

UNIT 2

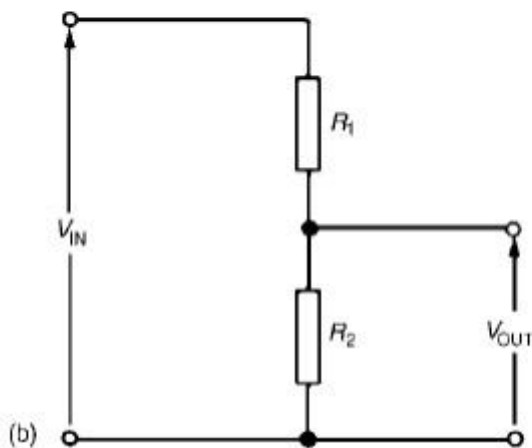
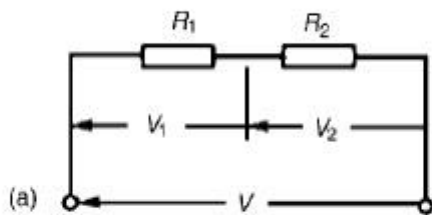
NETWORK REDUCTION AND NETWORK THEOREMS FOR DC AND AC CIRCUITS

POTENTIAL DIVIDER:

The voltage distribution for the circuit shown in Figure

$$V_1 = \left(\frac{R_1}{R_1 + R_2} \right) V$$

$$V_2 = \left(\frac{R_2}{R_1 + R_2} \right) V$$

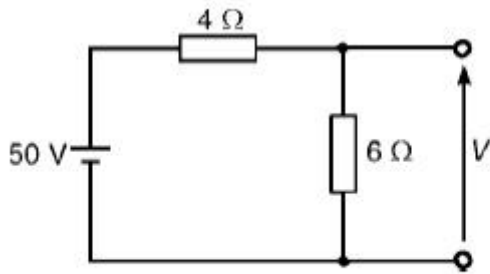


The circuit shown in Figure (b) is often referred to as a potential divider circuit. Such a circuit can consist of a number of similar elements in series connected across a voltage source, voltages being taken from connections between the elements. Frequently the divider consists of two resistors as shown in Figure (b), where

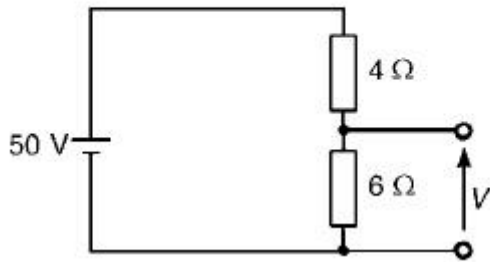
$$V_{\text{OUT}} = \left(\frac{R_2}{R_1 + R_2} \right) V_{\text{IN}}$$

A potential divider is the simplest way of producing a source of lower e.m.f. from a source of higher e.m.f., and is the basic operating mechanism of the potentiometer, a measuring device for accurately measuring potential differences

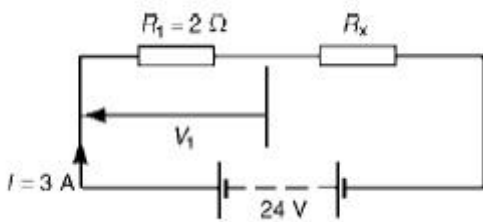
Problem 1: Determine the value of voltage V shown in Figure



$$V = \left(\frac{6}{6+4} \right) (50) = 30 \text{ V}$$



Problem 2: Two resistors are connected in series across a 24V supply and a current of 3A flows in the circuit. If one of the resistors has a resistance of 2Ω and (b) the p.d. across the 2Ω resistor. If the circuit is connected for 50 hours, how much energy is used?



(a) Total circuit resistance $R = V / I$

$$= 24/3 = 8 \Omega \text{ Value of unknown resistance, } R_x = 8 - 2 = 6 \Omega$$

(b) P.d. across 2Ω resistor, $V_1 = IR_1 = 3 \times 2 = 6\text{V}$ Alternatively, from above,

$$V_1 = \left(\frac{R_1}{R_1 + R_x} \right) V = \left(\frac{2}{2 + 6} \right) (24) = 6\text{V} \text{ Energy used} = \text{power} \times \text{time}$$

$$= V \times I \times t$$

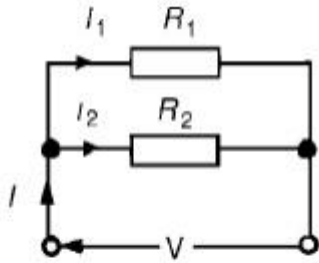
$$= (24 \times 3\text{W}) (50 \text{ h})$$

$$= 3600\text{Wh} = 3.6\text{kWh}$$

Current division:

For the circuit shown in Figure, the total circuit resistance, R_T is given by:

$$R_T = R_1 R_2 / R_1 + R_2$$



$$\text{and } V = IR_T = I \left(\frac{R_1 R_2}{R_1 + R_2} \right)$$

$$\text{Current } I_1 = \frac{V}{R_1} = \frac{I}{R_1} \left(\frac{R_1 R_2}{R_1 + R_2} \right) = \left(\frac{R_2}{R_1 + R_2} \right) (I)$$

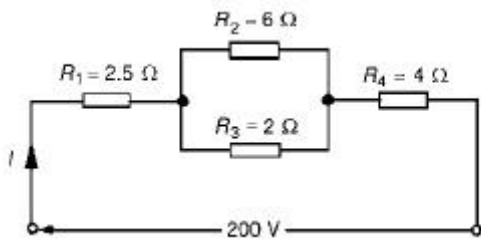
Similarly,

$$\text{current } I_2 = \frac{V}{R_2} = \frac{I}{R_2} \left(\frac{R_1 R_2}{R_1 + R_2} \right) = \left(\frac{R_1}{R_1 + R_2} \right) (I)$$

Summarizing, with reference to Figure 5.20

$$I_1 = \left(\frac{R_2}{R_1 + R_2} \right) (I) \quad \text{and} \quad I_2 = \left(\frac{R_1}{R_1 + R_2} \right) (I)$$

Problem 1: For the series-parallel arrangement shown in Figure, find (a) the supply current, (b) the current flowing through each resistor and (c) the p.d. across each resistor.



(a) The equivalent resistance R_x of R_2 and R_3 in parallel is: $R_x = 6 \times 2 / 6 + 2$

$$= 12/8$$

$$= 1.5 \Omega$$

The equivalent resistance R_T of R_1 , R_x and R_4 in series is:

$$R_T = 2.5 + 1.5 + 4 = 8 \Omega \quad \text{Supply current } I = V/R_T$$

$$= 200/8$$

$$= 25A$$

The current flowing through R_1 and R_4 is 25A The current flowing through R_2

$$= \left(\frac{R_3}{R_2 + R_3} \right) I = \left(\frac{2}{6 + 2} \right) 25$$

$$= 6.25\text{A}$$

The current flowing through R3

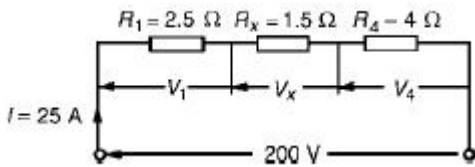
$$= \left(\frac{R_2}{R_2 + R_3} \right) I = \left(\frac{6}{6 + 2} \right) 25$$

$$= 18.75\text{A}$$

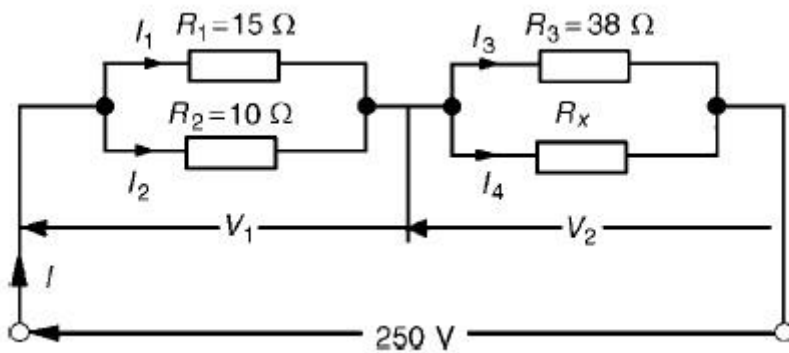
(c) The equivalent circuit of Figure is

p.d. across R1, i.e. $V_1 = IR_1 = (25)(2.5) = 62.5\text{V}$ p.d. across Rx, i.e. $V_x = IR_x = (25)(1.5) = 37.5\text{V}$ p.d. across R4, i.e. $V_4 = IR_4 = (25)(4) = 100\text{V}$

Hence the p.d. across R2 = p.d. across R3 = 37.5V



Problem 2: For the circuit shown in Figure 5.23 calculate (a) the value of resistor Rx such that the total power dissipated in the circuit is 2.5kW, and (b) the current flowing in each of the four resistors.



(a) Power dissipated $P = VI$ watts, hence $2500 = (250)(I)$ i.e. $I = 2500/250$

$$= 10\text{A}$$

From Ohm's $R_T = V/I = \text{law}, 250/10$

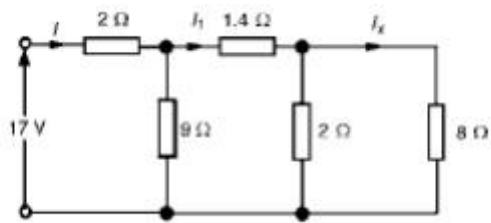
$= 25 \Omega$, where R_T is the equivalent circuit resistance. The equivalent resistance of R_1 and R_2 in parallel is $= 15 \times 10 / 15 + 10$

$$= 150/25$$

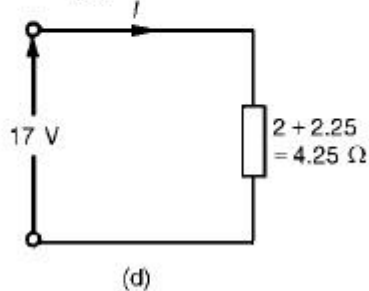
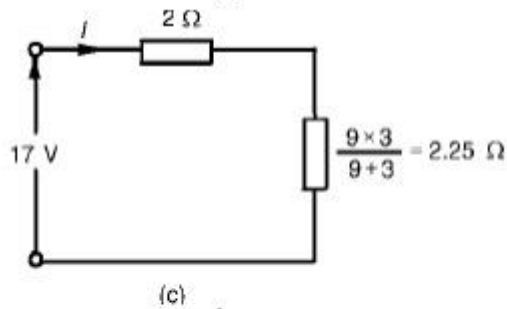
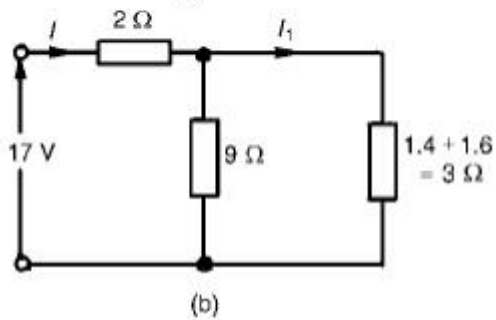
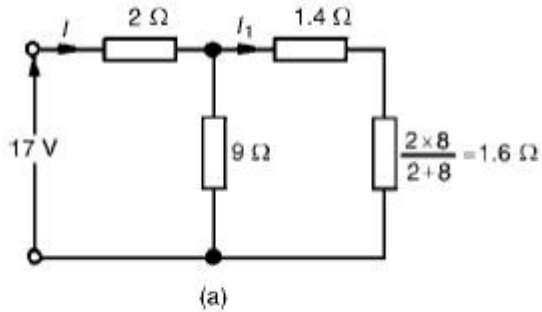
$$= 6 \Omega$$

The equivalent resistance of resistors R_3 and R_x in parallel is equal to $25 \Omega - 6 \Omega$, i.e. 19Ω . There are three methods whereby R_x can be determined.

Problem 3: For the arrangement shown in Figure find the current I_x .



Commencing at the right-hand side of the arrangement shown in Figure, the circuit is gradually reduced in stages as shown in Figure



From Figure (d), $I = 17/4.25 = 4A$

From Figure (b), $I_1 = 9/9 + 3(I) = 12/ (4) = 3A$ From Figure, $I_x = 2/2 + 8(I_1) = 2/10(3) = 0.6A$

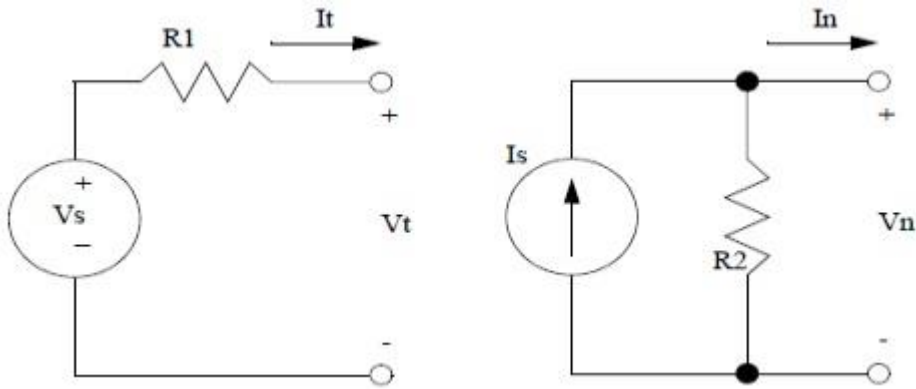
Source transformation:

Source transformation is defined as to convert the sources for easy analysis of circuit. In mesh analysis. it is easier if the circuit has voltage sources.

In nodal analysis. it is easier if the circuit has current sources.

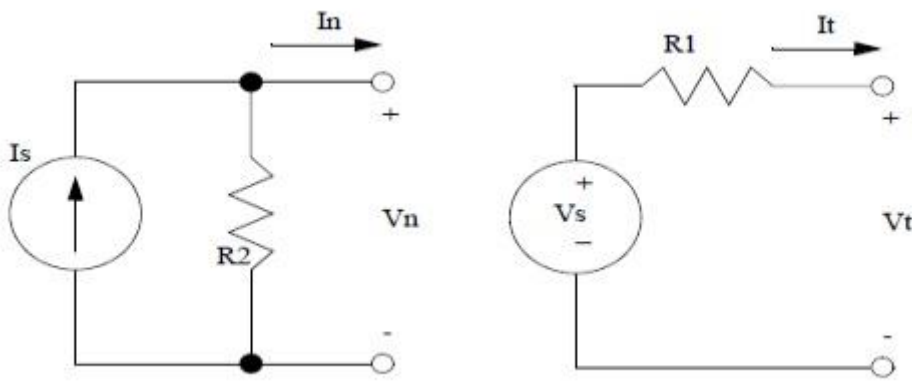
4. VOLTAGE SOURCE TO CURRENT SOURCE TRANSFORMATION:

If voltage source is converted to current source, then the current source $I = V/R_{se}$ with parallel resistance equal to R_{se} .



5. CURRENT SOURCE TO VOLTAGE SOURCE TRANSFORMATION:

If current source is converted to voltage source, then the voltage source $V = I \cdot R_{sh}$ with series resistance equal to R_{sh} .

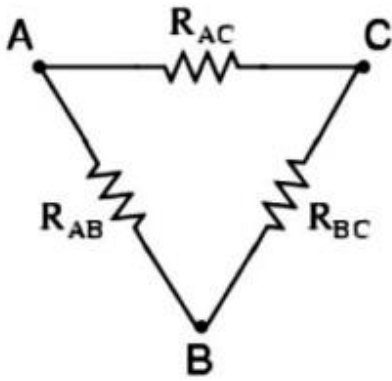


6. STAR DELTA CONVERSION:

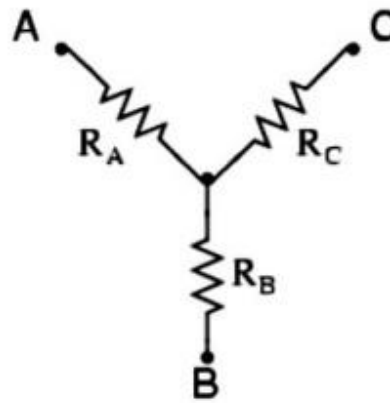
In many circuit applications, we encounter components connected together in one of two ways to

form a three-terminal network: the —Delta or (also the —Star (also known as the —Y) configuration

Delta (Δ) network



Wye (Y) network



To convert a Delta (Δ) to a Wye (Y)

$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

To convert a Wye (Y) to a Delta (Δ)

$$R_{AB} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_A}$$

$$R_{AC} = \frac{R_A R_B + R_A R_C + R_B R_C}{R_B}$$

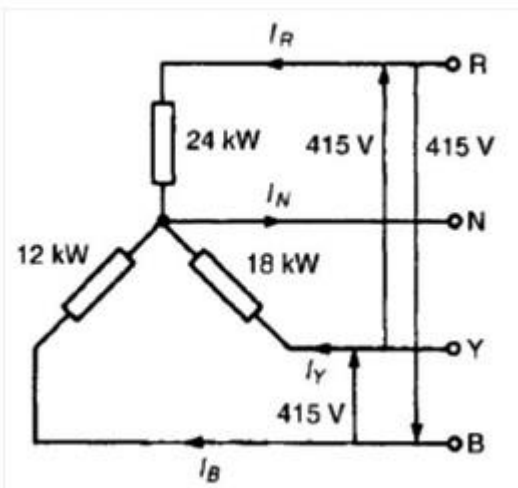
Problem 1: A star-connected load consists of three identical coils each of resistance 30Ω and inductance 127.3 mH . If the line current is 5.08 A , calculate the line voltage if the supply frequency is 50 Hz .
Inductive reactance $X_L = 2\pi fL$

$$= 2\pi(50)(127.3 \times 10^{-3}) = 40 \Omega$$

Impedance of each phase $Z_p = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 40^2} = 50 \Omega$ For a star connection $I_L = I_p = V_p / Z_p$

Hence phase voltage $V_p = I_p Z_p = (5.08)(50) = 254 \text{ V}$ Line voltage $V_L = \sqrt{3} V_p = \sqrt{3}(254) = 440 \text{ V}$

Problem 2: A 415 V , 3-phase, 4 wire, star-connected system supplies three resistive loads as shown in Figure. Determine (a) the current in each line and (b) the current in the neutral conductor.



(a) For a star-connected system $V_L = \sqrt{3} V_p$

Hence
$$V_P = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240 \text{ V}$$

Since current $I = \text{Power } P / \text{Voltage } V$ for a resistive load then $I_R = P_R / V_R = 24\,000 / 240 = 100 \text{ A}$

$I_Y = P_Y / V_Y = 18\,000 / 240 = 75 \text{ A}$ and $I_B = P_B / V_B = 12\,000 / 240 = 50 \text{ A}$

(b) The three line currents are shown in the phasor diagram of Figure. Since each load is resistive the currents are in phase with the phase voltages and are hence mutually displaced by 120° . The current in the neutral conductor is given by:

$I_N = I_R + I_Y + I_B$ phasorially.

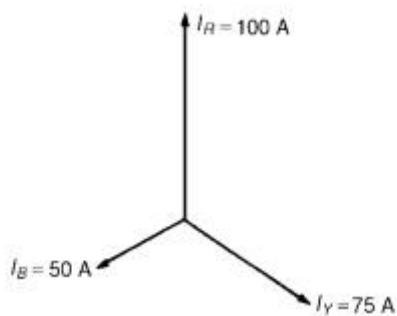


Figure shows the three line currents added phasorially. Oa represents I_R in magnitude and direction. From the nose of Oa , ab is drawn representing I_Y in magnitude and direction. From the nose of ab , bc is drawn representing I_B in magnitude and direction. Oc represents the resultant, I_N .

By measurement, $I_N = 43 \text{ A}$

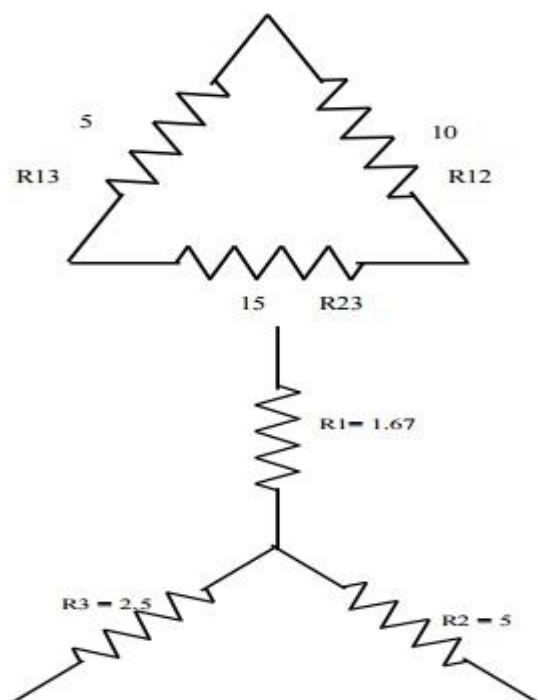
Alternatively, by calculation, considering I_R at 90° , I_B at 210° and I_Y at 330° :

Total horizontal component = $100 \cos 90^\circ + 75 \cos 330^\circ + 50 \cos 210^\circ = 21.65$

Total vertical component = $100 \sin 90^\circ + 75 \sin 330^\circ + 50 \sin 210^\circ = 37.50$

Hence magnitude of $I_N = \sqrt{(21.65^2 + 37.50^2)}$
 $= 43.3 \text{ A}$

Problem 3: Convert the given delta fig into equivalent star.



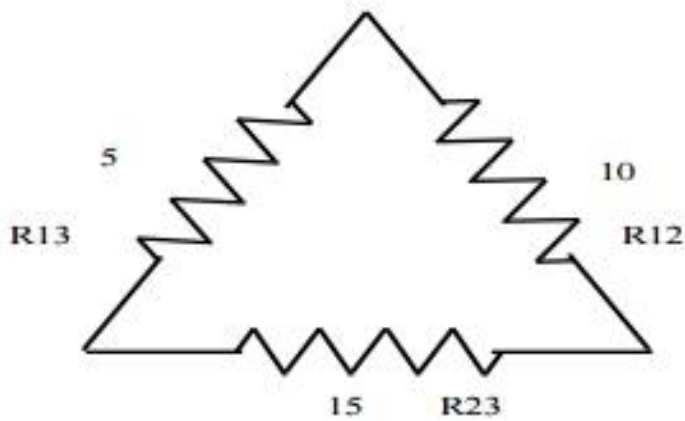
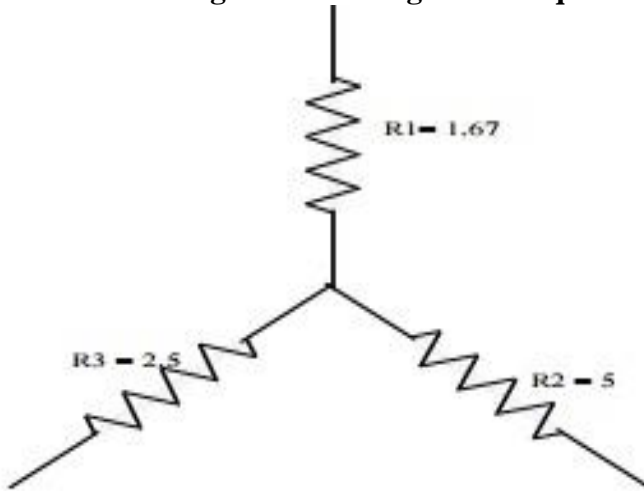
$$R_1 = R_{12}R_{13}/R_{12}+R_{13}+R_{23}$$

$$R_1 = 10 \times 5 / 30 = 1.67 \Omega$$

$$R_2 = 10 \times 15 / 30 = 5 \Omega$$

$$R_3 = 5 \times 15 / 30 = 2.5 \Omega$$

Problem 4: Convert the given star in fig into an equivalent delta.

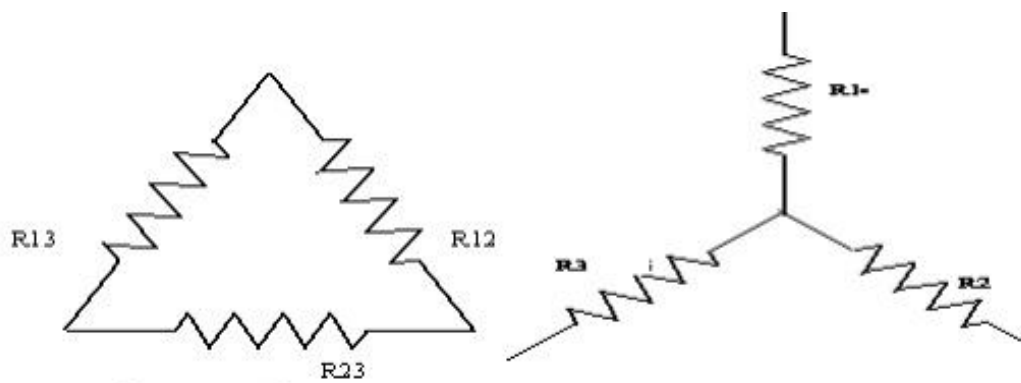


$$R_{12} = R_1 + R_2 + R_1 R_2 / R_3 = 1.67 + 5 + 1.67 \times 5 / 2.5 = 10 \Omega$$

$$R_{23} = 2.5 + 5 + 2.5 \times 5 / 1.67 = 15 \Omega$$

$$R_{31} = 2.5 + 1.67 + 2.5 \times 1.67 / 5 = 5 \Omega$$

Problem 5: Obtain the delta connected equivalent for the star connected circuit.



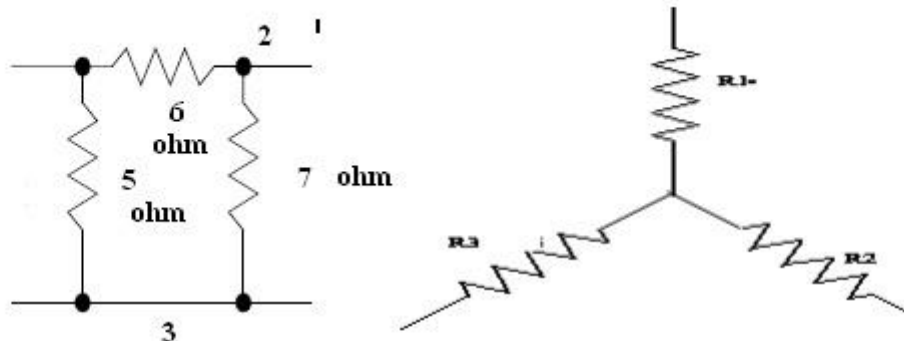
$$R_1 = 10\Omega, R_2 = 20\Omega, R_3 = 30\Omega$$

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{10 \times 20 + 20 \times 30 + 30 \times 10}{30} = 36.67\Omega$$

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 30 + 30 \times 10}{10} = 110\Omega$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{10 \times 20 + 20 \times 30 + 30 \times 10}{20} = 55\Omega$$

Problem 6: Obtain the star connected equivalent for the delta connected circuit.



$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{6 \times 5}{5 + 6 + 7} = 1.67\Omega$$

$$R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{6 \times 7}{5 + 6 + 7} = 2.33\Omega$$

$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} = \frac{5 \times 7}{5 + 6 + 7} = 1.94\Omega$$