CAI335 SOLAR AND WIND ENERGY SYSTEM

UNIT III NOTES

3.5 Wind power

The **energy pattern factor (EPF)** *is the ratio of power from speed distribution to the power from coverage speed of the turbine blades.*

i.e. $$EPF = \frac{\text{Power from speed distribution}}{\text{Power from average speed}}$$

Generally, *EPF* lies between 2 to 5.

Energy from wind stream is extracted by a wind turbine, by *converting the kinetic energy (K.E.) of the wind to rotational motion required to operate an electric generate.*

In order to compute the mathematical relationships, let us make the following assumptions:

1. The *flow of wind is 'incompressible'*, and hence the air stream *diverges* as it passes through the turbines
2. The *mass flow rate of wind is 'constant'* at *far upstream, at the rotor and at far down stream.*

Let,

p = Atmospheric wind pressure,

p_{us} = Pressure on *upstream* of wind turbine,

p_{ds} = Pressure on *downstream* of wind turbine,

U_w = Atmospheric wind velocity,

$(U_w)_{us}$ = Velocity of wind upstream of wind turbine,

$(U_w)_{bl}$ = Velocity of wind at blades,

$(U_w)_{ds}$ = Velocity of wind downstream of wind turbine before the wind front reforms and regains the atmospheric level,

A_{bl} = Area of blades,

$\dot{m}$ = Mass flow rate of wind, and

ρ = Density of air.

The kinetic energy of wind stream passing through the turbine rotor is given by:

$$\text{K.E.} = \frac{1}{2}\dot{m}(U_w)^2{}_{bl}$$

And, $$\dot{m} = \rho\, A_{bl}(U_w)_{bl}$$

$$\therefore \quad \text{K.E.} = \frac{1}{2}\rho A_{bl}(U_w)_{bl} \times (U_w)^2{}_{bl} = \frac{1}{2}\rho A_{bl}(U_w)^3_{bl}$$

The force on the rotor disc, F is given as:

$$F = (p_{us} - p_{ds})A_{bl}$$

Also, $$F = \dot{m}[(U_w)_{us} - (U_w)_{ds}]$$

[momentum per unit time from upstream to downstream winds]

Applying Bernoulli's equation to upstream and downstream sides, we get:

$$p + \frac{1}{2}\rho(U_w)_{us}^2 = p_{us} + \frac{1}{2}\rho(U_w)_{bl}^2$$

and, $$p_{ds} + \frac{1}{2}\rho(U_w)_{bl}^2 = p + \frac{1}{2}\rho(U_w)_{ds}^2$$

Solving the above equations, we obtain:

$$p_{us} - p_{ds} = \frac{1}{2}\rho\left[(U_w)_{us}^2 - (U_w)_{ds}^2\right]$$

Equating Eqns. (5.6 and 5.7), we get:

$$(p_{us} - p_{ds})\, A_{bl} = \dot{m}[(U_w)_{us} - (U_w)_{ds}] = \rho A_{bl}\, (U_w)_{bl}\, [(U_w)_{us} - (U_w)_{ds}]$$

Solving Eqns. (5.10 and 5.11), we get:

$$\frac{1}{2}\rho\left[(U_w)_{us}^2 - (U_w)_{ds}^2\right] = \rho(U_w)_{bl}\, [(U_w)_{us} - (U_w)_{ds}]$$

or, $$(U_w)_{bl} = \frac{[(U_w)_{us} + (U_w)_{ds}]}{2}$$

In a wind turbine system "*Speed flow work*", *W*, is equal to the difference in kinetic energy between upstream and downstream of turbine for unit mass flow, $\dot{m} = 1$. Therefore,

$$W = (K.E.)_{us} - (K.E.)_{ds}$$

or, $$W = \frac{1}{2}\left[(U_w)_{us}^2 - (U_w)_{ds}^2\right]$$

The *power output* '*P*' of wind turbine (rate of doing work) is given as:

$$P = \frac{1}{2}\dot{m}\left[(U_w)_{us}^2 - (U_w)_{ds}^2\right] = \dot{m}\left[\frac{(U_w)_{us}^2 - (U_w)_{ds}^2}{2}\right]$$

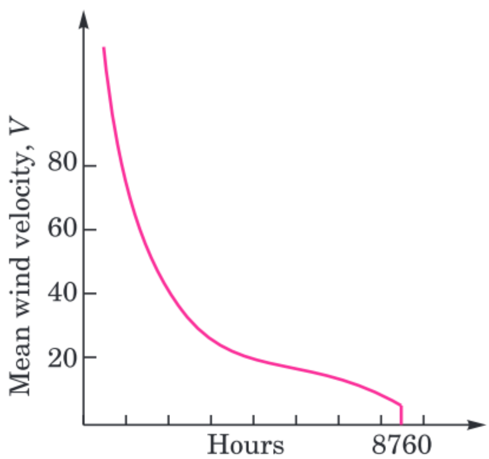
$$= \rho A_{bl}\left[\frac{(U_w)_{us} + (U_w)_{ds}}{2}\right]\left[\frac{(U_w)_{us}^2 - (U_w)_{ds}^2}{2}\right]$$

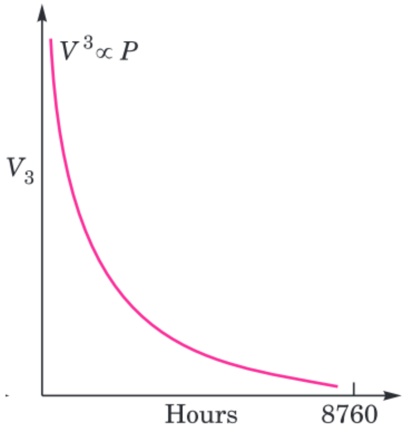
$$\left[\because \dot{m} = \rho A_{bl}\,(U_w)_{bl} = \rho A_{bl}\left\{\frac{(U_w)_{us} + (U_w)_{ds}}{2}\right\} \text{...using Eqn. (5.12)}\right]$$

or, $$P = \frac{1}{4}\rho A_{bl}\,[(U_w)_{us} + (U_w)_{ds}]\left[(U_w)_{us}^2 - (U_w)_{ds}^2\right]$$

3.6 Velocity and power duration curves

Velocity curve



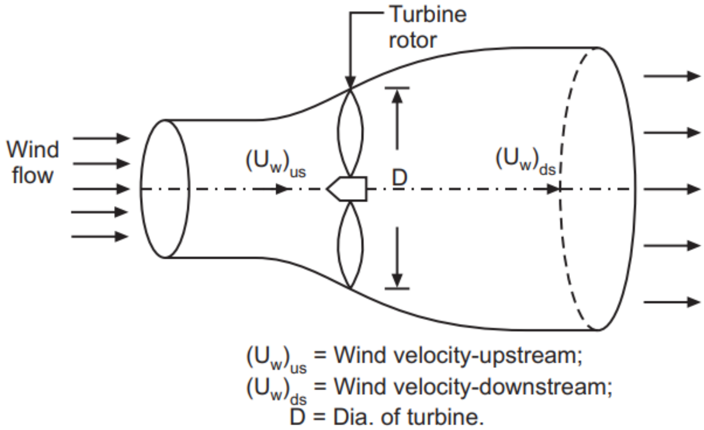


(b) Power duration curve

Both these curves are useful in interpreting wind energy potential.

3.7 Thrust on turbine Rotor

A turbine extracts wind energy, causing the difference in momentum of air streams between the upstream and downsteam sides



Wind flow across turbine rotor.

Actual force, $F_x = \rho A_{bl} (U_w)_{bl} [(U_w)_{us} - (U_w)_{ds}]$

$$= \rho A_{bl} \frac{[(U_w)_{us} + (U_w)_{ds}]}{2} [(U_w)_{us} - (U_w)_{ds}]$$

or $F_x = \frac{1}{2} \rho \frac{\pi}{4} D^4 \left[(U_w)_{us}^2 - (U_w)_{ds}^2\right]$

$$\because (U_w)_{bl} = \frac{[(U_w)_{us} + (U_w)_{ds}]}{2}$$

or, $$F_x = \frac{\pi}{8}\rho D^2\left[(U_w)_{us}^2 - (U_w)_{ds}^2\right]$$

For maximum output, $(U_w)_{ds} = \frac{1}{3}(U_w)_{us}$

$\therefore$ $$F_{x(max)} = \frac{\pi}{8}\rho D^2\left[(U_w)_{us}^2 - \frac{1}{9}(U_w)_{ds}^2\right]$$

i.e., $$F_{x(max)} = \frac{\pi}{9}\rho D^2 (U_w)_{us}^2$$

3.8 Torque

The torque on a turbine rotor would be maximum when maximum thrust can be ıpplied at the *blade tip farthest from the axis*, Maximum torque (T_{max}) in a propeller turbine ›f radius 'R' is given as:

$$T_{max} = F_{max} \cdot R$$

From Eqn. (5.19), F_x becomes maximum when $(U_w)_{ds} = 0$,

i.e., $$T_{max} = \frac{1}{2}\rho A_{bl}\,(U_w)_{us}^2 \,.R$$

(where, A_{bl} = area of blades)

For wind turbine producing a shaft torque T, the *torque coefficient* C_T is defined by:

$$T = C_T . T_{max}$$