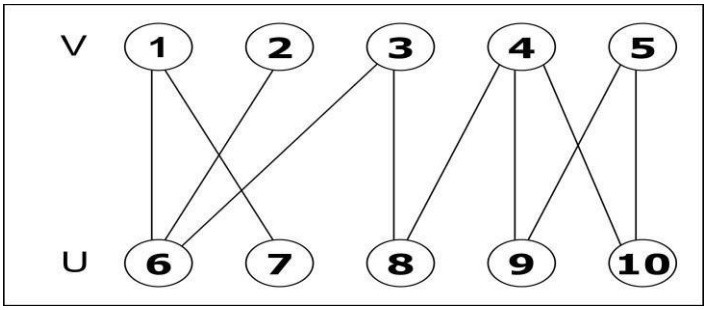
MAXIMUM MATCHING IN BIPARTITE GRAPHS

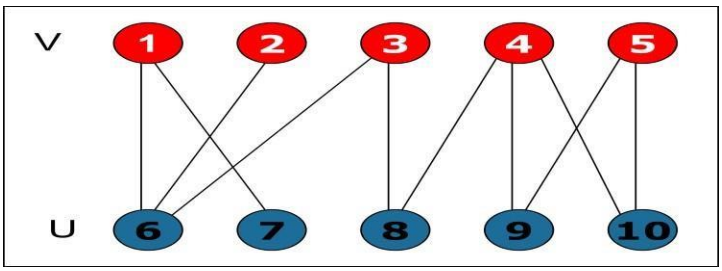
Bipartite Graphs

Bipartite graph: a graph whose vertices can be partitioned into two disjoint sets V and U, not necessarily of the same size, so that every edge connects a vertex in V to a vertex in U.

A graph is bipartite if and only if it does not have a cycle of an odd length.

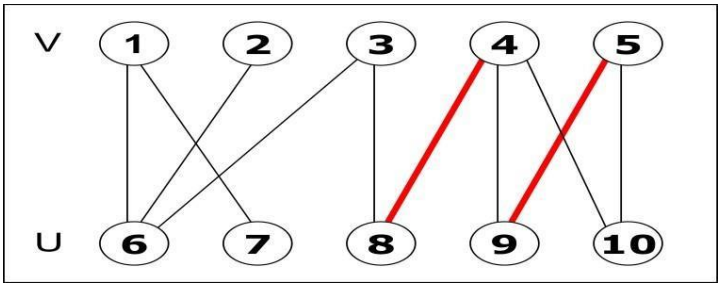


A bipartite graph is *2-colorable*: the vertices can be colored in two colors so that every edge has its vertices colored differently



Matching in a Graph

A *matching* in a graph is a subset of its edges with the property that no two edges



share a vertex

a matching in this graph M = {(4,8), (5,9)}

maximum (or *maximum cardinality*) *matching* is a matching with the largest number of edges

- Always exists
- not always unique

Free Vertices and Maximum Matching

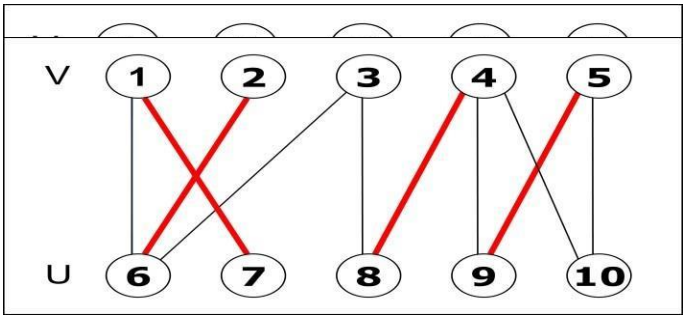
For a given matching M, a vertex is called *free* (or *unmatched*) if it is not an end point of any edge in M; otherwise, a vertex is said to be *matched*

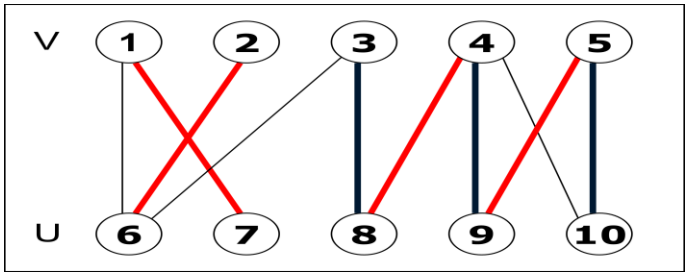
- If every vertex is matched, then M is a maximum matching
- If there are unmatched or free vertices, then M may be able to be improved
- We can immediately increase a matching by adding an edge connecting two free vertices (e.g., (1,6)above)
- Matched vertex = 4, 5, 8, 9. Free vertex = 1, 2, 3, 6, 7,10.

Augmenting Paths and Augmentation

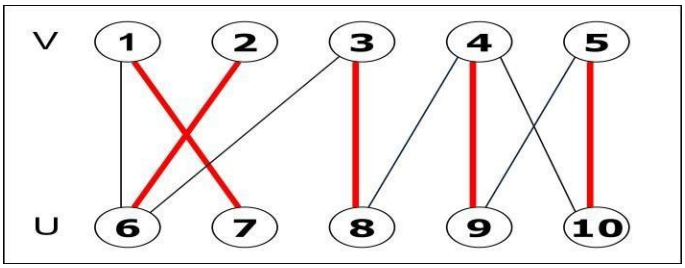
An *augmenting path* for a matching M is a path from a free vertex in V to a free vertex in U whose edges alternate between edges not in M and edges in M

- The length of an augmenting path is always odd
- Adding to M the odd numbered path edges and deleting from it the even numbered path edges increases the matching size by 1(*augmentation*)
- One-edge path between two free vertices is special case of augmentin gpath





Augmentation along path 2,6,1,7



Augmentation along 3, 8, 4, 9, 5, 10

Matching on the right is maximum (*perfect* matching).

<u>Theorem:</u> A matching M is maximum if and only if there exists no augmenting path with respect to M.

Augmenting Path Method (template)

- Start with some initial matching . e.g., the emptyset
- Find an augmenting path and augment the current matching along that path. e.g., using breadth-first search like method
- When no augmenting path can be found, terminate and return the last matching, which is maximum