ME3491 THEORY OF MACHINES

NOTES

1.8.Displacement, velocity and acceleration analysis

Important Concepts in Velocity Analysis

1. The absolute velocity of any point on a mechanism is the velocity of that point with reference to ground.

2. Relative velocity describes how one point on a mechanism moves relative to another point on the mechanism.

3. The velocity of a point on a moving link relative to the pivot of the link is given by the equation: V = ωr, where = angular velocity of the link and r = distance from pivot.

Acceleration Components

- ***Normal Acceleration:*** $\mathbf{A}^n$ = Points toward the centre of rotation
- ***Tangential Acceleration:*** $\mathbf{A}^t$ = In a direction perpendicular to the link
- ***Coriolis Acceleration:*** $\mathbf{A}^c$ = In a direction perpendicular to the link
- ***Sliding Acceleration:*** $\mathbf{A}^s$ = In the direction of sliding.

A rotating link will produce normal and tangential acceleration components at any point a distance, r, from the rotational pivot of the link. The total acceleration of that point is the vector sum of the components. A slider attached to ground experiences only sliding acceleration.

The total acceleration of a point is the vector sum of all applicable acceleration components:

$$\mathbf{A} = \mathbf{A}^n + \mathbf{A}^t + \mathbf{A}^c + \mathbf{A}^s$$

These vectors and the above equation can be broken into x and y components by applying sines and cosines to the vector diagrams to determine the x and y components of each vector. In this way, the x and y components of the total acceleration can be found.

1.9.Graphical Method, Velocity and Acceleration polygons:

*** Graphical velocity analysis:**

It is a very short step (using basic trigonometry with sines and cosines) to convert the graphical results into numerical results. The basic steps are these:

4. Set up a velocity reference plane with a point of zero velocity designated.
5. Use the equation, V = ωr, to calculate any known linkage velocities.
6. Plot your known linkage velocities on the velocity plot. A linkage that is rotating about ground gives an absolute velocity. This is a vector that originates at the zero-velocity point and runs perpendicular to the link to show the direction of motion. The vector, **V**A, gives the velocity of point A.
7. Plot all other velocity vector directions. A point on a grounded link (such as point B) will produce an absolute velocity vector passing through the zero-velocity point and perpendicular to the link. A point on a floating link (such as B relative to point A) will produce a relative velocity vector. This vector will be perpendicular to the link AB and pass through the reference point (A) on the velocity diagram.
8. One should be able to form a closed triangle (for a 4-bar) that shows the vector equation: **V**B = **V**A + **V**B/A where **V**B = absolute velocity of point B, **V**A = absolute

velocity of point A, and **V**B/A is the velocity of point B relative to point A.

Velocity and Acceleration analysis of mechanisms (Graphical Methods):

Velocity and acceleration analysis by vector polygons: Relative velocity and accelerations of particles in a common link, relative velocity and accelerations of coincident particles on separate link, Coriolis component of acceleration.

Velocity and acceleration analysis by complex numbers: Analysis of single slider crank mechanism and four bar mechanism by loop closure equations and complex numbers.

Coincident points, Coriolis Acceleration:

- ***Coriolis Acceleration:*** $\mathbf{A}^{c}$ = 2 (dr/dt). In a direction perpendicular to the link. A slider attached to ground experiences only sliding acceleration.

Problem: ***The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine : 1. velocity of piston, 2. angular velocity of connecting rod, 3. velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.***

Solution

Since the crank length OB = 0.5 m, therefore linear velocity of B with respect to O or velocity of B (because O is a fixed point), $v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426$ m/s . .

1.Velocity of piston

First of all draw the space diagram, to some suitable scale, as shown in Fig. 7.8 (a). Now the velocity diagram, as shown in Figure a is drawn as discussed below

1. Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or velocity of B such that vector ob = $v_{BO} = v_B = 9.426$ m/s

2. From point b, draw vector bp perpendicular to BP to represent velocity of P with respect to B (i.e. v_{PB}) and from point o, draw vector op parallel to PO to represent velocity of P with respect to O (i.e. v_{PO} or simply v_P). The vectors bp and op intersect at point p.

By measurement, we find that velocity of piston P, v_P = vector op = 8.15 m/s

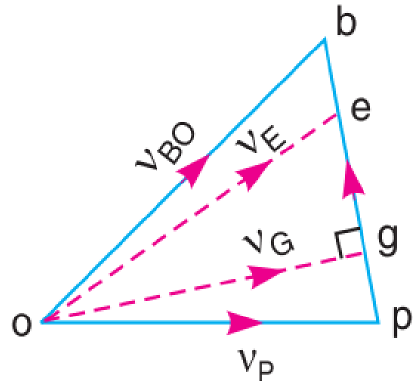

Figure :Velocity Analysis

2. Angular velocity of connecting rod

From the velocity diagram, we find that the velocity of P with respect to B, v PB = vector bp = 6.8 m/s.

Since the length of connecting rod PB is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s (Anticlockwise)}$$

3. Velocity of point E on the connecting rod

The velocity of point E on the connecting rod 1.5 m from the gudgeon pin (i.e. PE = 1.5 m) is determined by dividing the vector bp at e in the same ratio as E divides PB in Figure below.

This is done in the similar way as discussed in Art 7.6. Join oe. The vector oe represents the velocity of E.

By measurement, we find that velocity of point E, v E = vector oe = 8.5 m/s

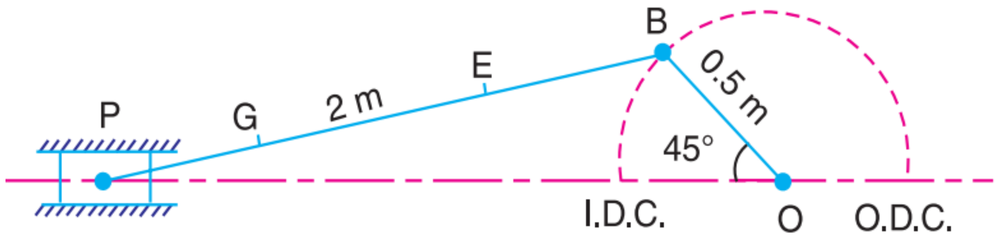

Figure:Space diagram

4. Velocity of rubbing

We know that diameter of crank-shaft pin at O, dO = 50 mm = 0.05 m

Diameter of crank-pin at B, dB = 60 mm = 0.06 m and diameter of cross-head pin, dC = 30 mm = 0.03 m

We know that velocity of rubbing at the pin of crank-shaft = O BO 0.05 18.85 2 2 d ×ω = × = 0.47 m/s

Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = \frac{0.06}{2}(18.85 + 3.4) = 0.6675 \text{ m/s}$$

and velocity of rubbing at the pin of cross-head

$$= \frac{d_C}{2} \times \omega_{PB} = \frac{0.03}{2} \times 3.4 = 0.051 \text{ m/s}$$

5. Position and linear velocity of point G on the connecting rod which has the least velocity relative to crank-shaft

The position of point G on the connecting rod which has the least velocity relative to crank shaft is determined by drawing perpendicular from o to vector bp.

Since the length of og will be the least, therefore the point g represents the required position of G on the connecting rod. By measurement, we find that vector bg = 5 m/s

The position of point G on the connecting rod is obtained as follows:

$$\frac{bg}{bp} = \frac{BG}{BP} \quad \text{or} \quad BG = \frac{bg}{bp} \times BP = \frac{5}{6.8} \times 2 = 1.47 \text{ m}$$