - \frac{1}{r}\right][\gamma - 1]}$$

For the Diesel cycle, the expression for mep/p3 is as follows:

$$\frac{mep}{p_3} = \frac{mep}{p_1}\left(\frac{1}{r^{\gamma}}\right) \qquad (31)$$

Modern high speed diesel engines do not follow the Diesel cycle. The process of heat addition is partly at constant volume and partly at constant pressure. This brings us to the dual cycle.

7. A diesel engine working on air standard cycle takes in air at 1bar and 25^0C. The specify volume of air at inlet is 0.8^3/kg. The compression ratio is 14 and heat is added at constant pressure is 840kJ/kg. Find cut – off ratio and air standard efficiency.

Given data:

$p_1 = 1bar$

$T_1 = 25^0\ C = 25 + 273 = 293\ K$

$v_1 = 0.8m^3/kg$

$r = 14$

$Q_s = 840kJ/kg$

Solution:

$$\frac{v_1}{v_2} = 14$$

$$v_2 = \frac{0.8}{14} = 0.05714\ m^3 / kg$$

$$T_2 = T_1 \times \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

$$T_2 = 298 \times (14)^{1.4-1}$$

$$T_2 = 856.38K$$

We know that, $Q_s = m \times C_v (T_3 - T_2)$

$$T_3 = \frac{Q_s}{C_v} + T_2 = \frac{840}{0.718} + 856.38$$

$$T_3 = 2026.3K$$

Consider process 2-3:

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

$$v_3 = \frac{v_2}{T_2} \times T_3 = \frac{0.05714}{856.38} \times 2026.3$$

$$= 0.1352m^3/kg$$

Cut-off ratio, $\rho = \frac{v_3}{v_2} = \frac{0.1352}{0.05714}$

$$= 2.37$$ **Ans.**

$\therefore$ **Air Standard efficiency,** $\eta = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left(\frac{\rho^{\gamma} - 1}{\rho - 1} \right)$

$$= 1 - \frac{1}{1.4(14)^{1.4-1}} \left[\frac{(2.37)^{1.4} - 1}{(2.37 - 1)} \right]$$

= 57.42 % **Ans.**

8. An engine working on an ideal air standard diesel cycle has the compression ratio 15 and heat transfer 1400kJ/kg. Find the pressure and temperature at the end of the each process if the inlet conditions are 280K and 1.1 bar. Find also the air standard efficiency and mean effective pressure.

Given data:

r=15

Q_s = 1400 k J/kg

T_1 = 280 K

P_1=1.1bar = 110 k N/m^2

Solution :

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$T_2 = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \times T_1 = (15)^{1.4-1} \times 280$$

$$T_2 = 827.17 K$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma}{\gamma-1}} \times p_1 = \left(\frac{827.17}{280} \right)^{\frac{1.4}{1.4-1}} \times 110$$

$p_3 = p_2$ = 4874.39kN/m^2

Consider the process 2-3 (Constant pressure process);

$$Qs = m \times (T_3 - T_2) = 1400 \, kJ/kg$$

1400 = 1 × 1.005 × (T_3 -827.17)

T_3 = 2220.2K

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2220.2}{827.17} = 2.684$$

Cut-off ratio, $\rho = \frac{V_3}{V_2} = 2.684$

Consider the process 3-4 (adiabatic expansion);

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \times T_3 \qquad (\because V_4 = V_1)$$

$$= \left(\frac{V_3}{V_2} \times \frac{V_2}{V_1}\right)^{\gamma-1} \times T_3 = \left(\frac{2.684}{15}\right)^{1.4-1} \times 2220.2$$

T₄ **= 1115.5K** **Ans.**

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$p_4 = \left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} \times p_3$$

$$= \left(\frac{1115.5}{2220.2}\right)^{\frac{1.4}{0.4}} \times 4874.39$$

$p_4 = 438.24 kN/m^3$ **Ans.**

Air standard efficiency,

$$\eta = 1 \frac{1}{\gamma (r)^{\gamma-1}} \left(\frac{\rho^{\gamma} - 1}{\rho - 1}\right)$$

$$= 1 - \frac{1}{1.4(15)^{1.4-1}} \left(\frac{(2.684)^{1.4} - 1}{2.684 - 1}\right)$$

= 57.16% **Ans**

Mean effective pressure,

$$P_m = \frac{p_1 r^{\gamma} \left[\gamma(\rho - 1) - r^{1-\gamma}\left(\rho^{\gamma} - 1\right)\right]}{(\gamma - 1)(r - 1)}$$

$$= \frac{100 \times (15)^{1.4} \left[1.4(2.684 - 1) - (15)^{1-1.4}\left[(2.684)^{1.4} - 1\right]\right]}{(1.4 - 1)(15 - 1)}$$

***p_m* = 1066.33kN/m²** **Ans.**

9.In an engine working on diesel cycle, inlet pressure and temperature are l bar and 17^0 C respectively. Pressure at the end of adiabatic compression is 35 bar. The ratio of expansion i.e., after constant pressure heat addition is 5. Calculate the hear addition, heat rejection the efficiency of the cycle, and mean effective pressure.

Assume =1.4, C_p = 1.004kJ/kgK and C_v = 0.717kJ/kgK.

Given data

Diesel cycle

p_1 = 1 bar = 100 kN/m^2

T_1 = 17^{0}C = 290K

p_2 =35bar = 3500 kN/m^2

$$\frac{V_4}{V_3} = \frac{V_1}{V_3} = 5$$

$$\gamma = 1.4; C_P = 1.004 \text{ kJ/kgK};$$

C_v=0.717 kJ/kgK

Solution

Consider process 1-2 ;

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = \left(\frac{3500}{100}\right)^{\frac{1.4-1}{1.4}} \times 290 = 800.87K$$

$$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

Compression ratio, $r = \left(\frac{3500}{100}\right)^{\frac{1}{1.4}} = 12.67$

$$\frac{V_1}{V3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3}$$

$$5 = 12.67 \times \frac{V_2}{V_3}$$

Cut – off ratio, $\rho = \frac{V_3}{V_2} = \frac{12.67}{5} = 2.534$

Consider process: Constant pressure process;

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow T_3 = \frac{V_3}{V_2} \times T_2 = 2.534 \times 800.87$$

T_3 = 2090.4K

Heat supplied during 2 – 3

$Q_s = C_p (T_3 – T_2)$

= 1.004(2029.4 – 800.87)

Q_s = 1233.45kJ/kg **Ans.**

Consider process 3-4: Adiabatic process;

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \times T_3 = \left(\frac{1}{5}\right)^{1.4-1} \times 2029.4$$

T_4 = 1066K

Heat rejection during constant volume process 4-1

$Q_R = C_V (T_4 – T_1)$ = 0.717(1066-290)

Q_R = 556.43kJ/kg **Ans.**

Efficiency, $\eta = 1 - \frac{Q_R}{Q_S} = 1 - \frac{556.43}{1233.45}$

= 54.88% **Ans.**

Mean effective pressure, p_m

$$P_m = \frac{p_1 r^{\gamma}\left[\gamma(\rho-1) - r^{1-\gamma}\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)}$$

$$= \frac{1\times(12.67)^{1.4}\left[1.4(2.534-1) - (12.67)^{1-1.4}\left((2.534)^{1.4}-1\right)\right]}{(1.4-1)(12.67-1)}$$

P_m = 8.83bar **Ans.**

10. **In an air standard diesel cycle, the pressure and temperatures of air at the beginning of Cycle are 1bar and 40°C. The temperatures before and after the heat supplied are 400⁰C and 1500⁰C. Find the air standard efficiency and mean effective pressure of the cycle. What is the Power output if it makes 100cycles / min?**

Given data:

$p_1 = 1bar = 100kN/m^2$

$T_1 = 40^0C = 40 + 273 = 313K$

$T_2 = 400^0C = 673K$

$T_3 = 1500^0C = 1773K$

Solution:

Consider the process 1-2 (Isentropic compression)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

Compression ratio, $r = \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{673}{313}\right)^{\frac{1}{1.4-1}}$

= 6.779

Consider the process 2-3 (Constant pressure heating)

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

Cut off ratio, $\rho = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{1773}{673} = 2.634$

Efficiency, $\eta = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left(\frac{\rho^\gamma - 1}{\rho - 1}\right)$

$$= 1 - \frac{1}{1.4(6.779)^{1.4-1}}\left(\frac{2.634^{1.4} - 1}{2.634 - 1}\right)$$

= 0.4142 = 41.42 % **Ans.**

Mean effective pressure,

$$\rho_m = \frac{p_1 r^\gamma\left(\gamma(\rho-1) - r^{1-\gamma}\left(\rho^\gamma - 1\right)\right)}{(\gamma-1)(r-1)}$$

$$\frac{100\times(6.779)^{1.4}\left(1.4(2.634-1)-(6.779)^{1-1.4}\left((2.634)^{1.4}-1\right)\right)}{(1.4-1)(6.779-1)}$$

$\rho_m = 597.77kN/m^2$ **Ans.**

Heat supplied = m x C_p (T_3-T_2)

= 1 x 1.005 (1773 – 673)

Q_S = 1105.5*kJ/kg*

Work done $= \eta \times Q_S = 0.4142 \times 1105.5$

=457.89*kJ/kg* $\left[\because \eta = 1 - \frac{Q_R}{Q_S} = \frac{W}{Q_S}\right]$

Power = *W* x cycles/min = 457.89 x 100

= 45 x 10^{-3}*kJ/kg-min* = 763.16*kJ/kg-sec*

= *763.16kW/kg* **Ans.**

11) Find the air standard efficiency of a diesel cycle when the compression ratio andcut–off ratio are 15 & 1.84 respectively. Assume γ = 1.4.

Given:

Compression ratio (r) =

15 Cut – off ratio(ρ) = 1.84

γ = 1. 4

Required: η

Solution:

$$\eta = 1 - \frac{1}{\gamma r^{\gamma-1}}\left[\frac{\rho^{\gamma-1}-1}{\rho-1}\right]$$

$$= 1 - \frac{1}{1.4(15)^{1.4-1}}\left[\frac{1.84^{1.4-1}-1}{1.84-1}\right]$$

η= 0.612---- Ans

12)In a diesel engine the pressure at the beginning of compression is 1 bar. Compression ratio is 14 : 1 and cut–off takes place at 10% of the stroke. Calculatethe air standard efficiency and ideal mep of the cycle (γ = 1. 4 for air).

Given:

Initial pressure (p_1) = 1

bar.Compression ratio (r) =

14.

Cut – off takes place at 10 % of

stroke,i.e., $V_3 – V_2 = 0.1$

$(V_1 – V_2)$

$\gamma = 1.4$

***Required* : η & p_m**

Solution:

$$\eta = 1 - \frac{1}{\gamma r^{\gamma - 1}} \left[\frac{\rho^{\gamma - 1} - 1}{\rho - 1}\right]$$

ρ = cut – off ratio = V_3 /

V_2r = $V_1 / V_2 = 14$

$\therefore V_1 = 14 V_2$

$\therefore V_3 - V_2 = 0.1 (14 V_2 - V_2)$

$V_3 = 2.3 V_2$.

$V_3 / V_2 = \rho = 2.3$

$$\therefore \quad \eta = 1 - \frac{1}{1.4 (14)^{1.4 - 1}} \left[\frac{2.3^{1.4 - 1} - 1}{2.3 - 1}\right]$$

= 0.577 ------ Ans

Mean effective pressure (p_m)

$$p_m = p_1 r^{\gamma} \left[\frac{\gamma (\rho - 1) - r^{1-\gamma} (\rho^{\gamma} - 1)}{(\gamma - 1)(r - 1)}\right] \rightarrow \text{for Diesel cycle}$$

$$= 1 \times 14^{1.4} \left[\frac{1.4 (2.3 - 1) - 14^{1 - 1.4} (2.3^{1.4} - 1)}{(1.4 - 1)(14 - 1)}\right] = 8.133 \text{ bar --------- Ans}$$