UNIT 4

ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

4.1 SPRINGS

4.1.1 Introduction

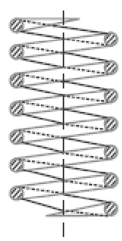
A spring is defined as an elastic body, whose functionis to distort when loaded and to recover its original shape
when the load is removed. The various importantapplications of springs are as follows :

1. To cushion, absorb or control energy due to eithershock or vibration as in car springs, railwaybuffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and springloadedvalves.
3. To control motion by maintaining contact betweentwo elements as in cams and followers.
4. To measure forces, as in spring balances andengine indicators.
5. To store energy, as in watches, toys, etc.

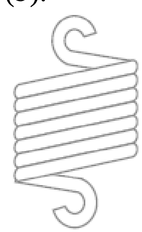
4.1.2 Types of Springs

Though there are many types of the springs, yet thefollowing, according to their shape, are important from the
subject point of view.

Helical springs*.** The helical springs are made up of a wire coiled in the form of a helix andis primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compressionhelical spring as shown in Fig. 4.1 (*a*) and ***tension helical spring*** as shown in Fig. 4.1 (*b*).



(*a*) Compression helical spring.



(*b*) Tension helical spring.

Fig.4.1 Helical springs.

In ***open coiled helical springs*,** the spring wire is coiled in such a way that there is a gap between
the two consecutive turns, as a result of which the helix angle is large.

The helical springs have the following advantages:
(*a*) These are easy to manufacture.
(*b*) These are available in wide range.
(*c*) These are reliable.
(*d*) These have constant spring rate.
(*e*) Their performance can be predicted more accurately.
(*f*) Their characteristics can be varied by changing dimensions.

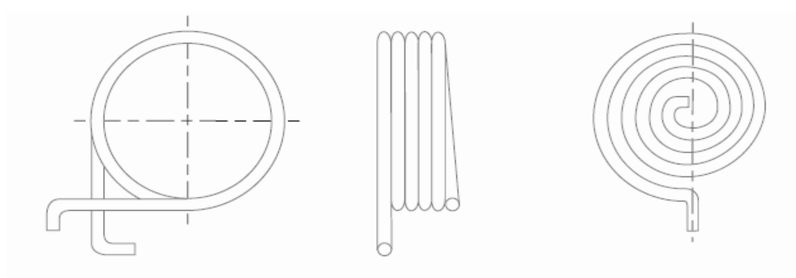
2. *Conical and volute springs*.
The conical spring, as shown in Fig. 4.2 (*a*), is wound with a uniform pitch whereasthe volute springs, as shown in Fig. 4.2 (*b*), are wound in the form of paraboloid with constant pitch. The major stresses produced in conical and volute springs are also shear stresses due to twisting.

(*a*) Conical spring. (*b*) Volute spring.

Fig.4.2 Conical and volute springs.

3. *Torsion springs*. These springs may be of ***helical*** or ***spiral*** type as shown in Fig. 4.3. The**helical type** may be used only in applications where the load tends to wind up the spring and are usedin various electrical mechanisms. The **spiral type** is also used where the load tends to increase thenumber of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Fig.4.3 Torsion springs.

4. *Laminated or leaf springs*. The laminated or leaf spring (also known as ***flat spring*** or ***carriagespring***) consists of a number of flat plates (known as leaves) of varying lengths held together bymeans of clamps and bolts, as shown in Fig. 4.4. These are mostly used in automobiles.The major stresses produced in leaf springs are tensile and compressive stresses.

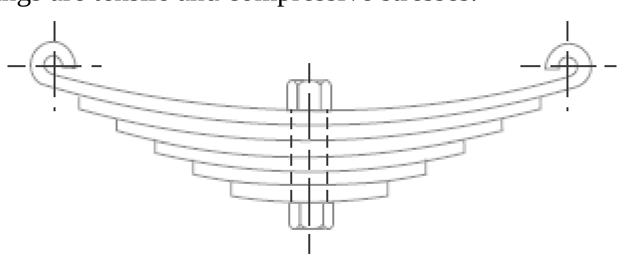


Fig.4.4 Laminated or leaf springs

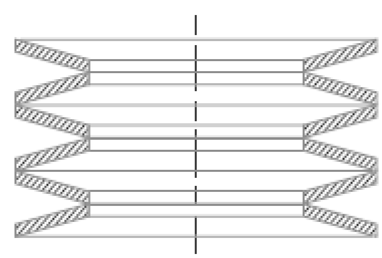


Fig.4.5 Disc or bellevile springs.

5. ***Disc or bellevile springs*.** These springs consist of a number of conical discs held togetheragainst slipping by a central bolt or tube as shown in Fig. 4.5. These springs are used in applicationswhere high spring rates and compact spring units are required.The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

4.1.3 Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience andit should be creep resistant. It largely depends upon the service for which they are used *i.e*. severeservice, average service or light service.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrousmaterials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in specialcases to increase fatigue resistance, temperature resistance and corrosion resistance.

4.1.4 Terms used in Compression Springs

The following terms used in connection with compression springs are important.

1. ***Solid length*.** When the compression spring is compressed until the coils come in contactwith each other, then the spring is said to be ***solid*.** The solid length of a spring is the product of totalnumber of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$L_S = n'.d$

wheren' = Total number of coils, and

d = Diameter of the wire.

2. ***Free length*.** The free length of a compression spring, as shown in Fig. 4.6, is the length ofthe spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection \or compression of the spring and the clearance between the adjacent coils (when fully compressed).

Mathematically,

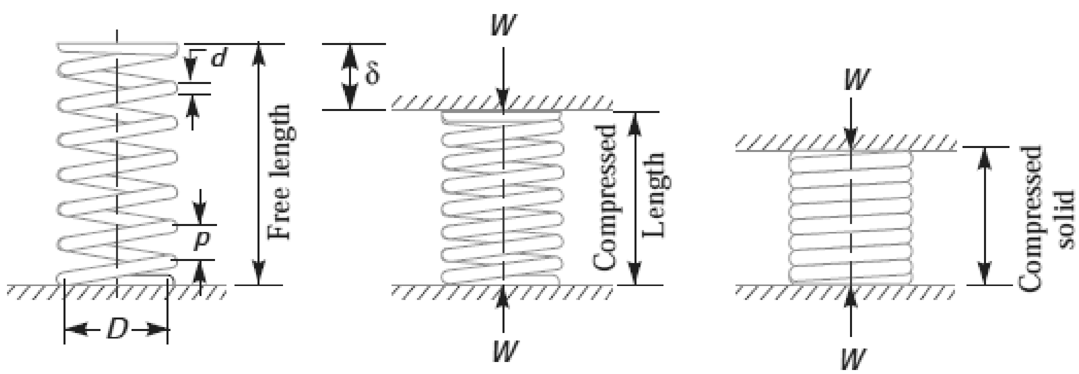


Fig 4.6. Compression spring nomenclature.

Free length of the spring,

L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta max + 0.15\ \delta max$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta max + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. *Spring index*. The spring index is defined as the ratio of the mean diameter of the coil to thediameter of the wire. Mathematically,
Spring index, $C = D / d$

where$D$ = Mean diameter of the coil, and
d = Diameter of the wire.

4. *Spring rate*. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$
where$W$ = Load, and
δ = Deflection of the spring.

5. *Pitch*. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state.
Mathematically,

Pitch of the coil, $p = \frac{FreeLength}{n^2 - 1}$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $p = \frac{L_{F-L_S}}{n'} + d$

whereL_F = Free length of the spring,
L_S = Solid length of the spring,
n' = Total number of coils, and
d = Diameter of the wire.

4.1.5 End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply theload. Various forms of end connections are shown in Fig. 4.7.

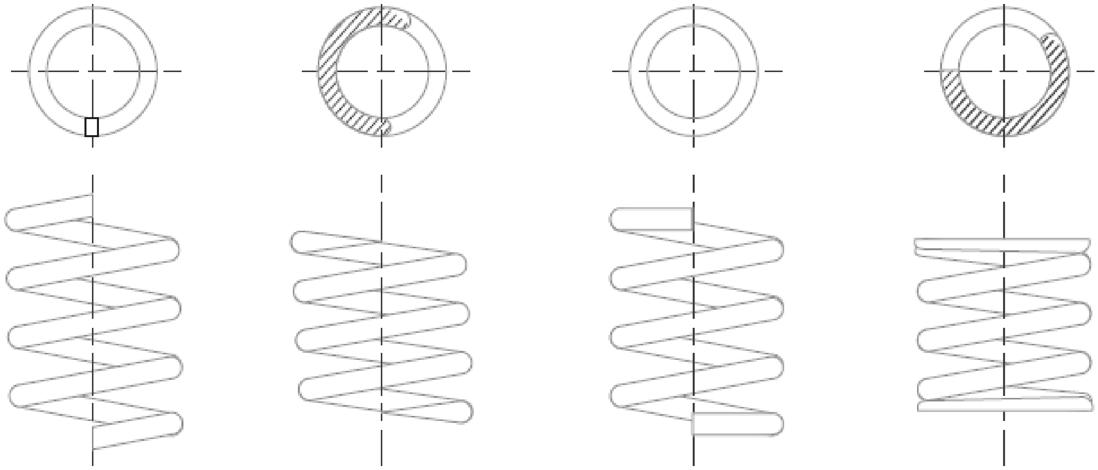


Fig 4.7 End connections for compression helical spring.

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and
hence are termed as ***inactive coils***. The turns which impart spring action are known as ***active turns*.** As the load increases, the number of inactive coils also increases due to seating of the end coils andthe amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table showsthe total number of turns, solid length and free length for different types of end connections.

Table 4.1. Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\, d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\, d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\, d$	$p \times n + 2d$

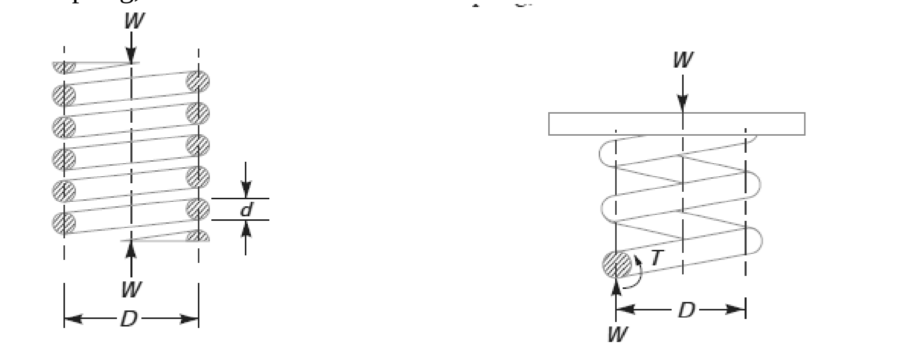
where n = Number of active turns,
p = Pitch of the coils, and
d = Diameter of the spring wire.

4.1.6 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 23.10 (*a*).

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ= Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and
δ= Deflection of the spring, as a result of an axial load W.



(*a*) Axially loaded helical spring.

(*b*) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig 4.8.

Now consider a part of the compression spring as shown in Fig. 23.10 (*b*). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.
A little consideration will show that part of the spring, as shown in Fig. 23.10 (*b*), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi d^3} \qquad ...(i)$$

The torsional shear stress diagram is shown in Fig. 23.11 (*a*).
In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

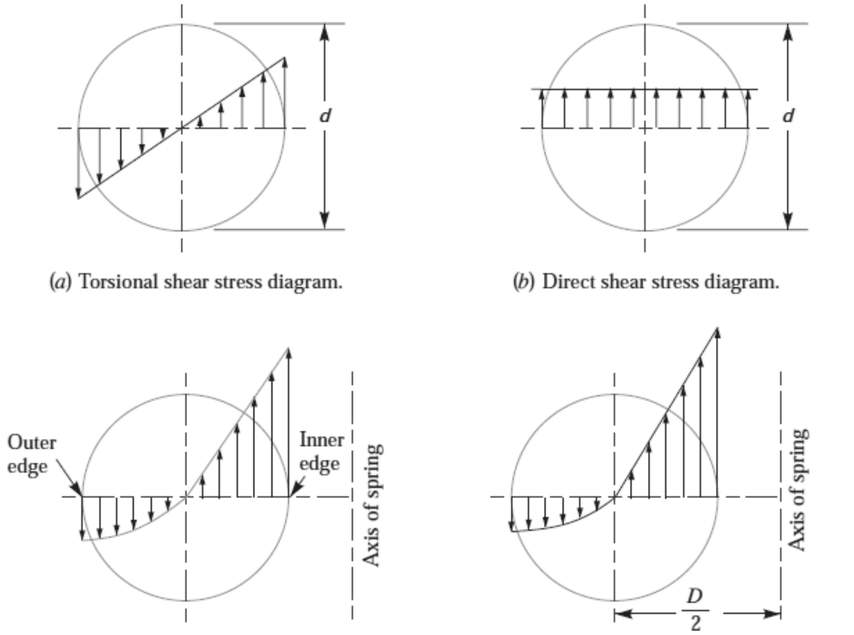
1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \qquad ...(ii)$$

The direct shear stress diagram is shown in Fig. 4.9 (*b*) and the resultant diagram of torsionalshear stress and direct shear stress is shown in Fig. 4.9 (*c*).



(*a*) Torsional shear stress diagram.
(*b*) Direct shear stress diagram.
(*c*) Resultant torsional shear and direct shear stress diagram.
(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

Fig. 4.9. Superposition of stresses in a helical spring.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outeredge of the wire. Since the stress is maximum at the inner edge of the wire, thereforeMaximum shear stress induced in the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8\,WD}{\pi d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index *C*. Also we have neglected the effect of wire curvaturein equation ***(iii)***. It may be noted that when the springs are subjected to static loads, the effect of wirecurvature may be neglected, because yielding of the material will relieve the stresses.In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl'sstress factor (*K*) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear,direct shear and curvature shear stress is shown in Fig. 4.9 (*d*). Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad ...(iv)$$

where

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

The values of *K* for a given spring index (*C*) may be obtained from the graph as shown in Fig. 4.10.

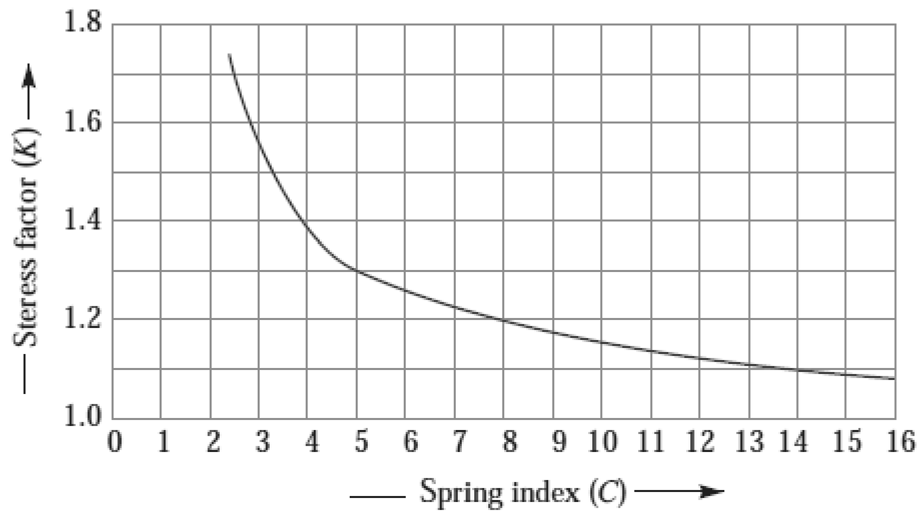


Fig. 4.10. Wahl's stress factor for helical springs.

We see from Fig. 4.10 that Wahl's stress factor increases very rapidly as the spring indexdecreases. The spring mostly used in machinery have spring index above 3.

Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

l = Length of one coil × No. of active coils = $\pi D \times n$

Let θ = Angular deflection of the wire when acted upon by the torque T.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \quad \text{...}(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \quad \text{...}\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16 W.D^2.n}{G.d^4} \quad \text{...}(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \quad \text{... } (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

4.1.7 Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e*. the springis subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, thestiffness of the spring is also affected. The eccentric load on the spring increases the stress on one sideof the spring and decreases on the other side. When the load is offset by a distance e from the springaxis, then the safe load on the spring may be obtained by multiplying the axial load by the factor

$$\frac{D}{2\,e + D}$$, where D is the mean diameter of the spring.

4.1.8 Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (*LF*) is more than fourtimes the mean or pitch diameter (*D*), then the spring behaves like a column and may fail by bucklingat a comparatively low load as shown in Fig. 4.11. The critical axial load (*Wcr*) that causes bucklingmay be calculated by using the following relation, *i.e*.

$$W_{cr} = k \times K_B \times L_F$$

where
k = Spring rate or stiffness of the spring = W/δ,
L_F = Free length of the spring, and
K_B = Buckling factor depending upon the ratio L_F / D.

The buckling factor (KB) for the hinged end and built-in end springs may be taken from the following table.

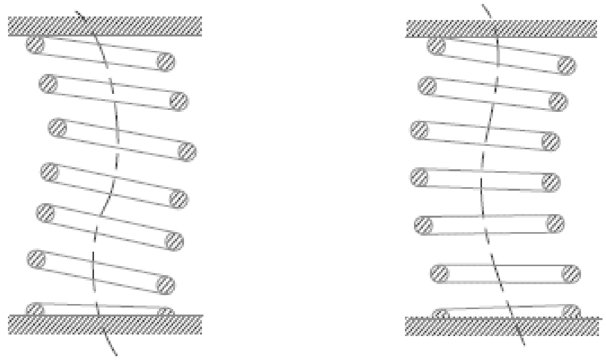


Fig. 4.11. Buckling of compression springs.

Table 4.2. Values of buckling factor (KB).

L_F/D	*Hinged end spring*	*Built-in end spring*	L_F / D	*Hinged end spring*	*Built-in end spring*
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19

It may be noted that a ***hinged end spring*** is one which is supported on pivots at both ends as incase of springs having plain ends where as a ***built-in end spring*** is one in which a squared and groundend spring is compressed between two rigid and parallel flat plates.It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube.When the spring is located on a tube, the clearance between the tube walls and the spring should bekept as small as possible, but it must be sufficient to allow for increase in spring diameter duringcompression.

4.1.9 Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loadedsuddenly, then all the coils of the spring will not suddenly deflect equally, because some time isrequired for the propagation of stress along the spring wire. A little consideration will show that in thebeginning, the end coils of the spring in contact with the applied load takes up whole of the deflectionand then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compressionpropagates through the coils to the supported end from where it is reflected back to the deflected end.This wave of compression travels along the spring indefinitely. If the applied load is of fluctuatingtype as in the case of valve spring in internal combustion engines and if the time interval between theload applications is equal to the time required for the wave to travel from one end to the other end,then resonance will occur. This results in very large deflections of the coils and correspondingly veryhigh stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon iscalled ***surge*.**

It has been found that the natural frequency of spring should be atleast twenty times the frequencyof application of a periodic load in order to avoid resonance with all harmonic frequencies uptotwentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,
D = Mean diameter of the spring,
n = Number of active turns,
G = Modulus of rigidity,
g = Acceleration due to gravity, and
ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 4.1. *A compression coil spring made of an alloy steel is having the followingspecifications : Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.*

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given :
D = 50 mm
d = 5 mm
n = 20
W = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$$= 534.7 \text{ MPa } \textbf{Ans.}$$

Example 4.2. *Design a spring for a balance to measure 0 to 1000 N over a scale of length80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turnsis 30. The modulus of rigidity is 85 kN/mm2. Also calculate the maximum shear stress induced.*

Solution.
Given :
W = 1000 N
δ = 80 mm
n = 30
G = 85 kN/mm^2 = 85 × 10^3 N/mm^2

Design of spring

Let D = Mean diameter of the spring coil,
d = Diameter of the spring wire, and
C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($Do = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\;C^3}{85\;d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm } \textbf{Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

$\therefore$ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$
$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa } \textbf{Ans.}$$

Example 4.3. *Design a helical spring for a spring loaded safety valve (Ramsbottom safetyvalve) for the following conditions :*
Diameter of valve seat = 65 mm ; Operating pressure = 0.7N/mm^2; Maximum pressure when the valve blows off freely = 0.75N/mm^2; Maximum lift of the valve when the pressure rises from 0.7 to0.75 N/mm^2 = 3.5 mm ; Maximum allowable stress = 550 MPa ;Modulus of rigidity = 84 kN/mm^2; Spring index = 6. Draw a neat sketch of the free spring showing the maindimensions.

Solution. Given :
$D1$ = 65 mm
P_1 = 0.7 N/mm^2
P_2 = 0.75N/mm^2
δ = 3.5 mm
τ = 550 MPa = 550 N/mm^2

G = 84 kN/mm^2 = 84 × 103 N/mm^2
C = 6

1. ***Mean diameter of the spring coil***

Let D = Mean diameter of the spring coil, and
d = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore thespring will be under tension. We know that initial tensile force actingon the spring (*i.e*. before the valve lifts),

$$W_1 = \frac{\pi}{4}(D_1)^2\, p_1 = \frac{\pi}{4}(65)^2\, 0.7 = 2323 \text{ N}$$

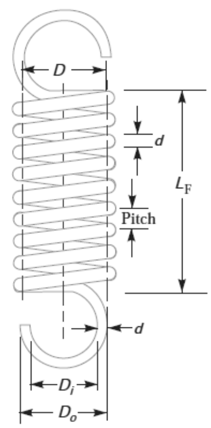


Fig. 4.12

and maximum tensile force acting on the spring (*i.e*. when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(D_1)^2\, p_2 = \frac{\pi}{4}(65)^2\, 0.75 = 2489 \text{ N}$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W_2 - W_1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load (W_2), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6\,d}{2} = 7467\, d \qquad \ldots (\because C = D/d = 6)$$

We know that maximum twisting moment (T),

$$7467\, d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108\, d^3$$

$$\therefore \qquad d^2 = 7467 / 108 = 69.14 \quad \text{or} \quad d = 8.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 2/0 having diameter (d) = 8.839 mm **Ans.**

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm } \textbf{Ans.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm } \textbf{Ans.}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coil*

Let n = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 166 \times 6^3 \times n}{84 \times 10^3 \times 8.839} = 0.386\,n$$

∴ n = 3.5 / 0.386 = 9.06 say 10 **Ans.**

For a spring having loop on both ends, the total number of turns,

n' = n + 1 = 10 + 1 = 11 **Ans.**

3. *Free length of the spring*

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the free length of the tension spring,

L_F = $n.d$ + (n – 1) 1 = 10 × 8.839 + (10 – 1) 1 = 97.39 mm **Ans.**

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n-1} = \frac{97.39}{10-1} = 10.82 \text{ mm } \textbf{Ans.}$$

The tension spring is shown in Fig. 4.12.