

## ME3491 THEORY OF MACHINES

## UNIT III - FRICTION IN MACHINE ELEMENTS

## Friction

Friction is a measure of how hard it is to slide one object over another.

1. Static friction. It is the friction, experienced by a body, when at rest.
2. Dynamic friction. It is the friction, experienced by a body, when in motion. The dynamic friction is also called kinetic friction and is less than the static friction.
It is of the following three types:
(a) Sliding friction. It is the friction, experienced by a body, when it slides over another body.
(b) Rolling friction. It is the friction, experienced between the surfaces which have balls or rollers interposed between them.
(c) Pivot friction. It is the friction, experienced by a body, due to the motion of rotation as in case of foot step bearings.

The friction may further be classified as:

1. Friction between unlubricated surfaces, and
2. Friction between lubricated surfaces.

These are discussed in the following articles.
Laws of Static Friction
Following are the laws of static friction:

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.
3. The magnitude of the limiting friction $(F)$ bears a constant ratio to the normal reaction $\left(R_{\mathrm{N}}\right)$ between the two surfaces. Mathematically

$$
F / R_{\mathrm{N}}=\text { constant }
$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

## Coefficient of friction

It is defined as the ratio of the limiting friction $(F)$ to the normal reaction $\left(R_{\mathrm{N}}\right)$ between the two bodies. It is generally denoted by $\mu$. Mathematically, coefficient of friction,

$$
\mu=F / R_{N}
$$

Consider that a body $A$ of weight ( $W$ ) is resting on a horizontal plane $B$, as shown in Fig.
If a horizontal force $P$ is applied to the body, no relative motion will take place until the applied force $P$ is equal to the force of friction $F$, acting opposite to the direction of motion. The magnitude of this force of friction is

$$
F=\mu \cdot W=\mu \cdot R
$$

## N , where $R$

N is the normal reaction.
In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces :

1. Weight of the body ( $W$ ),
2. Applied horizontal force $(P)$, and
3. Reaction $(R)$ between the body $A$ and the plane $B$.

The reaction $R$ must, therefore, be equal and opposite to the resultant of $W$ and $P$ and will be inclined at an angle _ to the normal reaction $R_{\mathrm{N}}$. This angle _ is known as the limiting angle of friction.
It may be defined as the angle which the resultant reaction $R$ makes with the normal reaction $R_{\mathrm{N}}$.

$$
\text { From, } \tan \Phi=F / R
$$

## Angle of Repose

Consider that a body A of weight $(W)$ is resting on an inclined plane B, If the angle of inclination of the plane to the horizontal is such that the body begins to move down the plane, then the angle $\alpha$ is called the angle of repose.

## Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as external threads.

- But if the threads are cut on the internal surface of a hollow rod, these are known as internal threads.
- The screw threads are mainly of two types i.e. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V threads are used for the purpose of tightening pieces together.

1. Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.
2. Pitch. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw.
3. Lead. It is the distance, a screw thread advances axially in one turn.
4. Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of a thread).
5. Single-threaded screw. If the lead of a screw is equal to its pitch, it is known as single threaded screw.
6. Multi-threaded screw. If more than one thread is cut in one lead distance of a screw, it is known as multithreaded screw e.g. in a double threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

$$
\text { Lead }=\text { Pitch } \times \text { Number of threads }
$$

Helix angle. It is the slope or inclination of the thread with the horizontal. Mathematically,

$$
\begin{aligned}
\tan \alpha & =\frac{\text { Lead of screw }}{\text { Circumference of screw }} \\
& =p / \pi d \quad \ldots(\text { In single-threaded screw }) \\
& =n \cdot p / \pi d \quad \ldots(\text { In multi-threaded screw }) \\
\alpha & =\text { Helix angle, } \\
p & =\text { Pitch of the screw }, \\
d & =\text { Mean diameter of the screw, and } \\
n & =\text { Number of threads in one lead. }
\end{aligned}
$$

1. An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of $300 \mathrm{~mm} / \mathrm{min}$. The screw has a single square thread of 6 mm pitch on a major diameter of 40 $\mathbf{m m}$. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor.

Solution. Given : $W=75 \mathrm{kN}=75 \times 10^{3} \mathrm{~N} ; v=300 \mathrm{~mm} / \mathrm{min} ; p=6 \mathrm{~mm} ; d_{0}=40 \mathrm{~mm}$ $\mu=\tan \phi=0.1$

We know that mean diameter of the screw,

$$
d=d_{0}-p / 2=40-6 / 2=37 \mathrm{~mm}=0.037 \mathrm{~m}
$$

$$
\tan \alpha=\frac{p}{\pi d}=\frac{6}{\pi \times 37}=0.0516
$$

$\therefore$ Force required at the circumference of the screw,

$$
\begin{aligned}
P & =W \tan (\alpha+\phi)=W\left[\frac{\tan \alpha+\tan \phi}{1-\tan \alpha \cdot \tan \phi}\right] \\
& =75 \times 10^{3}\left[\frac{0.0516+0.1}{1-0.0516 \times 0.1}\right]=11.43 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

and torque required to overcome friction,

$$
T=P \times d / 2=11.43 \times 10^{3} \times 0.037 / 2=211.45 \mathrm{~N}-\mathrm{m}
$$

We know that speed of the screw,

$$
N=\frac{\text { Speed of the nut }}{\text { Pitch of the screw }}=\frac{300}{6}=50 \text { r.p.m. }
$$

and angular speed,
$\therefore$ Power of the motor

$$
\begin{aligned}
\omega & =2 \pi \times 50 / 60=5.24 \mathrm{rad} / \mathrm{s} \\
& =T . \omega=211.45 \times 5.24=1108 \mathrm{~W}=1.108 \mathrm{~kW} \mathrm{Ans}
\end{aligned}
$$

2. The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm . The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 $k N$, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.

Solution. Given : $d=50 \mathrm{~mm} ; p=12.5 \mathrm{~mm} ; \mu=\tan \phi=0.13 ; W=25 \mathrm{kN}=25 \times 10^{3} \mathrm{~N}$
We know that, $\quad \tan \alpha=\frac{p}{\pi d}=\frac{12.5}{\pi \times 50}=0.08$
and force required on the screw to raise the load,

$$
\begin{aligned}
P & =W \tan (\alpha+\phi)=W\left[\frac{\tan \phi-\tan \alpha}{1+\tan \phi \cdot \tan \alpha}\right] \\
& =25 \times 10^{3}\left[\frac{0.08+0.13}{1-0.08 \times 0.13}\right]=5305 \mathrm{~N}
\end{aligned}
$$

Torque required on the screw
We know that the torque required on the screw to raise the load,

$$
T_{1}=P \times d / 2=5305 \times 50 / 2=132625 \text { N-mm Ans. }
$$

Ratio of the torques required to raise and lower the load
We know that the force required on the screw to lower the load,

$$
\begin{aligned}
P & =W \tan (\phi-\alpha)=W\left[\frac{\tan \phi-\tan \alpha}{1+\tan \phi \cdot \tan \alpha}\right] \\
& =25 \times 10^{3}\left[\frac{0.13+0.08}{1+0.13 \times 0.08}\right]=1237 \mathrm{~N}
\end{aligned}
$$

and torque required to lower the load

$$
T_{2}=P \times d / 2=1237 \times 50 / 2=30905 \mathrm{~N}-\mathrm{mm}
$$

$\therefore$ Ratio of the torques required,

$$
=T_{1} / T_{2}=132625 / 30925=4.3 \mathrm{Ans}
$$

## Efficiency of the machine

We know that the efficiency,

$$
\begin{aligned}
\eta & =\frac{\tan \alpha}{\tan (\alpha+\phi)}=\frac{\tan \alpha(1-\tan \alpha \cdot \tan \phi)}{\tan \alpha+\tan \phi}=\frac{0.08(1-0.08 \times 0.13)}{0.08+0.13} \\
& =0.377=37.7 \% \text { Ans. }
\end{aligned}
$$

Over Hauling and Self-Locking Screws
The torque required to lower the load

$$
T=P \times \frac{d}{2}=W \tan (\phi-\alpha) \frac{d}{2}
$$

In the above expression, if $\varphi<\alpha$, then torque required to lower the load will be negative. In other words, the load will start moving downward without the application of any torque. Such a condition is known as over hauling of screws. If however $\varphi>\alpha$, the torque required to lower the load will positive, indicating that an effort is applied to lower the load. Such a screw is known as self-locking screw. In other words, a screw will be self-locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle i.e. $\mu$ or $\tan \varphi>\tan \alpha$.
3. A load of 10 kN is raised by means of a screw jack, having a square threaded screw of 12 mm pitch and of mean diameter 50 mm . If a force of 100 N is applied at the end of a lever to raise the load, what should be the length of the lever used? Take coefficient of friction $=0.15$. What is the mechanical advantage obtained? State whether the screw is self locking.

We know that $\quad \tan \alpha=\frac{p}{\pi d}=\frac{12}{\pi \times 50}=0.0764$
$\therefore$ Effort required at the circumference of the screw to raise the load,

$$
\begin{aligned}
P & =W \tan (\alpha+\phi)=W\left[\frac{\tan \alpha+\tan \phi}{1-\tan \alpha \cdot \tan \phi}\right] \\
& =10 \times 10^{3}\left[\frac{0.0764+0.15}{1-0.0764 \times 0.15}\right]=2290 \mathrm{~N}
\end{aligned}
$$

and torque required to overcome friction,

$$
\begin{equation*}
T=P \times d / 2=2290 \times 50 / 2=57250 \mathrm{~N}-\mathrm{mm} \tag{i}
\end{equation*}
$$

We know that torque applied at the end of the lever,

$$
\begin{equation*}
T=P_{1} \times l=100 \times l \mathrm{~N}-\mathrm{mm} \tag{ii}
\end{equation*}
$$

Equating equations (i) and (ii)

$$
l=57250 / 100=572.5 \mathrm{~mm} \text { Ans. }
$$

## Mechanical advantage

We know that mechanical advantage,

## Self locking of the screw

$$
M \cdot A \cdot=\frac{W}{P_{1}}=\frac{10 \times 10^{3}}{100}=100 \mathrm{Ans}
$$

We know that efficiency of the screw jack,

$$
\begin{aligned}
\eta & =\frac{\tan \alpha}{\tan (\alpha+\phi)}=\frac{\tan \alpha(1-\tan \alpha \cdot \tan \phi)}{\tan \alpha+\tan \phi} \\
& =\frac{0.0764(1-0.0764 \times 0.15)}{0.0764+0.15}=\frac{0.0755}{0.2264}=0.3335 \text { or } 33.35 \%
\end{aligned}
$$

Friction of Pivot and Collar Bearing
The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.
The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.
The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface
When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig. 10.16 (c). The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar.


A conical pivot supports a load of 20 kN , the cone angle is $120^{\circ}$ and the intensity of normal pressure is not to exceed $0.3 \mathrm{~N} / \mathrm{mm} 2$. The external diameter is twice the internal diameter. Find the outer and inner radii
of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution. Given : $W=20 \mathrm{kN}=20 \times 10^{3} \mathrm{~N} ; 2 \alpha=120^{\circ}$ or $\alpha=60^{\circ} ; p_{n}=0.3 \mathrm{~N} / \mathrm{mm}^{2}$ $N=200 \mathrm{r} . \mathrm{p} . \mathrm{m}$. or $\omega=2 \pi \times 200 / 60=20.95 \mathrm{rad} / \mathrm{s} ; \mu=0.1$

## Outer and inner radii of the bearing surface

Let $\quad r_{1}$ and $r_{2}=$ Outer and inner radii of the bearing surface, in mm .
Since the external diameter is twice the internal diameter, therefore

$$
r_{1}=2 r_{2}
$$

We know that intensity of normal pressure $\left(p_{n}\right)$,

$$
\begin{aligned}
& 0.3=\frac{W}{\pi\left[\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}\right]}=\frac{20 \times 10^{3}}{\pi\left[\left(2 r_{2}\right)^{2}-\left(r_{2}\right)^{2}\right]}=\frac{2.12 \times 10^{3}}{\left(r_{2}\right)^{2}} \\
\therefore \quad\left(r_{2}\right)^{2} & =2.12 \times 10^{3} / 0.3=7.07 \times 10^{3} \text { or } r_{2}=84 \mathrm{~mm} \text { Ans. }
\end{aligned}
$$

and $\quad r_{1}=2 r_{2}=2 \times 84=168 \mathrm{~mm}$ Ans.

## Power absorbed in friction

We know that total frictional torque (assuming uniform pressure),

$$
\begin{aligned}
T & =\frac{2}{3} \times \mu \cdot W \cdot \operatorname{cosec} \alpha\left[\frac{\left(r_{1}\right)^{3}-\left(r_{2}\right)^{3}}{\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}}\right] \\
& =\frac{2}{3} \times 0.1 \times 20 \times 10^{3} \times \operatorname{cosec} 60^{\circ}=\left[\frac{(168)^{3}-(84)^{3}}{(168)^{2}-(84)^{2}}\right] \mathrm{N}-\mathrm{mm} \\
& =301760 \mathrm{~N}-\mathrm{mm}=301.76 \mathrm{~N}-\mathrm{m}
\end{aligned}
$$

$\therefore$ Power absorbed in friction,

$$
P=T . \omega=301.76 \times 20.95=6322 \mathrm{~W}=6.322 \mathrm{~kW} \text { Ans. }
$$

## Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces

Single Disc or Plate Clutch
A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel.


## 1. Considering uniform pressure

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$
\begin{equation*}
p=\frac{W}{\pi\left[\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}\right]} \tag{i}
\end{equation*}
$$

where
$W=$ Axial thrust with which the contact or friction surfaces are held together.
We have discussed above that the frictional torque on the elementary ring of radius $r$ and thickness $d r$ is

$$
T_{r}=2 \pi \mu \cdot p \cdot r^{2} d r
$$

Integrating this equation within the limits from $r_{2}$ to $r_{1}$ for the total frictional torque.
$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$
T=\int_{r_{1}}^{r_{2}} 2 \pi \mu \cdot p \cdot r^{2} \cdot d r=2 \pi \mu p\left[\frac{r_{3}}{3}\right]_{r_{2}}^{r_{1}}=2 \pi \mu p\left[\frac{\left(r_{1}\right)^{3}-\left(r_{2}\right)^{3}}{3}\right]
$$

Substituting the value of $p$ from equation (i),

$$
\begin{aligned}
T & =2 \pi \mu \times \frac{W}{\pi\left[\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}\right]} \times \frac{\left(r_{1}\right)^{3}-\left(r_{2}\right)^{3}}{3} \\
& =\frac{2}{3} \times \mu \cdot W\left[\frac{\left(r_{1}\right)^{3}-\left(r_{2}\right)^{3}}{\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}}\right]=\mu \cdot W \cdot R
\end{aligned}
$$

where

$$
R=\text { Mean radius of friction surface }
$$

$$
=\frac{2}{3}\left[\frac{\left(r_{1}\right)^{3}-\left(r_{2}\right)^{3}}{\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}}\right]
$$

2. Considering uniform wear

$$
\begin{align*}
& n=\text { Number of pairs of friction or contact surfaces, and } \\
& R=\text { Mean radius of friction surface } \\
&=\frac{2}{3}\left[\frac{\left(r_{1}\right)^{3}-\left(r_{2}\right)^{3}}{\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}}\right]  \tag{Foruniformpressure}\\
&=\frac{r_{1}+r_{2}}{?} \quad \ldots \text { (For uniform pressure) } \\
& \text {..... } p \text { (For uniform wear) }
\end{align*}
$$

$\therefore$ Total frictional torque on the friction surface,
e

$$
\begin{aligned}
T & =\int_{r_{2}}^{r_{1}} 2 \pi \mu \cdot C \cdot r \cdot d r=2 \pi \mu \cdot C\left[\frac{r^{2}}{2}\right]_{r_{2}}^{r_{2}}=2 \pi \mu \cdot C\left[\frac{\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}}{2}\right] \\
& =\pi \mu \cdot C\left[\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}\right]=\pi \mu \times \frac{W}{2 \pi\left(r_{1}-r_{2}\right)}\left[\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}\right] \\
& =\frac{1}{2} \times \mu \cdot W\left(r_{1}+r_{2}\right)=\mu \cdot W \cdot R
\end{aligned}
$$

$$
R=\text { Mean radius of the friction surface }=\frac{r_{1}+r_{2}}{2}
$$

;: 1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$
T=n \cdot \mu \cdot W \cdot R
$$

## Multiple Disc Clutch



Example 10.22. Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN . The inside radius of the contact surface is 50 mm and the outside radius is 100 mm . Assume uniform wear.

Solution. Given : $W=4 \mathrm{kN}=4 \times 10^{3} \mathrm{~N} ; r_{2}=50 \mathrm{~mm} ; r_{1}=100 \mathrm{~mm}$
Maximum pressure
Let $\quad p_{\max }=$ Maximum pressure.
Since the intensity of pressure is maximum at the inner radius $\left(r_{2}\right)$, therefore

$$
p_{\max } \times r_{2}=C \text { or } C=50 p_{\max }
$$

We know that the total force on the contact surface $(W)$,

$$
\begin{array}{rlrl} 
& & 4 \times 10^{3} & =2 \pi \mathrm{C}\left(r_{1}-r_{2}\right)=2 \pi \times 50 p_{\max }(100-50)=15710 p_{\max } \\
& \therefore \quad p_{\max } & =4 \times 10^{3} / 15710=0.2546 \mathrm{~N} / \mathrm{mm}^{2} \text { Ans. }
\end{array}
$$

## Minimum pressure

Let $\quad p_{\min }=$ Minimum pressure.
Since the intensity of pressure is minimum at the outer radius $\left(r_{1}\right)$, therefore

$$
p_{\min } \times r_{1}=C \quad \text { or } \quad C=100 p_{\min }
$$

We know that the total force on the contact surface $(W)$,

$$
\begin{aligned}
& & 4 \times 10^{3} & =2 \pi C\left(r_{1}-r_{2}\right)=2 \pi \times 100 p_{\min }(100-50)=31420 p_{\text {min }} \\
& \therefore & p_{\min } & =4 \times 10^{3} / 31420=0.1273 \mathrm{~N} / \mathrm{mm}^{2} \text { Ans }
\end{aligned}
$$

Average pressure
We know that average pressure,

$$
\begin{aligned}
p_{a v} & =\frac{\text { Total normal force on contact surface }}{\text { Cross-sectional area of contact surfaces }} \\
& =\frac{W}{\pi\left[\left(r_{1}\right)^{2}-\left(r_{2}\right)^{2}\right]}=\frac{4 \times 10^{3}}{\pi\left[(100)^{2}-(50)^{2}\right]}=0.17 \mathrm{~N} / \mathrm{mm}^{2} \text { Ans. }
\end{aligned}
$$

Example 10.25. A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The axial pressure is limited to $0.07 \mathrm{~N} / \mathrm{mm}^{2}$. If the coefficient of friction is 0.25 , find 1 . Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, and 2. Outer and inner radii of the clutch plate.

Solution. Given : $P=7.5 \mathrm{~kW}=7.5 \times 10^{3} \mathrm{~W} ; N=900 \mathrm{r} . \mathrm{p} . \mathrm{m}$ or $\omega=2 \pi \times 900 / 60=94.26 \mathrm{rad} / \mathrm{s}$; $p=0.07 \mathrm{~N} / \mathrm{mm}^{2} ; \mu=0.25$

1. Mean radius and face width of the friction lining

Let $\quad R=$ Mean radius of the friction lining in mm , and
$w=$ Face width of the friction lining in mm ,
Ratio of mean radius to the face width,

$$
\begin{equation*}
R / w=4 \tag{Given}
\end{equation*}
$$

We know that the area of friction faces,

$$
A=2 \pi R . w
$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$
W=A \times p=2 \pi R . w . p
$$

We know that torque transmitted (considering uniform wear),

$$
\begin{align*}
T & =n . \mu W \cdot R=n \cdot \mu(2 \pi R \cdot w \cdot p) R \\
& =n \cdot \mu\left(2 \pi R \times \frac{R}{4} \times p\right) R=\frac{\pi}{2} \times n \cdot \mu \cdot p \cdot R^{3} \\
& =\frac{\pi}{2} \times 2 \times 0.25 \times 0.07 R^{3}=0.055 R^{3} \mathrm{~N}-\mathrm{mm}
\end{align*}
$$ $\ldots(\because n=2$, for single plate clutch $)$

We also know that power transmitted $(P)$,

$$
\begin{align*}
& & 7.5 \times 10^{3} & =T . \omega=T \times 94.26 \\
\therefore & & T & =7.5 \times 10^{3} / 94.26=79.56 \mathrm{~N}-\mathrm{m}=79.56 \times 10^{3} \mathrm{~N}-\mathrm{mm} \tag{ii}
\end{align*}
$$

From equations (i) and (ii),

$$
R^{3}=79.56 \times 10^{3} / 0.055=1446.5 \times 10^{3} \text { or } R=113 \mathrm{~mm} \text { Ans. }
$$

and

$$
w=R / 4=113 / 4=28.25 \mathrm{~mm} \text { Ans. }
$$

2. Outer and inner radii of the clutch plate

Let $\quad r_{1}$ and $r_{2}=$ Outer and inner radii of the clutch plate respectively.
Since the width of the clutch plate is equal to the difference of the outer and inner radii, therefore

$$
\begin{equation*}
w=r_{1}-r_{2}=28.25 \mathrm{~mm} \tag{iii}
\end{equation*}
$$

Also for uniform wear, the mean radius of the clutch plate,

$$
\begin{equation*}
R=\frac{r_{1}+r_{2}}{2} \quad \text { or } \quad r_{1}+r_{2}=2 R=2 \times 113=226 \mathrm{~mm} \tag{iv}
\end{equation*}
$$

From equations (iii) and (iv),

$$
r_{1}=127.125 \mathrm{~mm} ; \text { and } r_{2}=98.875 \text { Ans. }
$$

Example 10.28. A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed $0.127 \mathrm{~N} / \mathrm{mm}^{2}$, find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction $=0.3$.

Solution. Given : $n_{1}+n_{2}=5 ; n=4 ; p=0.127 \mathrm{~N} / \mathrm{mm}^{2} ; N=500$ r.p.m. or $\omega=2 \pi \times 500 / 60$ $=52.4 \mathrm{rad} / \mathrm{s} ; r_{1}=125 \mathrm{~mm} ; r_{2}=75 \mathrm{~mm} ; \mu=0.3$

Since the intensity of pressure is maximum at the inner radius $r_{2}$, therefore

$$
p . r_{2}=C \quad \text { or } \quad C=0.127 \times 75=9.525 \mathrm{~N} / \mathrm{mm}
$$

We know that axial force required to engage the clutch,

$$
W=2 \pi C\left(r_{1}-r_{2}\right)=2 \pi \times 9.525(125-75)=2990 \mathrm{~N}
$$

and mean radius of the friction surfaces,

$$
R=\frac{r_{1}+r_{2}}{2}=\frac{125+75}{2}=100 \mathrm{~mm}=0.1 \mathrm{~m}
$$

We know that torque transmitted,

$$
T=n . \mu \cdot W \cdot R=4 \times 0.3 \times 2990 \times 0.1=358.8 \mathrm{~N}-\mathrm{m}
$$

$\therefore$ Power transmitted,

$$
P=T . \omega=358.8 \times 52.4=18800 \mathrm{~W}=18.8 \mathrm{~kW} \text { Ans. }
$$

Example 10.29. A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm . Assuming uniform wear and coefficient of friction as 0.3 , find the maximum axial intensity of pressure between the discs for transmitting 25 kW at 1575 r.p.m.

Solution. Given : $n_{1}=3 ; n_{2}=2 ; d_{1}=240 \mathrm{~mm}$ or $r_{1}=120 \mathrm{~mm} ; d_{2}=120 \mathrm{~mm}$ or $r_{2}=60 \mathrm{~mm}$; $\mu=0.3 ; P=25 \mathrm{~kW}=25 \times 10^{3} \mathrm{~W} ; N=1575 \mathrm{r} . \mathrm{p} . \mathrm{m}$. or $\omega=2 \pi \times 1575 / 60=165 \mathrm{rad} / \mathrm{s}$

Let $\quad T=$ Torque transmitted in $\mathrm{N}-\mathrm{m}$, and
$W=$ Axial force on each friction surface.
We know that the power transmitted $(P)$,

$$
25 \times 10^{3}=T . \omega=T \times 165 \text { or } T=25 \times 10^{3} / 165=151.5 \mathrm{~N}-\mathrm{m}
$$

Number of pairs of friction surfaces,

$$
n=n_{1}+n_{2}-1=3+2-1=4
$$

and mean radius of friction surfaces for uniform wear,
We know that torque transmitted ( $T$ ),

$$
\begin{array}{rlrl} 
& & 151.5 & =n \cdot \mu \cdot W \cdot R=4 \times 0.3 \times W \times 0.09=0.108 \mathrm{~W} \\
\therefore & W & =151.5 / 0.108=1403 \mathrm{~N} \\
\text { Let } & p & =\text { Maximum axial intensity of pressure. }
\end{array}
$$

Since the intensity of pressure $(p)$ is maximum at the inner radius $\left(r_{2}\right)$, therefore for uniform wear

$$
p . r_{2}=C \quad \text { or } \quad C=p \times 60=60 p \mathrm{~N} / \mathrm{mm}
$$

We know that the axial force on each friction surface $(W)$,

$$
\begin{array}{rlrl} 
& & 1403 & =2 \pi \cdot C\left(r_{1}-r_{2}\right)=2 \pi \times 60 p(120-60)=22622 p \\
\therefore & p & =1403 / 22622=0.062 \mathrm{~N} / \mathrm{mm}^{2} \text { Ans. }
\end{array}
$$

