3.5TRIGONOMETRIC LEVELLING

Trigonometrical levelling is the process of determining the differences of elevations of stations from observed vertical angles and known distances, which are assumed to be either horizontal or geodetic lengths at mean sea level. The vertical angles may be measured by means of ·an accurate theodolite and the horizontal distances may either be measured (in the case of plane surveying) or computed (in the case of geodetic observations).

We shall discuss the trigonometrical levelling under two beads:

(I) Observations fur heights and distances, and

(2) Geodetical observations

In the first case, the principles of plane surveying will be used. It is assumed that the distances between the points observed are not large so that either the effect of curvature and refraction n1ay be neglected or proper corrections may be applied linearly to the calculated difference in elevation. Under this head fall the various methods of angular levelling for determining the elevations of particular points such as top of chimney, or church spire etc.

In the geodetical observations of trigonometrical levelling, the distance between the points measured is geodetic and is large. The ordinary principles of plane surveying are not applicable. The corrections for curvature and refraction are applied in angular measure directly to the observed angles. The geodetical observations of trigonometrical levelling have been dealt with in the second volume.

HEIGHTS AND DISTANCES

In order to get the difference in elevation between the instrument station and the object under observation, we shall consider the following cases:

Case 1: Base of the object accessible.

Case 2: Base of the object inaccessible: Instrument stations in the same vertical plane as the elevated object. Case 3: Base of the object inaccessible: Instrument stations not in the same vertical plane as the elevated object.

BASE OF THE OBJECT ACCESSIBLE

Let it be assumed that the horizontal distance between the instrument and the object can be measured accurately. In Fig. 15.1,

P = instrument station

Q = point co be observed

A =centre of the instrument

Q' = projection of Q on horizontal plane through A

D = A Q ' = horizontal distance

between P and Q

h' = height of the instrument at P

h = QQ’

S = reading of staff kept ac B.M., with line of sight horizontal

a = angle of elevation from A to Q.

From mangle AQQ' ; h = D tan α

R. L. of Q = R. L. of instrument axis + D tan α

If the R.L. of P is known,

R.L. of Q= R. L. of P+ h' +D tan α **... (15.1)**

If the reading on the staff kept at the B. M. is S with the line of sight horizontal,

R.L. of Q = R.L. of B.M. + S + D tan α

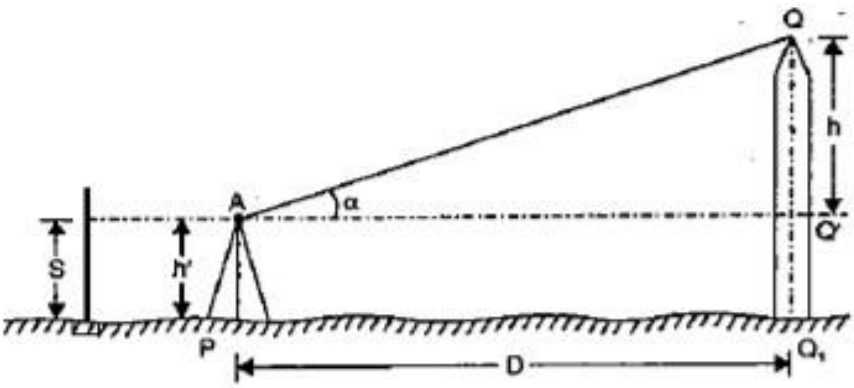

FIG. 15.1. BASE ACCESSIBLE

The method is usually employed when the distance A is small. However, if D large, the combined correction for curvature and refraction can be applied.

BASE OF THE OBJECT INACCESSIBLE:

Instrument Stations in the Same Vertical Plane as the Elevated Object.

If the horizontal distance between the instrument and the object can be measured due to obstacles etc., two instrument stations are used so that they are in the same vertical plane as the elevated object (Fig. 15.5).

Procedure

1. Set up the theodolite at P and level it accurately with respect to the altitude bubble

2. Direct the telescope towards Q and bisect it accurately. Clamp both the plates. Read the vertical angle α_1.

3.Transit the telescope so that the line of sight is reversed. Mark the second instrument station R on the ground. Measure the distance RP accurately. Repeat steps (2) and (3) for both face observations. The mean values should be adopted.

4. With the vertical vernier set to zero reading, and the altitude bubble in the centre of its run, take the reading on the staff kept at the nearby B.M.

5. Shift the instrument to R and set up the theodolite there. Measure the vertical angle α_1, to Q with both face observations.

6. With the vertical vernier set to zero reading, and the altitude bubble in the centre of its run, take the reading on the staff kept at the nearby B.M.

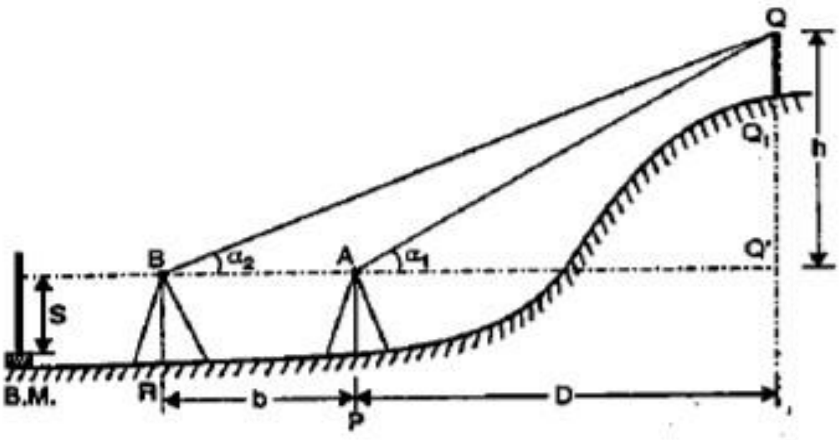

FIG. 15.5. INSTRUMENT AXES AT THE SAME LEVEL

In order to calculate the R.L. of Q. we will consider three cases:

(a) when the instrument axes at A and B are at the same level.

(b) when they are at different levels but the difference is small, and

(c) when they are at very different levels.

(d) instrument axes at the same level (Fig. 15.5)

Let h= QQ'

a, = angle of elevation from A to Q

a, = angle of elevation from B to Q.

S = staff reading on B.M., taken from both A and B, the reading being the same in both the cases.

b = horizontal distance between the instrument stations.

D = horizontal distance between P and Q

From triangle AQQ', h = D tan α_1 ... (1)

From triangle BQQ', h = (b +D) tan α_1 ... (2)

Equating (1) and (2), we get

' D tan α_1= (b +D) tan $\alpha 1$, or D (tan α_1- tan α_2) = b tan α_2, or

$$D = \frac{b \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} \qquad ...(15.2)$$

$$h = D \tan \alpha_1 = \frac{b \tan \alpha_1 \tan \alpha_2}{\tan \alpha_1 - \tan \alpha_2} = \frac{b \sin \alpha_1 \sin \alpha_2}{\sin (\alpha_1 - \alpha_2)} \qquad ...(15.3)$$

R.L. of Q=R.L. of B.M. +S+h

b) Instrument axes at different levels (Fig. 15.6 and 15.7)

Figs. 15.6 and 15.7 illustrate the cases, when the instrument axes are at different levels. If S, and S, are the corresponding staff readings on the staff kept at B.M., the difference in levels of the instrument axes will be either (S2 - S1) if the axis at B is higher or (S1, - S2,) if the axis at A is higher.

Let Q' be the projection of Q on horizontal line through A and Q'' be the projection on horizontal line through B.

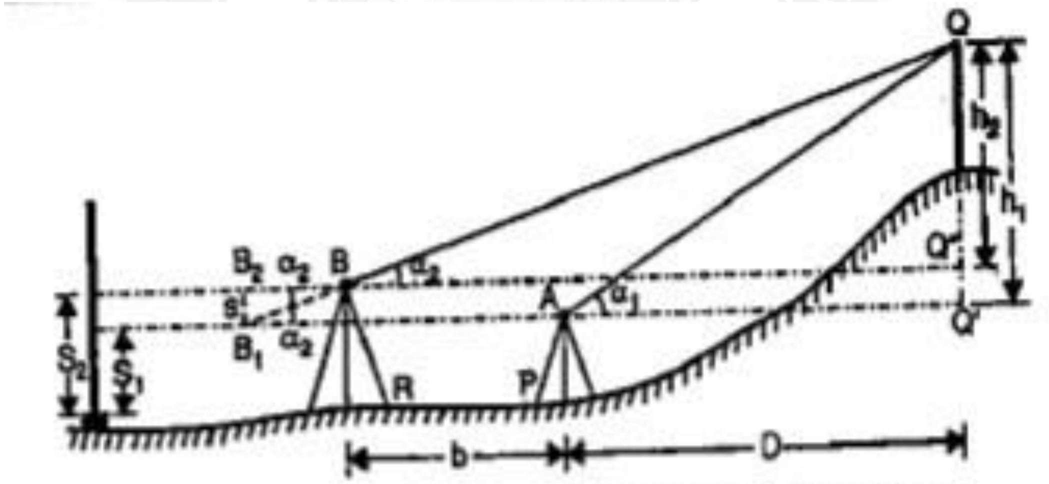

FIG. 15.6. INSTRUMENT AT DIFFERENT LEVELS.

Let us derive the expressions for Fig. 15.6 when S2, is greater than S1,

From triangle QAQ', h, = D tan**α1**, ... (1)

From triangle BQQ'', h, = (b + D) tan**α1**, ... (2)

Subtracting (2) from (1), we get

(h1 – h2) = D tan**α1**- (b +D) tan**α2**,

H1, - h2, =difference in level of instrument axes = S2- S1, = s (say)

s = D tan**α1**, - b tanα2 - D tan**α2**

or D (tan**α1**, - tan**α2**,) = s + b tan**α2** or $$D = \frac{s + b\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} = \frac{(b + s\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \quad ...[15.4\ (a)]$$

Now $$h_1 = D\tan\alpha_1$$

$\therefore$ $$h_1 = \frac{(b + s\cot\alpha_2)\tan\alpha_1\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} = \frac{(b + s\cot\alpha_2)\sin\alpha_1\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)} \quad ...[15.5\ (a)]$$

Expression 15.4 (a) could also be obtained by producing the lines of sight BQ backwards to meet the line Q'A in B1 • Drawing B1 B2, as vertical to meet the horizontal line Q'' B in B2, it is clear that with the same angle of elevation if the instrument axis were at B1, the instrument axes in both the cases would have been at the same elevation. Hence the distance at which the axies are at the same level is $AB_1 = b + BB2 = b + s \cot$ **α2.**

Substituting this value of the distance between the instrument stations in equation 15.2

we get,

$$D = \frac{(b + s\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$ which is the same as equation [15.4 (*a*)].

Proceeding on the same line for the case fig. 15.7 where the instrument axis at D is higher, it can be proved that

$$D = \frac{(b - s\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \quad ...[15.4\ (b)]$$

and

$$h_1 = \frac{(b - s\cot\alpha_2)\sin\alpha_1\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)} \quad ...[15.5\ (b)]$$

Thus, the general expressions For D and h_1 can be written as

$$D = \frac{(b \pm s\cot\alpha_2)\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2} \quad ...(15.4)$$

and $$h_1 = \frac{(b \pm s\cot\alpha_2)\sin\alpha_1\sin\alpha_2}{\sin(\alpha_1 - \alpha_2)} \quad ...(15.5)$$

Use + sign with s cot **α2** when the instrument axis at A is lower and - sign when higher than at B.

R.L. of Q = R.L. of B.M. + S1 + h1,

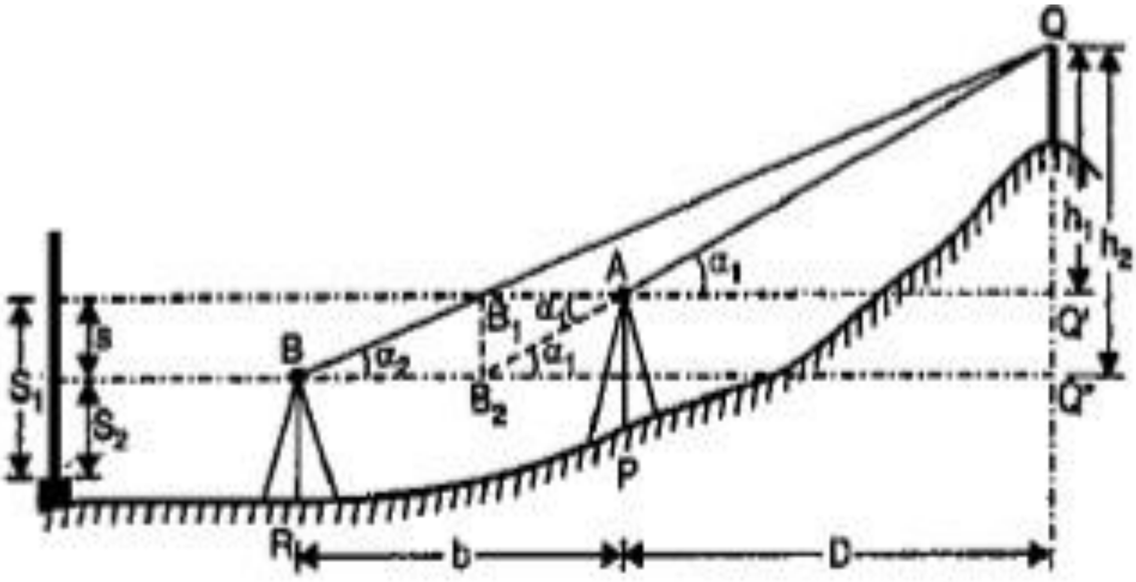

FIG. 15.7. INSTRUMENT AXES AT DIFFERENT LEVELS.

(c) Instrument axes at very different levels

If S_2 -S_1 or s is too great to be measured on a staff kept at the B.M., the following procedure is adopted (Fig. 15.8 and 15.9):

(1) Set the instrument at P (Fig. 15.8), level it accurately with respect to the altitude bubble and measure theangle **α,** to the point Q.

(2) Transit the telescope and establish a point R at a distance b from P.

(3) Shift the instrument to R. Ser the instrument and level it with respect to the altitude bubble, and measurethe angle α_2, to Q.

(4) Keep a vane of height r, at P (or a staff) and measure the angle to the top of the vane for to reading r, if a staff is used (Fig.15.9), vertical projection of Q. Thus, AQQ' is a vertical plane. Similarly, BQQ" is a vertical plane, Q" being the vertical projection of Q on a horizontal line through B. PRQ, is a horizontal plane, Q1, being the vertical projection of Q, and R vertical projection of B on a horizontal plane passing through P. θ_1 and θ_2 are the horizontal angles, and α_1 and α_2 are the vertical angles measured at A and B respectively.

From triangle AQQ', $\quad QQ' = h_1 = D \tan \alpha_1 \qquad ...(1)$

From triangle PRQ_1, $\quad \angle PQ_1R = 180° - (\theta_1 + \theta_2) = \pi - (\theta_1 + \theta_2)$

From the sine rule, $\quad \dfrac{PQ_1}{\sin \theta_2} = \dfrac{RQ_1}{\sin \theta_1} = \dfrac{RP}{\sin [\pi - (\theta_1 + \theta_2)]} = \dfrac{b}{\sin (\theta_1 + \theta_2)}$

$$\therefore \quad PQ_1 = D = \frac{b \sin \theta_2}{\sin (\theta_1 + \theta_2)} \qquad ...(2)$$

and
$$RQ_1 = \frac{b \sin \theta_1}{\sin (\theta_1 + \theta_2)} \qquad ...(3)$$

Substituting the value of D in (1), we get

$$h_1 = D \tan \alpha_1 = \frac{b \sin \theta_2 \tan \alpha_1}{\sin (\theta_1 + \theta_2)} \qquad ...(15.6)$$

$\therefore \quad$ R.L. of Q = R.L. of B.M. $+ s + h_1$

As a check, $\quad h_2 = RQ_1 \tan \alpha_2 = \dfrac{b \sin \theta_1 \tan \alpha_2}{\sin (\theta_1 + \theta_2)}$

If a reading on B.M. is taken from B, the R.L. of Q can be known by adding h_2 to R.L. of B.

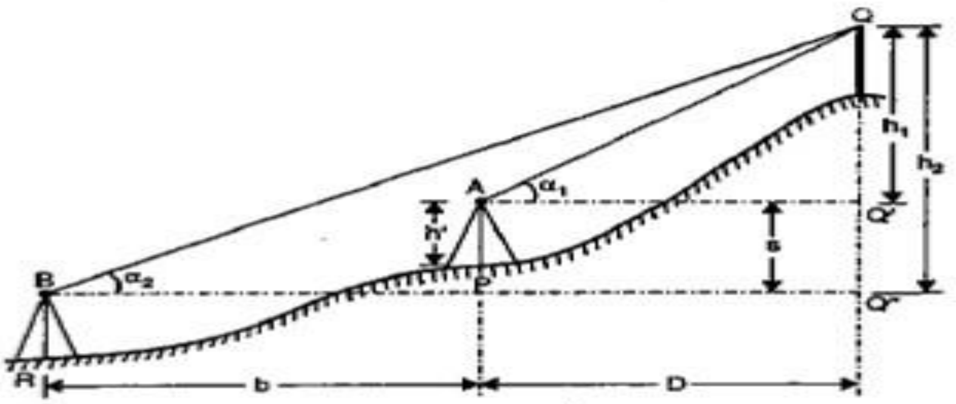

FIG. 15.8. INSTRUMENT AXES AT VERY DIFFERENT LEVELS.