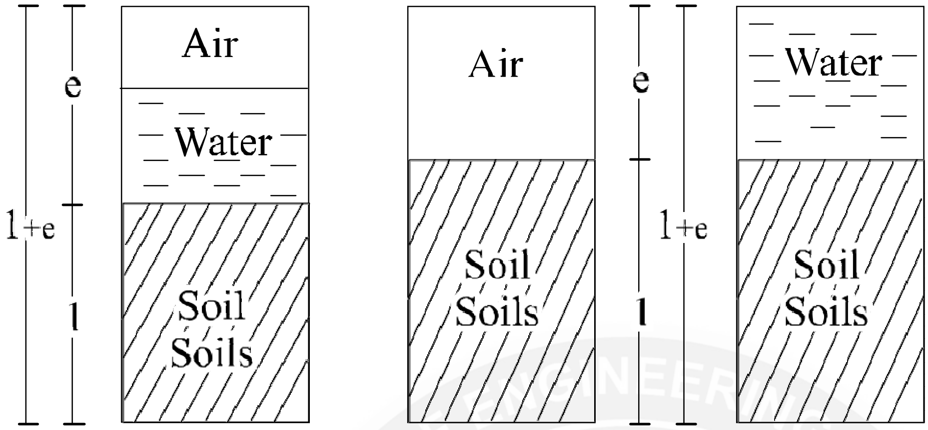
1.4. INTER RELATIONS:-



Soil Element interms of 'e' **Soil Element interms of 'e'**

Fig – 1.2 Phase diagram

1) Relation between 'e' and 'n'

$$n = \frac{V_v}{V}$$

$$V = V_v + V_s$$

$$e = \frac{V_v}{V_s}$$

$$e = \frac{V_v}{V - V_v}$$

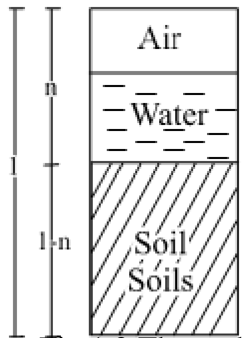


Fig 1.3 Three phase diagram

Divide V_v on top and bottom

$$e = \frac{1}{\frac{V}{V_v} - 1}$$

$$= \frac{1}{\frac{1}{n} - 1}$$

$$= \frac{1}{\frac{1-n}{n}}$$

$$e = \frac{n}{1-n}$$

2) Relation between ' e , s, w ' and ' G '

Refer fig 1.4 e - water voids ratio

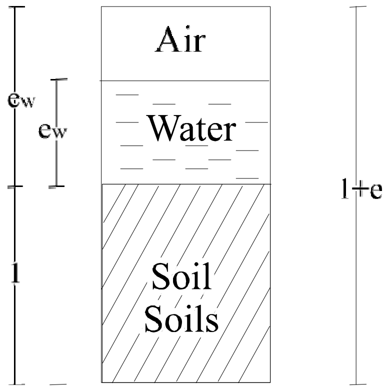


Fig 1.4 Three phase diagram

$$G = \frac{\frac{W_s}{V_s}}{\frac{W_w}{V_w}}$$

$$= \frac{W_s}{V_s} x \frac{V_w}{W_w}$$

$$= \frac{W_s}{W_w} X \frac{V_w}{V_s}$$

$$= \frac{1}{w} X \frac{V_w}{V_s}$$

Multiply and divide by V_v

$$G = \frac{1}{w} X \frac{V_w}{V_v} X \frac{V_v}{V_s}$$

$$G = \frac{1}{w} x S_r X e$$

$$W \text{ x } G = S_r X e$$

For fully saturated S_r=1

e=w.G

3) <u>Relation between ' γ , γ_d' and ' w '</u>

We have

$$w = \frac{W_w}{W_s}$$

$$1 + w = \frac{W_w}{W_s} + 1$$

$$= \frac{W_w + W_s}{W_s}$$

$$= \frac{W}{W_s}$$

$$W_s = \frac{w}{1 + w}$$

Dividing both sides by 'V' then we get,

$$\frac{W_s}{V} = \frac{w}{V(1 + w)}$$

Finally,

$$\gamma_d = \frac{\gamma}{1 + \text{w}}$$

2) Relation between ' γ , γ_d, γ_{sat}' and 'γ'

Refer fig 1.4,

$$\text{We have } \gamma = \frac{W}{V}$$

$$= \frac{w_s + W_w}{V}$$

$$= \frac{V_s\gamma_s + V_w\gamma_w}{V}$$

$$\text{Therefore } \gamma = \frac{1.\gamma_s + e_w\gamma_w}{1+e}$$

$$= \frac{G\gamma_w + e.s.\gamma_w}{1 + e}$$

$$= \frac{(G + es)\gamma_w}{1 + e}$$

For dry soil mass, s = 0

$$\gamma = \frac{G.\gamma_w}{1 + e}$$

For fully saturated soil mass, s = 1 , $\gamma = \gamma_{sat}$

So that,

$$\gamma_{sat} = \frac{(G + e)\gamma_w}{1 + e}$$

For submerged condition, $\gamma' = \gamma_{sat} - \gamma_w$

$$\gamma' = \frac{(G + e)\gamma_w}{1 + e} - \gamma_w$$

$$= \frac{(G + e)\gamma_w - \gamma_w(1 + e)}{1 + e}$$

$$= \frac{G\gamma_w - \gamma_w}{1 + e}$$

finally,

$$\gamma' = \frac{(G - 1)\gamma_w}{1 + e}$$

4) Relation between ' e , s ' and ' n_a'

$$n_a = \frac{e(1-s)}{1+e}$$

5) Relation between ' n_a , a_c ' and ' n'

$$n_a = n \, . \, a_c$$

6) Relation between ' γ_d , G ' and ' n'

$$\gamma_d = (1-n)\, G\, \gamma_w$$

7) Relation between ' γ_{sat} , G ' and ' n'

$$\gamma_{sat} = (1-n)\, G\, \gamma_w + \; n.\gamma_w$$

8) Relation between ' γ' , γ_d ' and ' n'

$$\gamma' = \gamma_d - \; (1-n) \; \gamma_w$$

9) Relation between ' γ_{sat} , γ , γ_d ' and ' s '

$$\gamma = \gamma_d + s\;(\gamma_{sat} - \gamma_d)$$

10) Relation between ' γ_d , G, w ' and ' s'

$$\gamma_d = \frac{G.\gamma_w}{1+\frac{w}{s}} \text{ if S=1 } \gamma_d = \frac{G.\gamma_w}{1+w_{sat}.G}$$

11) Relation between ' γ_d , G , w ' and ' n_a'

$$\gamma_d = \frac{(1-n_a)G.\gamma_w}{1+w.G}$$

12) Density Index and Relative compaction

It is defined as the ratio of the difference between the voids ratio of the soil in its *loosest* state 'e_{max}' and its natural voids ratio 'e' to the difference between the void ratio in the *loosest and densest* states. (e_{min})

$$\text{ie,} I_D = \frac{e_{max}-e}{e_{max}-e_{min}}$$

It is also referred as Density Index (or) Relative density (or) Degree of density. It is used to express the relation the compactness or degree of compaction of a natural cohesion less soil deposit. Degree of compaction is also sometimes expressed interims of an index called relation compaction (R_c).

$$\text{ie, } R_c = \frac{\gamma_d}{\gamma_{dmax}} \text{ or } = \frac{1+e_{min}}{1+e}$$

γ_d max – maximum dry density from compaction test.

PROBLEM

1) A soil sample in its undisturbed state was found to have volume of 105cm^3 and mass of 201 g. After oven drying the mass got reduced to 168g. Compute (i) water content (ii) void ratio (iii) porosity (iv) degree of saturation (v) air content. Take G = 2.7

Given:

Volume of soil mass, V = 105 cm^3

Mass of soil mass, M = 201 g

Dry mass of soil, M_d = 168 g

Specific gravity, G = 2.7

Solution :

(i) Water content,

$$w = \frac{W_w}{W_s or\ W_d} = \frac{201 - 168}{168}$$

= 0.196 => 19.6%

(ii) Void ratio,

$$e = \frac{G.\gamma_w}{\gamma_d} - 1$$

$$\gamma_d = \frac{G.\gamma_w}{1 + e}$$

ie, Dry density, $\gamma_d = \frac{W_s or\ W_d}{V}$

$$= \frac{168}{105} = 1.6\ g/cm^3$$

void ratio,

$$e = \frac{G.\gamma_w}{\gamma_d} - 1$$

$$= \frac{2.7x1}{\gamma_d} - 1 = 0.69$$

Porosity (n)

$$n = \frac{e}{1 + e}$$

$$= \frac{0.69}{1+0.69}$$

$$= 0.408 => 40.8\%$$

(iii) Degree of saturation (s)

$$\text{Wkt,} \quad e.s = w.G$$

$$S = \frac{wG}{e}$$

$$= \frac{0.190X2.7}{0.69}$$

$$= 0.767 => 76.7\%$$

(iv) Air Content (a_c)

$$\text{Wkt,} \ a_c = 1 - s => 1 - 0.767 => 0.233 => 23.3\%$$

2)A natural soil deposit has a bulk unit weight of 18.44 KN/m^3 and water content of 5%. Calculate the amount of water required to be added to 1m^3 of soil to raise the water content to 15% Assume the voids ratio to remain constant. What will then be the degree of saturation? Assume G = 2.67

<u>Given :</u>

γ = 18.44 KN/m^3,

V = 1 m^3,

w_1 = 5%,

w_2 = 15%,

e = constant,

G = 2.67

Solution :--

$$\gamma_d = \frac{\gamma}{1+w}$$

$$= \frac{18.44}{1+0.05}$$

$$=17.56 KN/m^3$$

$$\gamma_d = \frac{W_d}{V} = 17.56 = \frac{W_d}{1}$$

$$W_d = 17.56 \text{ KN}$$

If W=0.05(5%)

$$w = \frac{W_w}{W_s}$$

$$0.05 = \frac{W_w}{17.56}$$

$W_w = 0.88$ KN

$$\gamma_w = \frac{w_w}{V_w}$$

$$9.81 = \frac{0.88}{V_w}$$

$V_w=0.089\ m^3$

If W=0.15(15%)

$$w = \frac{W_d}{W_w}$$

$$0.15 = \frac{W_w}{17.56}$$

$W_w = 2.634$ KN

$$\gamma_w = \frac{w_w}{V_d}$$

$$9.81 = \frac{2.634}{V_w}$$

$V_w=0.27\ m^3$

The amount of water required to raise the water content = 0.270 – 0.089 = 0.181 m^3

$$\gamma_d = \frac{G.\gamma_w}{1+e}$$

$$17.56 = \frac{2.67X9.81}{1+e}$$

e=0.49

$$e = \frac{w.G}{S_r}$$

$$0.49 = \frac{0.15X2.67}{S_r}$$

$$S = 0.81 = 81\%$$

3)A soil specimen has a water content of 10% and a wet unit weight of 20kN/m3.If the specific gravity of solids is 2.70, determine the dry unit weight, void ratio, and the degree of saturation. Take γ_w=10kN/m^3.

Give data:

w=10%

γ=20kN/m^3

G=2.7

γ_w=10kN/m^3.

To Find:

γd=?

e =?

Sr=?

Solution:

$$\gamma_d = \frac{\gamma}{1+w}$$

$$= \frac{20}{1+w} = 18.18KN/m^3$$

$$1+e = \frac{G.\gamma_w}{\gamma_d}$$

$$1+e = \frac{2.7X10}{18.18} = 1.49$$

e =1.49-1=0.49

$$S = \frac{w\ G}{e}$$

$$= \frac{0.1X2.7}{0.49} = 0.551 = 55.1\%$$

4)A soil to be excavated from a borrow pit which has a density of 1.75 g/cc and water content of 12%. The specific gravity of soil is 2.7. the soil is compacted so that the water content is 18% and dry density 1.65g/cc for 1000 m^3 of soil, in fill estimate,

a)The quantity of soil excavated from the pit in m^3

b)The amount of water to be added.Also determine the void ratio soil in borrow pit and fill.

<u>Given:</u>

ρ = 1.75 g/cc, w = 12%, G = 2.67, V = 1000 m^3, after compaction, γ_d = 1.65g/cc,w=18%

Solution :

Let us use suffix '1' for borrow pit and '2' for the fill

For the borrow pit $\rho = \frac{G\rho_w(1+w)}{1+e}$

$$e_1 = \frac{G\rho_W(1+W)}{\rho} - 1$$

$$= \frac{2.7X(1+0.12)}{1.75} - 1 = 0.728$$

For the fill

$$\rho_d = \frac{G\rho_w}{1+e}$$

$$e_2 = \frac{G\rho_w}{\rho_d} - 1$$

$$= \frac{2.7X1}{1.65} - 1 = 0.636$$

Since the volume of soilds remains constant

$$V_s = \frac{V_1}{1+e_1} = \frac{V_2}{1+e_2}$$

$$= \frac{1000}{1+0.636}$$

$$= 611.1m^3$$

$$V_1 = 611.1(1+0.728) = 1056m^3$$

$$w = \frac{M_w}{M_d}$$

$$M_d = M_s = V_s G.\rho_w = 611.1X2.7X1 = 1649.97$$

$$M_{w1} = M_d.w_1 = 1649.97X0.12 = 197.996m^3$$

$$M_{w2} = M_d.w_2 = 1649.97X0.18 = 296.995m^3$$

$$\text{Water to be added} = M_{w2} - M_{w1} = 296.995 - 197.996 = 99m^3$$

5)A soil sample has a porosity of 40% The specific gravity of solids is 2.70.Calculate (a)voids ratio, (b)dry density, (c)unit weight if the soil is 50% saturated, and (d)unit weight if the soil is completely saturated.

Given:

n=40%=0.4

G=2.7

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$$

$$\gamma_d = \frac{G.\gamma_w}{1+e}$$

$$= \frac{2.7X9.81}{1+0.667} = 15.89\ KN/m^3$$

$$e = \frac{wG}{S}$$

$$w = \frac{eS}{G} = \frac{0.667X0.5}{2.7} = 0.124$$

$$\gamma = \gamma_d(1+w)$$

$$= 15.89(1+0.124) = 17.86\ KN/m^3$$

When the soil is fully saturated e=w.G

$$w = \frac{e}{G}$$

$$= \frac{0.667}{2.7} = 0.247$$

$$\gamma_{sat} = \gamma_d(1+w)$$

$$=15.89(1+0.247)=19.81KN/m^3$$

6)The insitu- density of an embankment, compacted at a water content of 12% was determined with the help of a core cutter. The empty mass of the cutter was1286g and the cutter full of soil has a mass of 3195g, the volume of the cutter being 1000cm3 Determine the bulk density, dry density and the degree of saturation of the embankment.

If the embankment becomes fully saturated during rains, what would be its water content and saturated unit weight? Assume no volume changes in soil on saturation. Take the specific gravity of the soilas2.70.

Solution:

Mass of soil in cutter, M=3195-1286=1909g

Bulk density,$\rho = \frac{M}{V}$

$$= \frac{1909}{1000} = 1909\ g/cm^3$$

γ=9.81X1.909=18.73KN/m3

$$\gamma_d = \frac{\gamma}{1+w}$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1$$

$$= \frac{2.7x9.81}{16.72} - 1 = 0.584$$

$$S = \frac{wG}{e} = \frac{0.12X2.7}{0.584} = 0.55 = 55\%$$

At saturation since the volume remains the same ,the voids ratio also remains unchanged now,

e=w.G

$$w = \frac{e}{G} = \frac{0.584}{2.7} = 0.216 = 21.\%$$

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w$$

$$= \frac{2.7+0.584}{1+0.584} X9.81$$

$$=20.34KN/m^3$$

7)The insitu- percentage voids of a sand deposit is 34%. For determining the density index, dried sand from the stratum was first filled loosely in a 1000cm^3 mould and was then vibrated to give a maximum density. The loose dry mass in the mould was 1610 g and the dense dry mass at maximum compaction was found to be 1980g. Determine the density index if the specific gravity of the sand particles is 2.67.

Solution:

n=34%=0.34

$$e = \frac{n}{1-n} = \frac{0.34}{1-0.34} = 0.515$$

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

$$= \frac{2.67X9.81}{1+0.515} = 17.289$$

$$(\gamma_d)_{max} = \frac{1980}{1000}X9.81$$

$$= 19.42\ KN/m^3$$

$$(\gamma_d)_{min} = \frac{1610}{1000}X9.81$$

$$= 15.79\ KN/m^3$$

$$e_{min} = \frac{G\gamma_w}{(\gamma_d)_{max}} - 1$$

$$= \frac{2.67X9.81}{19.42} - 1 = 0.349$$

$$e_{max} = \frac{G\gamma_w}{(\gamma_d)_{min}} - 1$$

$$= \frac{2.67X9.81}{15.79} - 1 = 0.659$$

$$I_d = \frac{\mathrm{e_{max-e}}}{\mathrm{e_{max}} - \mathrm{e_{min}}}$$

$$= \frac{0.659-0.515}{0.659-0.349} = 0.465 = 46.5\%$$

8)Sandy soil in a borrow pit has unit weight of solids as 26.3 kN/m^3 equal to 11% and bulk unit weight equal to16.4kN/m^3 How many cubic meter of compacted fill could be constructed of 3500m^3 of sand excavated from the borrow pit, if the required value of porosity in the compacted fill is30%?Also compute the change in degree of saturation.

Let us use suffix I for the borrow pit soil and 2 for the compacted soil. Assuming that weight and water content do not change during construction, the change in the volume can be calculated from the change in the unit weight.

$$\frac{V_1}{V_2} = \frac{W/\gamma_1}{W/\gamma_2} = \frac{\gamma_2}{\gamma_1} = \frac{1+e_1}{1+e_2}$$

$$V_2 = V_1 X \frac{1+e_1}{1+e_2} = V_1 X \frac{1-n_1}{1-n_2} \text{-------(1)}$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1$$

$$= \frac{\gamma_s}{\gamma/(1+w)} - 1$$

$$= \frac{\gamma_s(1+w)}{\gamma} - 1$$

$$e_1 = \frac{\gamma_s(1+w)}{\gamma_1} - 1$$

$$= \frac{26.3(1+0.11)}{16.4} - 1 = 0.780$$

$$e_2 = \frac{n_2}{1-n_2}$$

$$= \frac{0.3}{1-0.3} = 0.429$$

$$V_2 = V_1 X \frac{1+e_1}{1+e_2}$$

$$= 3500X\frac{(1+0.429)}{(1+0.780)} = 2810m^3$$

$$S_1 = \frac{w\gamma_s}{e_1\gamma_w} = \frac{0.11X26.3}{0.78X9.81} = 0.378$$

$$S_2 = \frac{w\gamma_s}{e_2\gamma_w} = \frac{0.11X26.3}{0.429X9.81} = 0.687$$

Worked Example

1) A sample of dry soil weighs 68gm.Find the volume of voids if the total volume of the sample is 40ml and the specific gravity of solids is 2.65.Also determine the void ratio.
2) A moist soil sample weighs 3.52N.After drying in an oven, its weight is reduced to 2.9N.The specific gravity of solids and the mass specific gravity are,respectively2.65 and 1.85.Determine the water content ad voidratio,porosity and the degree of saturation. Take γ_w=10kN/m^3.
3) A soil has a porosity of 40%, the specific gravify of solids of 2.65and a water content of 12%.Determine the mass of water required to be added to 100m^3 of this soil for full saturation.
4) A sample of saturated soil has a water content of 25percnt and a bulk unit Weight of 20 kN/m^3 .Determine dry density, void ratio and specific gravity of

solid particles. What would be the bulk unit weight of the same soil at the same void ratio but at a degree of saturation of 80%?Take γ_w=10kN/m^3.

5) A sample of clay was coated with paraffin wax and its mass, including the1 mass of wax ,was found to be 697.5gm.The sample was immersed in water and the volume of water displaced was found to be 355ml.The mass of the sample without wax was 690.0g and the water content of the representative specimen was18%.Determie the bulk density ,dry density, void ratio and the degree of saturation. The specific gravity of the solids was 2.70 and that of the wax was 0.89.

6) In a compaction test of a soil, the mass of wet soil when compacted in the mould was 1.855kg. The water content of the soil was16%.If the volume of the mould was 0.945litres,determine the dry density,void ratio,degree of saturation and percentage air voids.TakeG=2.*68.*