Capacitance and Capacitors

We have already stated that a conductor in an electrostatic field is an Equipotential body and any charge given to such conductor will distribute themselves in such a manner that electric field inside the conductor vanishes. If an additional amount of charge is supplied to an isolated conductor at a given potential, this additional charge will increase the

surface charge density ρ_s. Since the potential of the conductor is given by $V = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_S ds'}{r}$, the potential of the conductor will also increase maintaining the ratio $\frac{Q}{V}$ same. Thus we can write $C = \frac{Q}{V}$

where the constant of proportionality C is called the capacitance of the isolated conductor. SI unit of capacitance is Coulomb/ Volt also called Farad denoted by F. It can It can be seen that if V=1, $C = Q$. Thus capacity of an isolated conductor can also be defined as the amount of charge in Coulomb required to raise the potential of the conductor by 1 Volt.

Of considerable interest in practice is a capacitor that consists of two (or more) conductors carrying equal and opposite charges and separated by some dielectric media or free space. The conductors may have arbitrary shapes. A two-conductor capacitor is shown in figure 2.19.

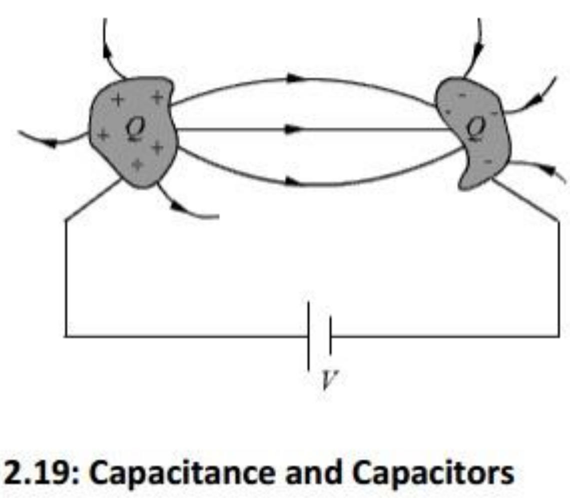

Fig 2.19: Capacitance and Capacitors

When a d-c voltage source is connected between the conductors, a charge transfer occurs which results into a positive charge on one conductor and negative charge on the other conductor. The conductors are equipotential surfaces and the field lines are perpendicular to the conductor surface. If V is the mean potential difference between the conductors, the capacitance

is given by $C = \frac{Q}{V}$. Capacitance of a capacitor depends on the geometry of the conductor and the permittivity of the medium between them and does not depend on the charge or potential difference between conductors. The capacitance can be computed by assuming Q(at the same time $-Q$ on the other conductor), first determining $\vec{E}$ using Gauss's theorem and then determining $V = -\int \vec{E}.d\vec{l}$. We illustrate this procedure by taking the example of a parallel plate capacitor.

Example: Parallel plate capacitor

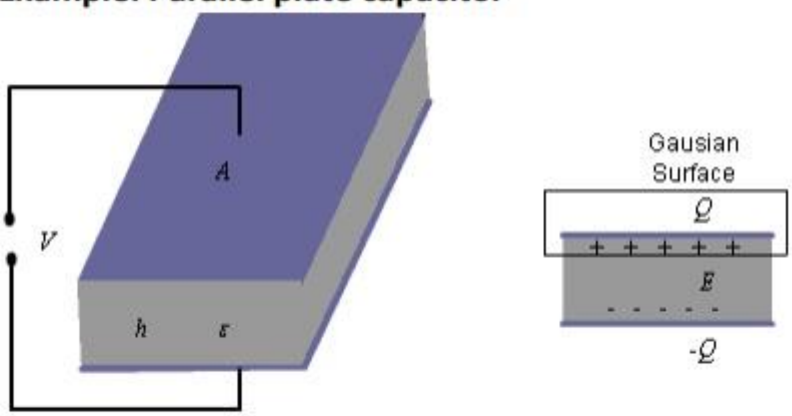

Fig 2.20: Parallel Plate Capacitor

For the parallel plate capacitor shown in the figure 2.20, let each plate has area A and a distance h separates the plates. A dielectric of permittivity ε fills the region between the plates. The electric field lines are confined between the plates. We ignore the flux fringing at the edges of the plates and charges are assumed to be uniformly distributed over the conducting plates with densities ρ_s and $-\rho_s$, $\rho_s = \frac{Q}{A}$.

By Gauss's theorem we can write, $E = \frac{\rho_s}{\varepsilon} = \frac{Q}{A\varepsilon}$ ....(2.85)

As we have assumed ρ_s to be uniform and fringing of field is neglected, we see that E is constant in the region between the plates and therefore, we can write $V = Eh = \frac{hQ}{\varepsilon A}$. Thus, for a parallel plate capacitor we have, $C = \frac{Q}{V} = \varepsilon \frac{A}{h}$ ....(2.86)

Series and parallel Connection of capacitors

Capacitors are connected in various manners in electrical circuits; series and parallel connections are the two basic ways of connecting capacitors. We compute the equivalent capacitance for such connections.

Series Case: Series connection of two capacitors is shown in the figure 2.21. For this case we can write,

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2}$$ ....(2.87)

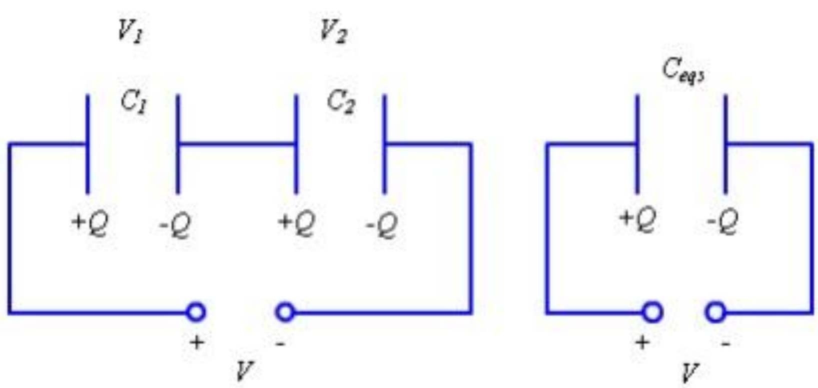

Fig 2.21: Series Connection of Capacitors

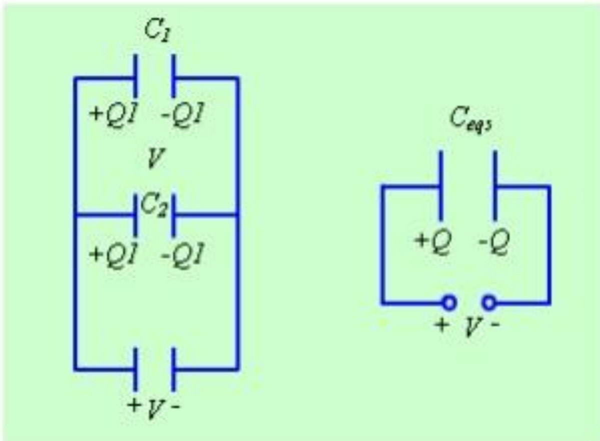

Fig 2.22: Parallel Connection of Capacitors

The same approach may be extended to more than two capacitors connected in series.

Parallel Case: For the parallel case, the voltages across the capacitors are the same.

The total charge $Q = Q_1 + Q_2 = C_1V + C_2V$

Therefore,

$$C_{eqp} = \frac{Q}{V} = C_1 + C_2$$

.......................(2.88)

Electrostatic Energy and Energy Density

We have stated that the electric potential at a point in an electric field is the amount of work required to bring a unit positive charge from infinity (reference of zero potential) to that point. To determine the energy that is present in an assembly of charges, let us first determine the amount of work required to assemble them. Let us consider a number of discrete charges $Q1$, $Q2$,......., QN are brought from infinity to their present position one by one. Since initially there is no field present, the amount of work done in bring Q1 is zero. Q2 is brought in the presence of the field of Q1, the work done $W1 = Q2V21$ where $V21$ is the potential at the location of Q2 due to Q1. Proceeding in this manner, we can write, the total work done

$$W = V_{21}Q_2 + (V_{31}Q_3 + V_{32}Q_3) + \ldots\ldots\ldots + (V_{N1}Q_N + \ldots\ldots + V_{N(N-1)}Q_N)$$

……………………………………………(2.89)

Had the charges been brought in the reverse order,

$$W = (V_{1N}Q_1 + \ldots\ldots + V_{12}Q_1) + \ldots\ldots\ldots + (V_{(N-2)(N-1)}Q_{N-2} + V_{(N-2)N}Q_{N-2}) + V_{(N-1)N}Q_{N-1}$$

……………..(2.90)

Therefore,

$$2W = (V_{1N} + V_{1(N-1)} + \ldots\ldots + V_{12})Q_1 + (V_{2N} + V_{2(N-1)} + \ldots\ldots + V_{23} + V_{21})Q_1 \ldots\ldots$$
$$\ldots\ldots\ldots + (V_{N1} + \ldots\ldots V_{N2} + V_{N(N-1)})Q_N$$

……………..(2.91)

Here V_{IJ} represent voltage at the I^{th} charge location due to J^{th} charge. Therefore,

$$2W = V_1Q_1 + \ldots\ldots\ldots + V_NQ_N = \sum_{I=1}^{N} V_IQ_I$$

Or, $W = \frac{1}{2}\sum_{I=1}^{N} V_IQ_I$ ……………..(2.92)

If instead of discrete charges, we now have a distribution of charges over a volume *v* then we can write,

$$W = \frac{1}{2}\int_v V\rho_v dv \qquad (2.93)$$

where ρ_v is the volume charge density and *V* represents the potential function.

Since, $\rho_v = \nabla \cdot \vec{D}$, we can write

$$W = \frac{1}{2}\int_v (\nabla \cdot \vec{D}) V dv \qquad (2.94)$$

Using the vector identity,

$\nabla.(V\vec{D}) = \vec{D}\cdot\nabla V + V\nabla\cdot\vec{D}$, we can write

$$W = \frac{1}{2}\int_v \left(\nabla.(V\vec{D}) - \vec{D}\cdot\nabla V\right) dv$$
$$= \frac{1}{2}\oint_s \left(V\vec{D}\right).d\vec{s} - \frac{1}{2}\int_v \left(\vec{D}\cdot\nabla V\right) dv \qquad (2.95)$$

In the expression $\frac{1}{2}\oint_s \left(V\vec{D}\right).d\vec{s}$, for point charges, since *V* varies as $\frac{1}{r}$ and D varies as $\frac{1}{r^2}$, the term $V\vec{D}$ varies as $\frac{1}{r^3}$ while the area varies as r^2. Hence the integral term varies at least as $\frac{1}{r}$ and the as surface becomes large (i.e. $r \to \infty$) the integral term tends to zero.

Thus the equation for *W* reduces to

$$W = -\frac{1}{2}\int_v \left(\vec{D}.\nabla V\right) dv = \frac{1}{2}\int_v \left(\vec{D}.\vec{E}\right) dv = \frac{1}{2}\int_v \left(\varepsilon E^2\right) dv = \int_v w_e dv \qquad (2.96)$$

$w_e = \frac{1}{2}\varepsilon E^2$, is called the energy density in the electrostatic field.