Maxwell’s Equations

Maxwell’s equations are like the instruction manual for how electricity and magnetism work. They were created by a smart scientist named James Clerk Maxwell in the 1800s. Since these equations help us understand everything from how lights work to how our gadgets and technology function, they are extremely significant. Maxwell’s equations describe how the electric field can create a magnetic field and vice versa.

Evolution of Maxwell’s Equations

Back in the 1800s, people were trying to figure out how electricity and magnetism were connected. Scientists like Michael Faraday and André-Marie Ampère made discoveries that got Maxwell thinking. He put together their ideas and made four special equations that explain how electricity and magnetism are related.

Derivations of Maxwell’s Equations

The four equations of Maxwell’s are :

- Maxwell’s First Equation (based on Gauss Law)
- Maxwell’s Second Equation (based on Gauss’s law on magnetostatics)
- Maxwell’s Third Equation (based on Faraday’s laws of Electromagnetic Induction)
- Maxwell’s Fourth Equation (based on Ampere’s Law)

Maxwell's equations

$$\nabla . D = \rho_V \qquad \nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times H = \frac{\partial D}{\partial t} + J$$

Maxwell First Equation

Maxwell's first equation is based on the Gauss law of electrostatic. This law states that "in a closed surface the integral of the electric flux density is equal to the charge enclosed ." The expression for Maxwell's first equation can be expressed mathematically as,

$\nabla . E dv = \rho v$

Derivation for Maxwell First Equation:

From the definition of Gauss Law, we have obtained,

$\oiint$ E.ds = Qenclosed ——(1)

As we know that any system is made up of composition of various surfaces but the volume of the system remains consistent. Thus for the convenience in calculation, let's convert surface integral to volume integral by taking the Divergence of the same vector .

$\oiint$ E.ds = $\iiint$ ∇. Edv —-(2)

On combining equation (1) and (2) we obtain,

$\iiint \nabla$. Edv = Qenclosed ——(3)

when we supply some amount of charges to the system, it will spread throughout its volume. Thus volume charge density (that is number of charges per unit volume) of the system can be expressed as

$$pv = \frac{dQ}{dv}$$

or dQ=pvdv ——(4)

Total charges enclosed can be obtained by integrating equation (3) i.e ,

Q=$\iiint \rho$vdv ——(5)

On Substituting the value of Q obtained in equation (5) to equation (3), we get,

$\iiint \nabla$. Edv = $\iiint \rho$vdv

Therefore, $\Rightarrow$ $\nabla . E dv = \rho v dv$

This is the required expression for Maxwell's First equation. This equation is also referred to as **Gauss's law of Electrostatic.**

Maxwell's Second Equation

Maxwell second equation is based on Gauss law on magnetostatics. This law states that " the sum of outer flux in the magnetic induction through any closed surface is zero". The expression for Maxwell's second equation can be expressed mathematically as:

$\boldsymbol{\nabla . H = 0}$

Derivation for Maxwell's Second Equation

According to Gauss law on magnetostatics

$\oiint B \,.\, ds = \phi enclosed$ ——(1)

Magnetic flux cannot be enclosed inside a surface
$\oiint B \,.\, ds = 0$ ——(2)

Converting surface integral to a volume integral using divergence of vectors

$\oiint B \,.\, ds = \iiint \Delta \,.\, Bdv$ ——(3)

On substituting (3) in (2) we get,

$\iiint \Delta \,.\, Bdv = 0$ ——(4)

The above equation can be satisfied using only the following two conditions:

- $\iiint dv = 0$
- $\Delta \,.\, B = 0$

However, the volume of an object cannot be 0, thus $\boldsymbol{\Delta \,.\, B = 0}$

where, $\boldsymbol{B = \mu H}$ is the flux density.

Therefore,
$\boldsymbol{\Delta \,.\, H = 0}$ *is the required expression.*

Maxwell's Third Equation

Maxwell's 3rd equation is derived from Faraday's laws of Electromagnetic Induction, which states that " the line integral of magnetic field in a closed circuit is equal to the closed current." The expression for Maxwell's third equation can be expressed mathematically as:

$$E = -\frac{B}{t}$$

Derivation for Maxwell's Third Equation

According to Faraday's law ,

$$emf_{alt} = -\frac{d}{dt} \text{——(1)}$$

Total magnetic flux on arbitrary surface S is

$\phi = \iint B.ds$

Substitute the value of magnetic flux in equation (1), we get,

$$emf_{alt} = \frac{(dB.ds)}{dt}$$

$$emf_{alt} = (\frac{B}{dt}).ds \text{——(2)}$$

Since, the induced alternative emf in the coil is basically a closed path, thus it can be expressed mathematically as closed integral as,

emfalt = $\oint E \,.\, dl$ ——(3)

From equation (2) and (3)

$\oint E \,.\, dl = \iint -(B/dt).ds$ ——(4)

By using stroke's Theorem contour integration can be converted to surface integration as

$\oint E \,.\, dl = \iint (\nabla\times E) .ds$

By substituting this value in equation (4) we get,

$\iint (\nabla\times E) ds = \iint -(B/dt).ds$

Therefore , $\nabla\times E = -(B/dt)$ ***is the required equation.***

This equation is Faraday's law of electromagnetic induction.

Maxwell's Fourth Equation

Maxwell's fourth equation is derived from Ampere's Law, which states that "the magnetic field divergence is always zero." The expression for Maxwell's fourth equation can be expressed mathematically as:

H = J +(D/t)

Derivation for Maxwell's fourth Equation

According to Ampere's circuital law

$\oint B.dl = \mu 0i$ —— (1)

According to Stoke's theorem–

$\oint B.dl = \oint S\ (\nabla \times B).ds$ —— (2)

From equation (1) and equation(2)

$\oint S\ (\nabla \times B).ds = \mu 0i$ —— (3)

where $i = \oint S\ J . ds$ —— (4)

So from equation (3) and equation (4)

$\oint S\ (\nabla \times B) .ds = \mu 0 \oint S\ (J . ds)$

$\oint S\ (\nabla \times B) .ds – \mu 0 \oint S\ (J . ds) = 0$

$\oint S\ [(\nabla \times B) – \mu 0\ J] .ds = 0$

$(\nabla \times B) − \mu 0\ J = 0$

$(\nabla \times B) = \mu 0\ J$

As we know that $B = \mu 0\ H$

$\boldsymbol{\nabla \times H = J}$

Modified Maxwell's Fourth Equation

The modified Maxwell's fourth equation is the differential form of the modified Ampere's circuital law.

We know the modified Ampere's circuital law-

$\oint B . dl = \mu 0\ i + id$

Where id = Displacement current

Therefore the modified Maxwell's fourth equation can be written as-

$\nabla \times H = J + Jd$ —— (1)

Where Jd = Displacement current density

And its value of Jd is :

$J_d = \epsilon_0 (\partial E/\partial t)$

and

$J_d = \partial D/\partial t$ ($\because D = \epsilon_0 E$)

Now substitute the value of Jd in equation (1)

Therefore , $\nabla \times H = J + (\partial D / \partial t)$.

This is the required equation

Application of Maxwell's Equation

There are many application and uses of Maxwell's equations in the field of electrodynamics.

- The equations act as a mathematical model for electric, optical, and radio technologies such as power production, electric motors, wireless communication, lenses, radar, and so on.
- They describe how charges, currents, and field changes produce electric and magnetic fields.
- According to Maxwell's equations, a changing magnetic field always produces an electric field, and a changing electric field always induces a magnetic field.

Advantages and Disadvantages of Maxwell's Equation

There are some list of Advantages and Disadvantages of Maxwell's Equation given below :

Advantages of Maxwell's Equation

- Maxwell's equations shows the connection between the theory of magnetism and electricity.
- Various electromagnetic phenomena like the propagation of electromagnetic waves, including light, behaviour of electric circuits were predicted and explained by Maxwell's equations .
- These equations are the foundation for classical electrodynamics. They are necessary to comprehend how electromagnetic fields are produced by charges and currents.
- The emergence of light as an electromagnetic wave, gave a way for the development of technologies like radio, television, and wireless communication, was made possible by Maxwell's equations, which predicted electromagnetic waves.

- Modern physics rely on Maxwell's equations, which have influenced development of other theories including quantum mechanics and special relativity.

Disadvantages of Maxwell's Equation

- Although Maxwell's equations are based on the principles of classical electrodynamics, more sophisticated theories are required when applying them to very tiny scales (quantum electrodynamics) and very high speeds (relativistic electrodynamics).
- Maxwell's equations do not account for quantum effects.
- Certain presumptions and simplifications, such as the lack of magnetic monopoles and the idealized characteristics of materials, form the foundation of Maxwell's equations. These presumptions might not always adequately represent occurrences that occur in the real world.
- Maxwell's equations are simple, but solving them for complicated geometries and boundary conditions can be challenging.
- Concepts of gravitational force quantum mechanics are not included in Maxwell's equations, which constitute a classical theory.