ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY

ME 3391 ENGINEERING THERMODYNAMICS

DIGITAL NOTES

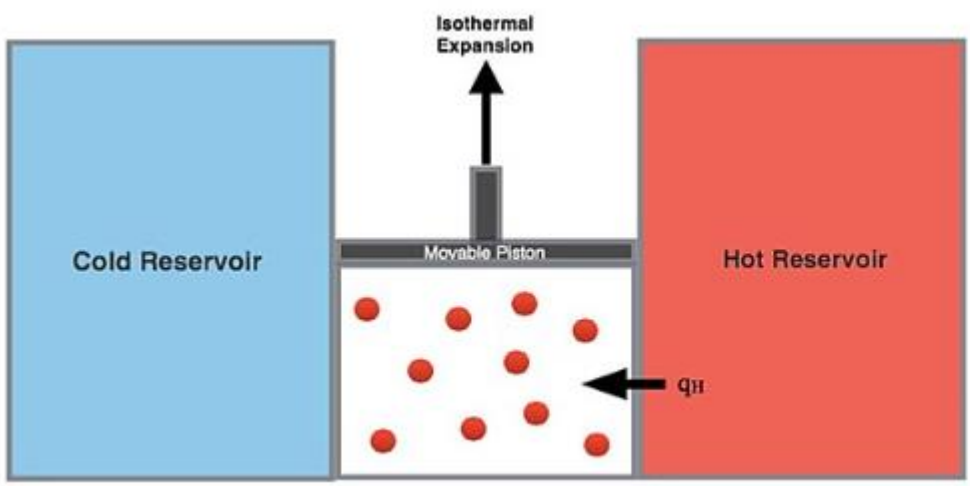
UNIT II CARNOT CYCLE

CARNOT CYCLE

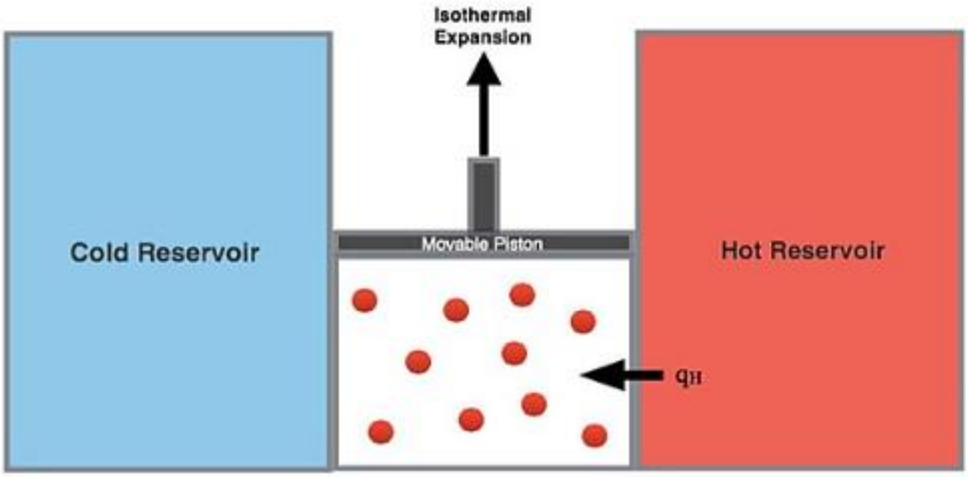
A Carnot cycle is defined as an ideal reversible closed thermodynamic cycle. Four successive operations are involved: isothermal expansion, adiabatic expansion, isothermal compression, and adiabatic compression. During these operations, the expansion and compression of the substance can be done up to the desired point and back to the initial state. . During these operations, the expansion and compression of the substance can be done up to the desired point and back to the initial state.

Stages

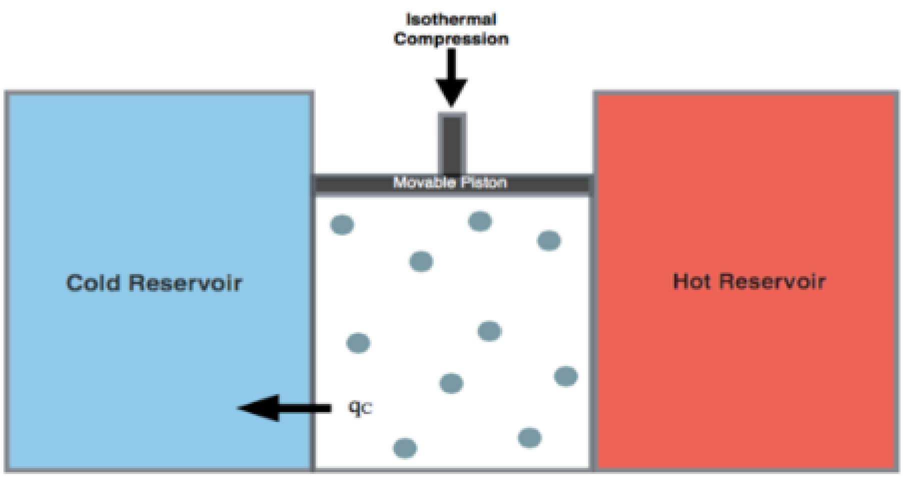
Isothermal expansion. Heat (as an energy) is transferred reversibly from hot temperature reservoir at constant temperature T_H to the gas at temperature infinitesimally less than T_H (to allow heat transfer to the gas without practically changing the gas temperature so isothermal heat addition or absorption). During this step, the gas is thermally in contact with the hot temperature reservoir (while thermally isolated from the cold temperature reservoir) and the gas is allowed to expand, doing work on the surroundings by gas pushing up the piston (stage 1 figure, right). Although the pressure drops from points 1 to 2 (figure 1) the temperature of the gas does not change during the process because the heat transferred from the hot temperature reservoir to the gas is exactly used to do work on the surroundings by the gas, so no gas internal energy changes (no gas temperature change for an ideal gas).



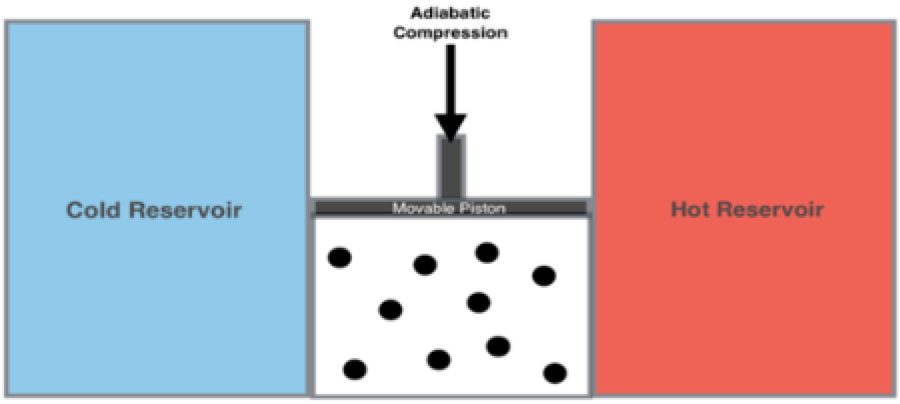
Isentropic (reversible adiabatic) expansion of the gas (isentropic work output). For this step the gas in the engine is thermally insulated from both the hot and cold reservoirs, thus they neither gain nor lose heat, an 'adiabatic' process. The gas continues to expand with reduction of its pressure, doing work on the surroundings (raising the piston; stage 2 figure, right), and losing an amount of internal energy equal to the work done. The gas expansion without heat input causes the gas to cool to the "cold" temperature (by losing its internal energy), that is infinitesimally higher than the cold reservoir temperature T_C. The entropy remains unchanged as no heat Q transfers ($Q = 0$) between the system (the gas) and its surroundings, so an isentropic process, meaning no entropy change in the process).

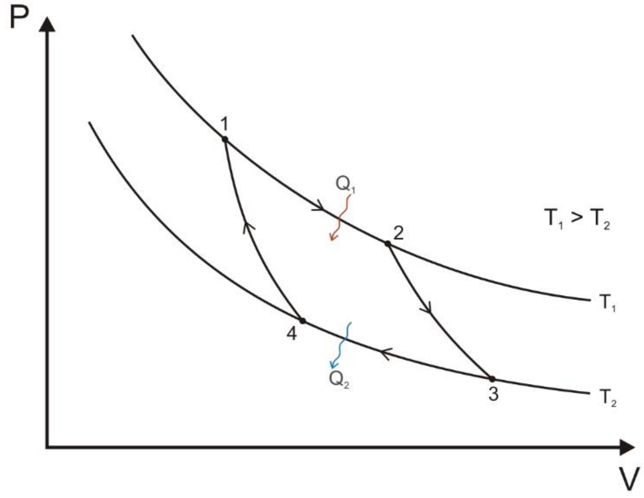


Isothermal compression. Heat transferred reversibly to low temperature reservoir at constant temperature T_C (isothermal heat rejection). In this step the gas in the engine is in thermal contact with the cold reservoir at temperature T_C (while thermally isolated from the hot temperature reservoir) and the gas temperature is infinitesimally higher than this temperature (to allow heat transfer from the gas to the cold reservoir without practically changing the gas temperature). The surroundings do work on the gas, pushing the piston down (stage 3 figure, right). An amount of energy earned by the gas from this work exactly transfers as a heat energy $Q_C < 0$ (negative as leaving from the system, according to the universal convention in thermodynamics) to the cold reservoir and entropy decreases due to compression.



Isentropic compression. Once again the gas in the engine is thermally insulated from the hot and cold reservoirs, and the engine is assumed to be frictionless and the process is slow enough, hence reversible. During this step, the surroundings do work on the gas, pushing the piston down further (stage 4 figure, right), increasing its internal energy, compressing it, and causing its temperature to rise back to the temperature infinitesimally less than T_H due solely to the work added to the system, but the entropy remains unchanged. At this point the gas is in the same state as at the start of step 1.





$$\Delta S_H + \Delta S_C = \Delta S_{\text{cycle}} = 0,$$

or,

$$\frac{Q_H}{T_H} = -\frac{Q_C}{T_C}.$$

Carnot cycle the maximum pressure and temperature are limited to 18 bar and 410°C. The volume ratio of isentropic compression is 6 and ermal expansion is 1.5. Assume the volume of the air at the beginning of isothermal expansion as 0.18m3. Show the cycle P-V and T- S am and determine (i) the pressure and temperature at main points (ii) Thermal efficiency of the cycle. [May/June -2016 / R-2008] (8 marks

Given Data :

The highest pressure P_2 = 18 bar

The highest temperature T_2 = 410 + 273 = 683 K = T3

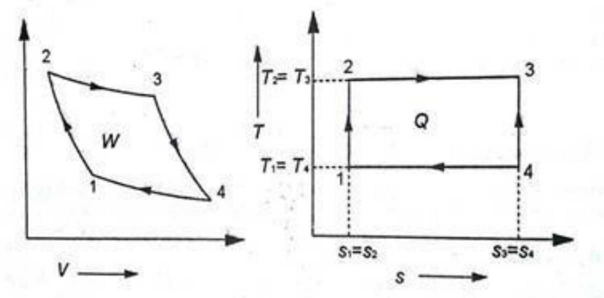
Volume V_2 = 0.18 m^3

$\frac{V_1}{V_2} = 6$; V_1 = 1.08 m^3

$\frac{V_3}{V_2}$ = 1.5 ; V_3 = 0.27 m^3

-SOLUTION:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$



(a) p-V diagram (b) T-s diagram

$$\frac{683}{T_1} = (6)^{1.4-1}$$

T_1 = 333.5 K = T_4

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\frac{18}{P_1} = (6)^{1.4}$$

P_2 = 1.46 bar

Process 2-3:

$P_2 V_2 = P_3 V_3$

$18 \times 0.18 = P_3 \times 0.27$

$P_3 = 12$ bar

Process 3-4 :

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$\frac{P_4}{12} = \left(\frac{1}{6}\right)^{1.4}$$

$P_4 = 0.977$

Thermal efficiency of cycle

$$\eta = \frac{T_2 - T_1}{T_3}$$

$$\eta = \frac{683 - 333.55}{683}$$

Thermal efficiency of the cycle = 51.16%

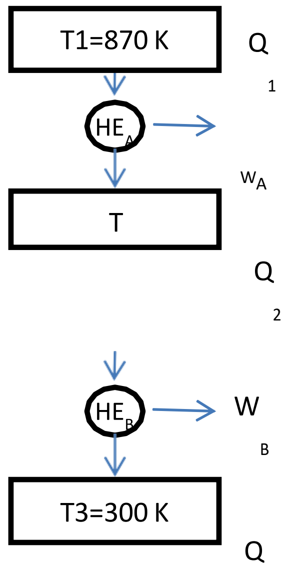
2. **Two Carnot engines A and B are operated in series. The first one receives heat at 870 K and reject to a reservoir at T. B receives heat rejected by the first engine and in turn rejects to a sink at 300 K. Find the temperature T for (a) Equal work outputs of both engine (b) Same efficiencies [Nov/Dec-2013 / R-2008] [12 –marks]**

Given Data :

$T_1 = 870$ K ; $T_3 = 300$K

To find : Intermediate Temperature T_2

Solution :   Case (a) Equal work output of Engine



$$\frac{Q_1 - Q_2}{T_1 - T_2} = \frac{Q_2 - Q_3}{T_2 - T_3}$$ here $W_A = Q_1 - Q_2$ and $W_B = Q_2 - Q_3$

$$\frac{W_A}{870 - T} = \frac{W_B}{T - 300}$$ here $W_A = W_B$ so both are cancelled

870 – T = T-300

2 T   = 1170

T = 585 K

Case (b) Same efficiency

$$\eta_A = \frac{T_1 - T}{T_1} ; \eta_B = \frac{T - T_3}{T}$$

here $\eta_A = \eta_B$

$$\frac{870 - T}{870} = \frac{T - 300}{} \qquad 870T - T^2 = 870T - 26100$$

$T^2 = 261000 \rightarrow T = 510.88$ K

PROBLEMS

1. A Reversible heat engine operates between two reservoirs at temperature of 600°C and 40°C. The engine drives a reversible refrigerator which operates between reservoirs at temperature of 40°C and -20°C. The heat transfer to the heat engine is 2000 kJ and the network output for the combined engine & refrigerator is 360 kJ. Calculate(1) the heat transfer to the refrigerant and the net heat transfer to the reservoir at 40°C. [Apr/May-15/ R-2013] [16 MARKS] (2)Reconsider (1) given that the efficiency of the heat engine and cop of the refrigerator are each 40% of their maximum possible value

Given Data:

$T_1 = 600 + 273 = 873$ K,

$T_2 = 40 + 273 = 313$ K $= T_4$;

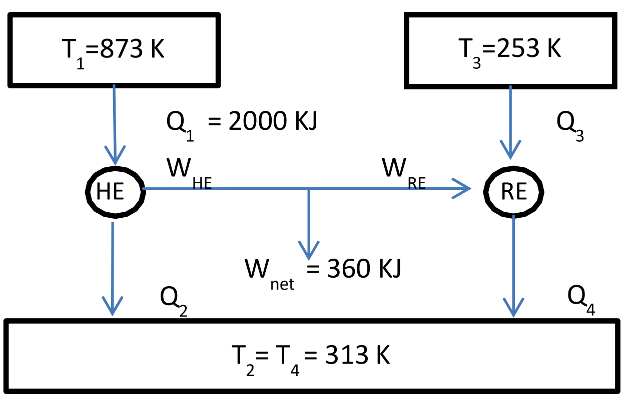
$T_3 = -20 + 273 = 253$ K;

Q1= 2000 KJ;

$W_{NET} = 360$ KJ

To find :

Q3 , $Q_{Rnet} = Q_2 + Q_4$



Tips : All four temperatures are known.

Find efficiency of HE and COP of RE

Wnet = W_{HE} - W_{Re}

Case (i)

Efficiency of Heat engine

$$\eta_{max} = \frac{T_1 - T_2}{T_1} = \frac{873-313}{873} = 0.642$$

Heat rejection by Heat engine

$.\eta_{max} = \frac{Q_1 - Q_2}{Q_1}; 0.642 = \frac{2000 - Q_2}{2000}; \; Q_2 = 716 \; kJ$

Work output of Heat engine

$W_{HE} = Q_1 - Q_2 = 2000 - 716 = 1284$ kJ

Work input of Refrigerator W_{NET}

$= W_{HE} - W_{RE}$; $W_{RE} = 924$ kJ

COP of refrigerator

$$COP_{max} = \frac{T_3}{T_4 - T_3} = \frac{253}{313 - 253} = 4.22$$

$COP_{max} = \frac{Q_3}{W_{RE}}; 4.22 = \frac{Q_3}{922.93}; \; Q_3 = 3899 \; kJ$;

$Q_4 = Q_3 + W_{RE} = 4823 \; KJ$

Net heat rejected to 40°C reservoir $Q_{Rnet} = Q_2 + Q_4 = 5539$ kJ

Case (ii)

Efficiency of the actual heat engine cycle

$\eta = 0.4 \times \eta_{max} = 0.4 \times 0.642$

$W_{HE} = 0.4 \times 0.642 \times 2000 = 513.6 kJ$

$W_{RE} = 513.6 - 360 = 153.6 kJ$

COP of the actual refrigeration cycle

$COP = 0.4 \times COP_{max} = 0.4 \times 4.22 = 1.69$

Therefore, $Q_3 = 153.4 \times 1.69 =$ **259.6 kJ**

$Q_4 = 259.6 + 153{,}6 =$ **413.2 kJ**

$Q_2 = Q_1 - W_{HE} = 2000\text{-}513.6 = \mathbf{1486.4\ kJ}$

Net heat rejected to 40°C reservoir $Q_{Rnet} = Q_2 + Q_4 =$ **1899.6 kJ**

2. A heat engine receives 800 kJ of heat from the reservoir at 1000 K and rejects 400 kJ at 400 K. If the surrounding is at 300 K. calculate the first and the second law efficiency, and the relative efficiency of the heat engine. [Apr/May-2016/R-2013] [6 marks]

Give Data: Q1 = 800 kJ; T1=1000 K; Q2=400 kJ; T2=400 K; T0=300 K

To find:

IST and IIndlaw efficiency, Relative efficiency

Solution:

Heat engine efficiency $\eta_{HE} = \frac{W}{T_1} = \frac{400}{800} = 50\%$

Efficiency of reversible heat engine operating with atmosphere as sink

The second law efficiency of heat engine

$$\eta_{II\ HE} = \frac{\eta_{HE}}{\eta_{RHEo}} = \frac{50}{70} = 71.43\%$$

The efficiency of reversible heat engine

$$\eta_{RHE} = \frac{T_1 - T_2}{T_1} = \frac{1000 - 40}{1000} = 60\%$$

The relative efficiency

$$\eta_{RHEo} = \frac{T_1 - T_0}{T_1} = \frac{1000 - 300}{1000} = 70\%$$

$$\eta_{HE\ R} = \frac{\eta_{HE}}{\eta_{RHE}} = \frac{50}{60} = 83.33\ \%$$

3.A heat pump operates on a carnot heat pump cycle with a COP of 8.7. it keeps a space at 24ºC by consuming 2.15 kw of power. Determine the temperature of the reservoir from which the heat is absorbed and the heating load provided by the heat pump. NOV/DEC 2016 (7 MARK)

Given data:

Carnot COP of heat pump = 8.7TH= 24ºC = 297 K

Power consumption or work done = 2.15 kW

Solution:

For reversible heat pump,

Actual COP of heat pump = carnot COP of heat pump

$$\frac{1}{1-\frac{Q_L}{Q_H}} = \frac{1}{1-\frac{T_L}{T_H}}$$

$$1-\frac{Q_L}{Q_H} = 1-\frac{T_L}{T_H}$$

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

$$Q = \frac{T_L}{T_H} \times Q_H$$

$$Q_L = \frac{262.86_L}{297} \times Q_H = 0.89 Q_H$$

But, work done = $Q_H - Q_L$

Substituting Q_L in work done,

$$2.15 = Q_H - 0.89 Q_H$$

$$= 0.11 Q_H$$

Heating load, $Q_H = 19.55 kW.$

4.)A heat pump working on the carnot cycle takes in heat from a reservoir at5 °C and delivers heat to a reservoir at 60 °C. The heat pump is driven by a reversible heat engine which takes in heat from reservoir at 840 °C and rejectsto a reservoir at 60 °C. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from5 °C reservoir, determine

(i) **the rate of heat supply from the 840 °C source, and**
(ii) the rate of heat rejection to the 60 °C sink

Given:

T_1 = 840+273 = 1113 K

T_2 = 60+273 = 333 K

T_3 = 5+273 = 278 K

T_4 = 60+273 = 333 K

Q_3 = 17kJ/s

W_3 = 30 kW

To fond:

(i) the rate of heat supply from the 840 °C source, and
(ii) the rate of heat rejection to the 60 °C sink

Solution:

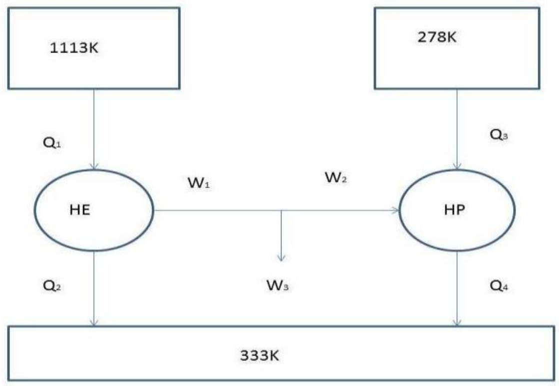
$$COP_{HP} = \frac{T_H}{T_H - T_L} = \frac{T_4}{T_4 - T_3} = \frac{333}{333 - 278} = 6.055$$

(COP) HP =QS2/QS2-QR2

6.055=Q4/Q4-17

Q4=20.36 KJ/S

Q



$$W_2 = Q_4 - Q_3 = 20.36 - 17 = 3.36\,kJ\,s$$
$$W_1 = W_2 + W_3 = 3.36 + 30 = 33.36kW$$

Maximum Efficiency of heat engine

$$\eta_{max} = \frac{T_H - T_L}{T_H} = \frac{T_1 - T_2}{T_1}$$

$$= \frac{1113 - 333}{1113} = 0.7 = 70\%$$

$$\eta_{max} = \frac{W_1}{Q_1}$$

$$Q_1 = \frac{W_1}{\eta_{max}} = \frac{33.36}{0.7} = 47.66kW$$

$$W_1 = Q_1 - Q_2$$

Substituting QL in work done,

$$2.15 = Q_H - 0.89Q_H$$
$$= 0.11Q_H$$

Heating load, $Q_H = 19.55kW$.