Normal Incidence at a Plane Dielectric Boundary

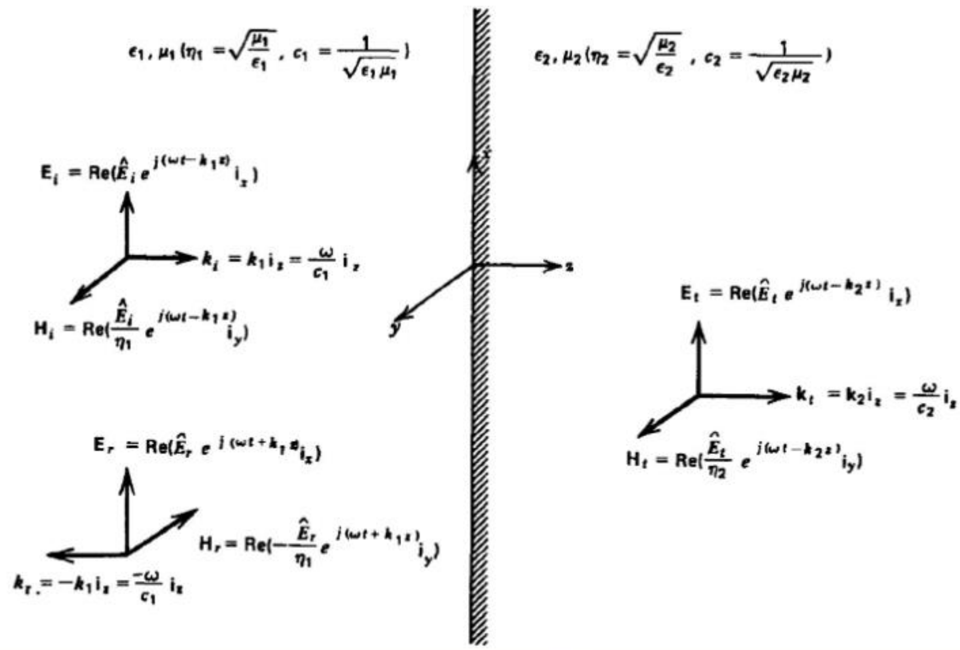
Lossless Dielectric

We replace the perfect conductor with a lossless dielectric of permittivity ε2 and permeability μ2 as in Figure below, with a uniform plane wave normally incident from a medium with permittivity ε1 and permeability μ1. In addition to the incident and reflected fields for z<0, there are transmitted fields which propagate in the +z direction within the medium for z>0.

$$\left.\begin{aligned}
\mathbf{E}_i(z,t) &= \mathrm{Re}\left[\hat{E}_i e^{j(\omega t - k_1 z)}\mathbf{i}_x\right], \qquad k_1 = \omega\sqrt{\varepsilon_1\mu_1} \\
\mathbf{H}_i(z,t) &= \mathrm{Re}\left[\frac{\hat{E}_i}{\eta_1} e^{j(\omega t - k_1 z)}\mathbf{i}_y\right], \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \\
\mathbf{E}_r(z,t) &= \mathrm{Re}\left[\hat{E}_r e^{j(\omega t + k_1 z)}\mathbf{i}_x\right] \\
\mathbf{H}_r(z,t) &= \mathrm{Re}\left[-\frac{\hat{E}_r}{\eta_1} e^{j(\omega t + k_1 z)}\mathbf{i}_y\right]
\end{aligned}\right\} \quad z < 0$$

$$\left.\begin{aligned}
\mathbf{E}_t(z,t) &= \mathrm{Re}\left[\hat{E}_t e^{j(\omega t - k_2 z)}\mathbf{i}_x\right], \qquad k_2 = \omega\sqrt{\varepsilon_2\mu_2} \\
\mathbf{H}_t(z,t) &= \mathrm{Re}\left[\frac{\hat{E}_t}{\eta_2} e^{j(\omega t - k_2 z)}\mathbf{i}_y\right], \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}
\end{aligned}\right\} \quad z > 0$$

It is necessary in (1) to use the appropriate wave number and impedance within each region. There is no wave traveling in the –z direction in the second region as we assume no boundaries or sources for z>0.



A uniform plane wave normally incident upon a dielectric interface separating two different materials has part of its power reflected and part transmitted.

The unknown quantities E^r and E^i, can be found from the boundary conditions of continuity of tangential E and H at z=0,

$$\hat{E}_i + \hat{E}_r = \hat{E}_t$$
$$\frac{\hat{E}_i - \hat{E}_r}{\eta_1} = \frac{\hat{E}_r}{\eta_2}$$

from which we find the reflection R and transmission T field coefficients as

$$R = \frac{\hat{E}_r}{\hat{E}_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$T = \frac{\hat{E}_t}{\hat{E}_i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

where from (2)

$$1+R=T$$

If both mediums have the same wave impedance, η1=η2, there is no reflected wave.

Time-Average Power Flow

The time-average power flow in the region z<0 is

$$\begin{aligned}< S_{xi} > &= \frac{1}{2}\text{Re}\left[\hat{E}_x(z)\,\hat{H}_y^*(z)\right] \\ &= \frac{1}{2\eta_1}\text{Re}\left[\hat{E}_i e^{-jk_1 z} + \hat{E}_r e^{+jk_1 z}\right]\left[\hat{E}_i^* e^{+jk_1 z} + \hat{E}_r^* e^{-jk_1 z}\right] \\ &= \frac{1}{2\eta_1}\left[\left|\hat{E}_i\right|^2 - \left|\hat{E}_r\right|^2\right] \\ &\quad + \frac{1}{2\eta_1}\underbrace{\text{Re}\left[\hat{E}_r\hat{E}_i^* e^{+2jk_1 z} - \hat{E}_r^*\hat{E}_i e^{-2jk_1 z}\right]}_{0}\end{aligned}$$

The last term on the right-hand side of (5) is zero as it is the difference between a number and its complex conjugate, which is pure imaginary and equals 2j times its imaginary part. Being pure imaginary, its real part is zero. Thus the time-average power flow just equals the difference in the power flows in the incident and reflected waves. The coupling terms between oppositely traveling waves have no time-average yielding the simple superposition of time-average powers:

$$\begin{aligned}< S_{xi} > &= \frac{1}{2\eta_1}\left[\left|\hat{E}_i\right|^2 - \left|\hat{E}_r\right|^2\right] \\ &= \frac{\left|\hat{E}_i\right|^2}{2\eta_1}\left[1 - R^2\right]\end{aligned}$$

This net time-average power flows into the dielectric medium, as it also equals the transmitted power;

$$< S_{xi} > = \frac{1}{2\eta_2}\left|\hat{E}_t\right|^2 = \frac{\left|\hat{E}_i\right|^2 T^2}{2\eta_2} = \frac{\left|\hat{E}_i\right|^2}{2\eta_1}\left[1 - R^2\right]$$

Lossy Dielectric

If medium 2 is lossy with Ohmic conductivity σ, the solutions of (3) are still correct if we replace the permittivity ε2 by the complex permittivity ε^2,

$$\hat{\varepsilon}_2 = \varepsilon_2 \left(1 + \frac{\sigma}{j\omega\varepsilon_2}\right)$$

so that the wave impedance in region 2 is complex:

$$\eta_2 = \sqrt{\mu_2/\hat{\varepsilon}_2}$$

We can easily explore the effect of losses in the low and large loss limits.

(a) Low Losses

If the Ohmic conductivity is small, we can neglect it in all terms except in the wave number k2:

$$\lim_{\sigma/\omega\varepsilon_2 \ll 1} k_2 \approx \omega\sqrt{\varepsilon_2\mu_2} - \frac{j}{2}\sigma\sqrt{\frac{\mu_2}{\varepsilon_2}}$$

The imaginary part of k2 gives rise to a small rate of exponential decay in medium 2 as the wave propagates away from the z=0 boundary.

(b) Large Losses

For large conductivities so that the displacement current is negligible in medium 2, the wave number and impedance in region 2 are complex:

$$\lim_{\sigma/\omega\varepsilon_2 \gg 1} \begin{cases} k_2 = \dfrac{1-j}{\delta}, \quad \delta = \sqrt{\dfrac{2}{\omega\mu_2\sigma}} \\ \eta_2 = \sqrt{\dfrac{j\omega\mu_2}{\sigma}} = \dfrac{1+j}{\sigma\delta} \end{cases}$$

The fields decay within a characteristic distance equal to the skin depth δ. This is why communications to submerged submarines are difficult. For seawater, $\mu 2 = \mu 0 = 4\pi\times10^{-7}$ henry/m and σ≈4siemens/m so that for 1MHz signals, δ≈0.25m. However, at 100Hz the skin depth increases to 25meters. If a submarine is within this distance from the surface, it can receive the signals. However, it is difficult to transmit these low frequencies because of the large free space wavelength, $\lambda \approx 3\times10^6$ m. Note that as the conductivity approaches infinity,

$$\lim_{\sigma\to\infty}\begin{cases} k_2=\infty \\ \eta_2=0 \end{cases} \Rightarrow \begin{cases} R=-1 \\ T=0 \end{cases}$$

so that the field solution approaches that of normal incidence upon a perfect conductor.