GRAPH TRAVERSAL ALGORITHMS

There are two standard methods of graph traversal, they are

1. Breadth-first search
2. Depth-first search

While breadth-first search uses a queue as an auxiliary data structure to store nodes for further processing, the depth-first search scheme uses a stack. But both these algorithms make use of a variable STATUS. During the execution of the algorithm, every node in the graph will have the variable STATUS set to 1 or 2, depending on its current state.

BREADTH-FIRST SEARCH ALGORITHM

- Breadth-first search (BFS) is a graph search algorithm that begins at the root node and explores all the neighbouring nodes. Then for each of those nearest nodes, the algorithm explores their unexplored neighbour nodes, and so on, until it finds the goal.
- That is, we start examining the node A and then all the neighbours of A are examined. In the next step, we examine the neighbours of neighbours of A, so on and so forth. This means that we need to track the neighbours of the node and guarantee that every node in the graph is processed and no node is processed more than once. This is accomplished by using a queue that will hold the nodes that are waiting for further processing and a variable STATUS to represent the current state of the node.

Algorithm to breadth-first Search

Step 1: SET STATUS = 1 (ready state) for each node in G

Step 2: Enqueue the starting node A and set its STATUS = 2 (waiting state)

Step 3: Repeat Steps 4 and 5 until QUEUE is empty

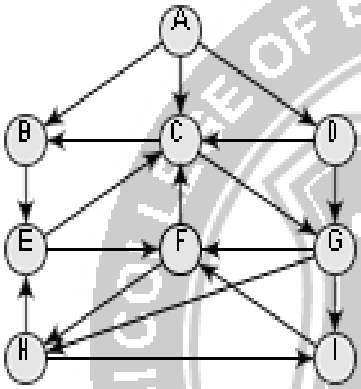
Step 4: Dequeue a node N. Process it and set its STATUS = 3 (processed state).

Step 5: Enqueue all the neighbours of N that are in the ready state (whose STATUS = 1) and set their STATUS = 2 (waiting state) [END OF LOOP]

Step 6: EXIT

Example

Consider the graph G given. The adjacency list of G is also given. Assume that G represents the daily flights between different cities and we want to fly from city A to I with minimum stops. That is, find the minimum path P from A to I given that every edge has a length of 1.



Adjacency List
A: B, C, D
B: E
C: B, G
D: C, G
E: C, F
F: C, H
G: F, H, I
H: E, I
I: F

The minimum path P can be found by applying the breadth-first search algorithm that begins at city A and ends when I is encountered. During the execution of the algorithm, we use two arrays:

QUEUE and ORIG. While QUEUE is used to hold the nodes that have to be processed,

ORIG is used to keep track of the origin of each edge.

Initially, FRONT = REAR = –1.

The algorithm for this is as follows:

(a) Add A to QUEUE and add NULL to ORIG.

FRONT = 0	QUEUE = A
REAR = 0	ORIG \0

(b) Dequeue a node by setting FRONT = FRONT + 1 (remove the FRONT element of QUEUE) and enqueue the neighbours of A. Also, add A as the ORIG of its neighbours.

FRONT = 1	QUEUE = A	B	C	D	
REAR = 3	ORIG	\0	A	A	A

(c) Dequeue a node by setting FRONT = FRONT + 1 and enqueue the neighbours of B. Also, add B as the ORIG of its neighbours.

FRONT = 2	QUEUE = A	B	C	D	E	
REAR = 4	ORIG	\0	A	A	A	B

d) Dequeue a node by setting FRONT = FRONT + 1 and enqueue the neighbours of C. Also, add C as the ORIG of its neighbours. Note that C has two neighbours B and G. Since B has already been added to the queue and it is not in the Ready state, we will not add B and only add G.

FRONT = 3	QUEUE = A	B	C	D	E	G	
REAR = 5	ORIG	\0	A	A	A	B	C

(e) Dequeue a node by setting FRONT = FRONT + 1 and enqueue the neighbours of D. Also, add D as the ORIG of its neighbours. Note that D has two neighbours C and G. Since both of them have already been added to the queue and they are not in the Ready state, we will not add them again.

FRONT = 4	QUEUE = A	B	C	D	E	G	
REAR = 5	ORIG	\0	A	A	A	B	C

(f) Dequeue a node by setting FRONT = FRONT + 1 and enqueue the neighbours of E. Also, add E as the ORIG of its neighbours. Note that E has two neighbours C and F. Since C has already been added to the queue and it is not in the Ready state, we will not add C and add only F.

FRONT = 5	QUEUE = A	B	C	D	E	G	F	
REAR = 6	ORIG	\0	A	A	A	B	C	E

(g) Dequeue a node by setting FRONT = FRONT + 1 and enqueue the neighbours of G. Also, add G as the ORIG of its neighbours. Note that G has three neighbours F, H, and I.

FRONT=6	QUEUE = A	B	C	D	E	G	F	H	I
REAR = 9	ORIG \0	A	A	A	B	C	E	G	G

Since F has already been added to the queue, we will only add H and I. As I is our final destination, we stop the execution of this algorithm as soon as it is encountered and added to the QUEUE. Now, backtrack from I using ORIG to find the minimum path P. Thus, we have P as A -> C -> G -> I

Features of Breadth-First Search Algorithm

Space complexity

- In the breadth-first search algorithm, all the nodes at a particular level must be saved until their child nodes in the next level have been generated. The space complexity is therefore proportional to the number of nodes at the deepest level of the graph. Given a graph with branching factor b (number of children at each node) and depth d, the asymptotic space complexity is the number of nodes at the deepest level O(bd).
- If the number of vertices and edges in the graph are known ahead of time, the space complexity can also be expressed as O (| E | + | V |), where | E | is the total number of edges in G and | V | is the number of nodes or vertices.

Time complexity

In the worst case, breadth-first search has to traverse through all paths to all possible nodes, thus the time complexity of this algorithm asymptotically approaches O(bd). However, the time complexity can also be expressed as O(| E | + | V |), since every vertex and every edge will be explored in the worst case.

Completeness

Breadth-first search is said to be a complete algorithm because if there is a solution, breadth-first search will find it regardless of the kind of graph. But in case of an infinite graph where there is no possible solution, it will diverge.

Optimality

Breadth-first search is optimal for a graph that has edges of equal length, since it always returns the result with the fewest edges between the start node and the goal node. But generally, in real-world applications, we have weighted graphs that have costs associated with each edge, so the goal next to the start does not have to be the cheapest goal available.

Applications of Breadth-First Search Algorithm

Breadth-first search can be used to solve many problems such as:

- Finding all connected components in a graph G.
- Finding all nodes within an individual connected component.
- Finding the shortest path between two nodes, u and v, of an unweighted graph.
- Finding the shortest path between two nodes, u and v, of a weighted graph