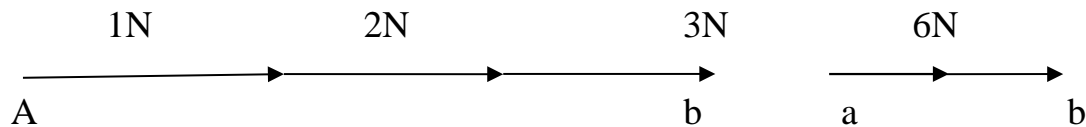


## Statics of Particles in Two Dimensions- Resultant Force

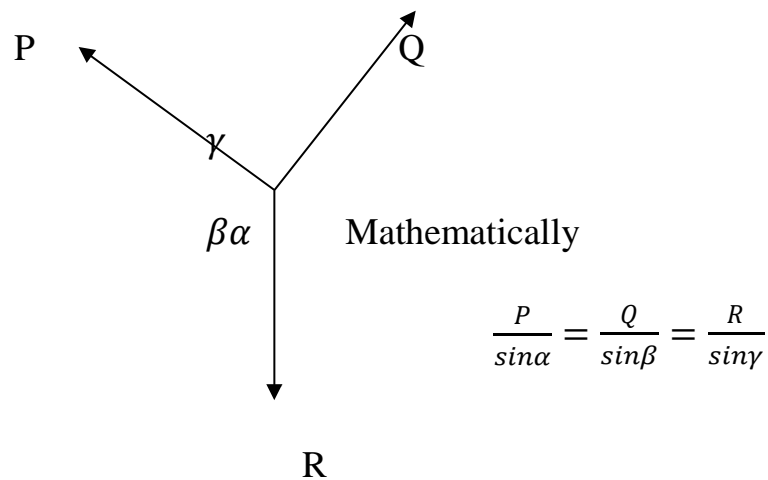
### Resultant Force:

If a number of forces acting on a particle simultaneously are replaced by a single force which could produce the same effort as produced by the given forces, that single force is called resultant force.



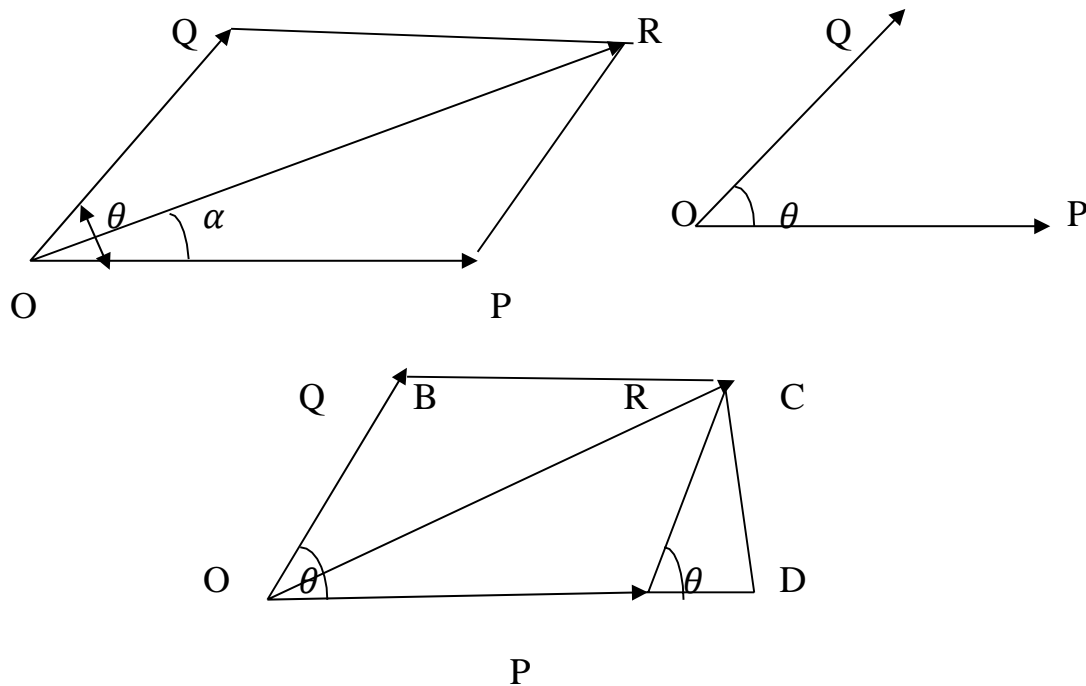
### Lami's Theorem:

It states that “if three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sin of the angle between the other two forces.



### Parallelogram Law of forces:

It states that “if the two forces acting simultaneously at a point represented in magnitude and direction by the two adjacent sides of the parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of the parallelogram originating from that point.



Let  $P$  and  $Q$  are two concurrent force acting on a point  $O$  at an angle of  $\theta$ .

The forces  $P$  and  $Q$  are graphically represented by the lines  $OA$  and  $OB$  respectively.

The parallelogram  $OACB$  is completed by drawing the lines  $BC$  and  $AC$  parallel to  $OA$  and  $OB$  respectively.

In parallelogram  $OACB$ , the diagonal  $OC$  represents the resultant force of  $P$  and  $q$ . by II Law of forces.

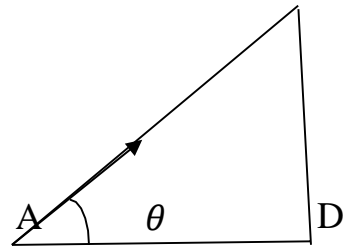
In order to prove the, parallelogram law of forces, extend the lines of action of force  $P$ , till its meet the perpendicular drawn from point  $C$ .

Let the point of intersection of these two lines be D. from the geometry of the parallelogram.

$$OB=AC$$

$$OA=BC$$

In triangle ACD



$$\cos\theta = \frac{AD}{Q} \quad \sin\theta = \frac{CD}{Q}$$

$$AD = Q \cos \theta \text{ ----- (1)}$$

$$CD = Q \sin\theta \text{----- (2)}$$

$$\text{Also } AD^2 + CD^2 = AC^2$$

$$= AD^2 + CD^2 = Q^2 \text{----- (3)}$$

In triangle OCD

$$OC^2 = OD^2 + CD^2$$

$$= (OA + AD)^2 + CD^2$$

$$= OA^2 + AD^2 + 2 \times OA \times AD + CD^2$$

$$= OA^2 + (AD^2 + CD^2) + 2OA AD$$

$$= OA^2 + AC^2 + 2OA AD$$

$$R^2 = P^2 + Q^2 + 2 \times PQ \cos \theta$$

$$R = \sqrt{P^2 + Q^2 + 2 \times PQ \cos \theta}$$

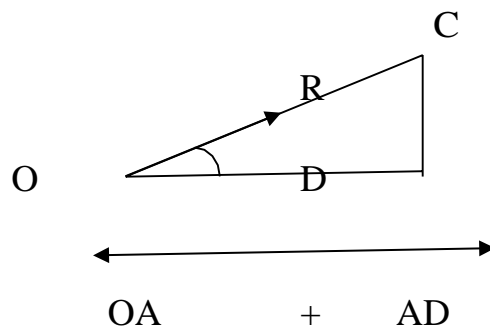
Inclination of the resultant force with the force P

Let the angle of inclination of R with the line of action of the force P be a

In triangle OCD

$$\tan a = \frac{CD}{OD} = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\therefore \tan a = \frac{Q \sin \theta}{P + Q \cos \theta}$$



### **Important results:**

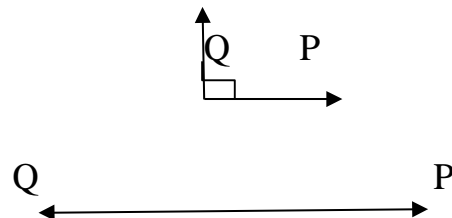
1. If  $\theta = 0^\circ$  then the resultant forces P and Q will be like collinear, then,

$$R = P + Q \quad P \longrightarrow \quad Q \longleftarrow$$

2. If  $\theta = 90^\circ$ , the forces P and Q are at right angles then  $R = \sqrt{P^2 + Q^2}$

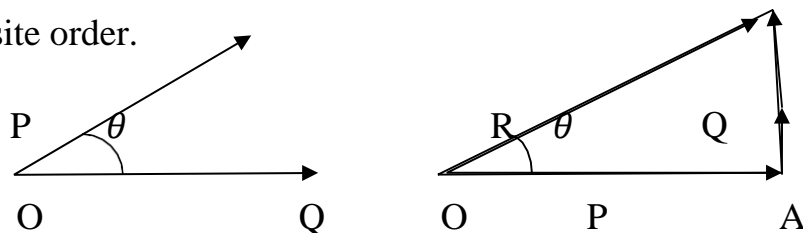
3. If  $\theta = 180^\circ$ , then the forces P and Q will be unlike collinear forces, then

$$R = P - Q.$$



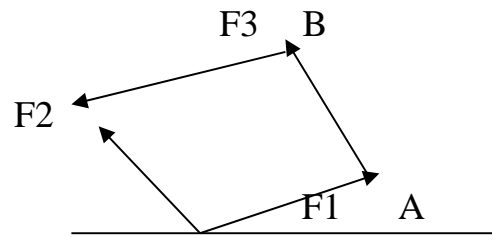
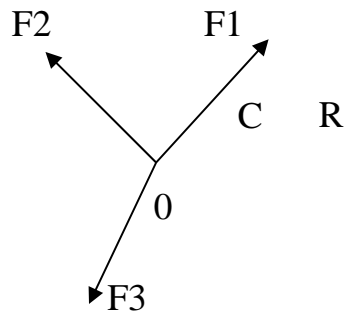
### **Triangle law of Forces:**

If two forces acting at a point are represented by two sides of a triangle taken in order, then their resultant force is represented by the third side taken in opposite order.



### **Polygon Law of forces:**

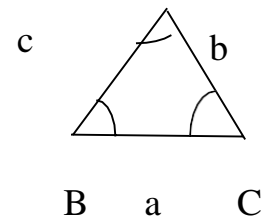
Polygon Law of forces states that, 'if a number of coplanar concurrent forces are represented in magnitude and direction by the sides of a polygon taken in an order then their resultant force is represented by the closing side of the polygon taken in the opposite order.'



Sine Law:

The law of sines can be used when two angles and a side are known a technique known as triangulation.

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$



Cosine Law:

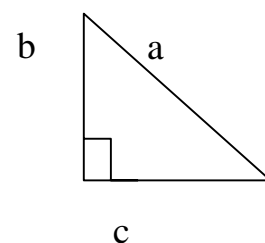
If two side and the angle between the sides are known,

Then the third is given by

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

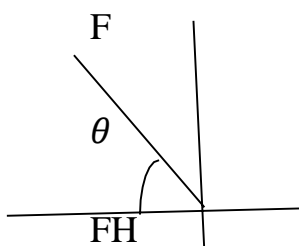
$$b^2 = c^2 + a^2 - 2ca \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



### Resolution of a force in to its horizontal and vertical part.

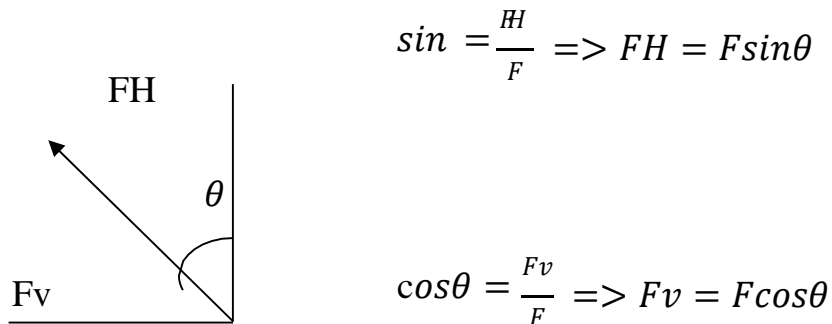
Method I



$$\sin \theta = \frac{F_V}{F} \Rightarrow F_V = F \sin \theta$$

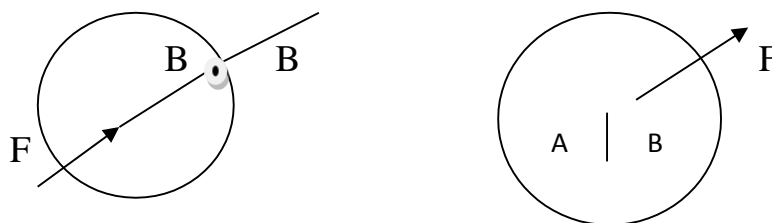
$$\cos \theta = \frac{F_H}{F} \Rightarrow F_H = F \cos \theta$$

## Method II



## Principle of transmissibility of Forces:

If a force act at any point of on a rigid body it may also be considered to act at any other point on its line of action.

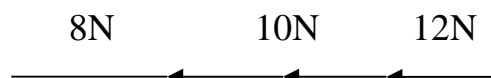


## Resultant force of two concurrent forces:

1. Resultant force of two concurrent force
2. Resultant force of more than two concurrent force

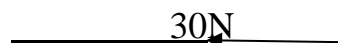
## Problem based on parallelogram & Resultant forces:

1. Find the resultant force of the collinear forces shown in fig.

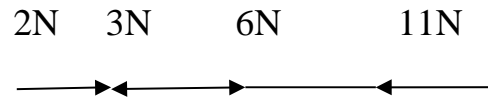


### Soln:

Resultant force  $R = 8 + 10 + 12 = 30\text{N}$



2. Find the resultant force of the collinear forces, shown in fig

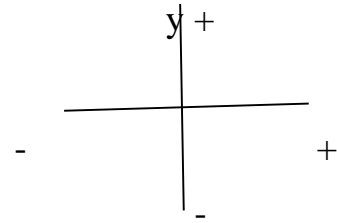


**Soln:**

Magnitude of resultant force

$$= 2-3+6-11$$

$$R = -6N$$

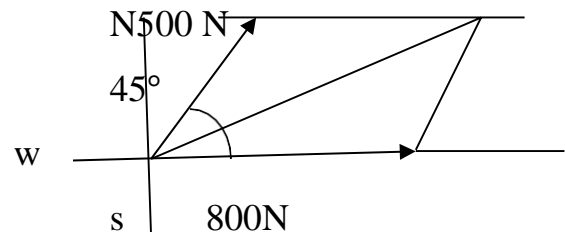


3. Find the resultant force an 800 N force acting towards eastern direction and a 500 n force acting towards north eastern direction.

by 1. Parallelogram Law

2. Triangle Law

Also find the direction



**Given**

$$P=800 \text{ N} \quad Q=500\text{N} \quad \theta=45^\circ$$

To find

Resultant force & direction

**Soln**

1. Parallelogram Law

$$\text{Resultant Force } R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{800^2 + 500^2 + 2 \times 800 \times 500 \times \cos 45}$$

$$R = 1206.52 \text{ N}$$

Direction of magnitude

$$Q = \tan^{-1} \left[ \frac{Q \sin \theta}{P + Q \cos \theta} \right]$$

$$Q = \tan^{-1} \left[ \frac{500 \sin 45}{800 + 500 \cos 45} \right]$$

$$Q = 17^\circ 04'$$

### **Summing of components:**

$$R = \sqrt{FH^2 + \sum FV^2}$$

$$\sum FH = 800 + 500 \sin 45 = 1153.55 \text{ N}$$

$$\sum FV = 500 \sin 45 = 353.55 \text{ N}$$

$$R = \sqrt{1153.55^2 + 353.55^2}$$

$$R = 1206.52 \text{ N}$$

$$\alpha = \tan^{-1} \frac{[\sum FV]}{\sum FH}$$

$$= \tan^{-1} \frac{[1153.55]}{353.55}$$

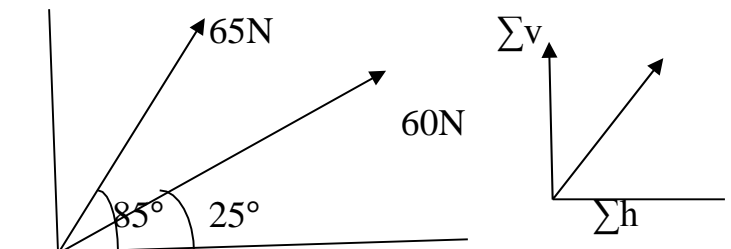
$$\alpha = 17^\circ 04'$$

4. Two forces 60 N and 65 N act on a screw at an angle of  $25^\circ$  and  $85^\circ$  from the base. Determine the magnitude and direction of their resultant.

### **Given:**

$$P_1 = 60 \text{ N}, \theta_1 = 25^\circ$$

$$P_2 = 65 \text{ N}, \theta_2 = 85^\circ$$



### **To find:**

Magnitude & direction of their resultant

### **Soln:**

1. Magnitude of resultant force

$$R = \sqrt{\sum FH^2 + \sum FV^2}$$



$$\Sigma FH = 60 \cos \theta_1 + 65 \cos \theta_2$$

$$= 60 \cos 25 + 65 \cos 85$$

$$\Sigma FH = 60 \text{ N}$$

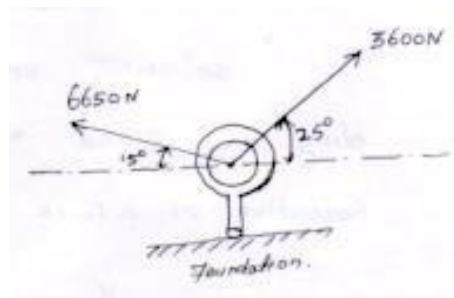
$$\Sigma FV = 60 \sin \theta_1 + 65 \sin \theta_2 = 60 \sin 25 + 65 \sin 85$$

$$\Sigma FV = 90 \text{ N}$$

$$R = \sqrt{60^2 + 90^2}$$

$$R = 108.17 \text{ N}$$

5. Two wires are attached to a bolt in a foundation as shown in fig. below.  
Determine the pull exerted by the bolt on the foundation.



**Soln:**

$$\text{Resultant force } R = \sqrt{\Sigma FH^2 + \Sigma FV^2}$$

$$\Sigma FH = 3600 \cos 25 - 6650 \cos 15$$

$$\Sigma FH = -3160 \text{ N}$$

$$\Sigma FV = 3600 \sin 25 + 6650 \sin 15$$

$$\Sigma FV = 3242 \text{ N}$$

$$R = \sqrt{-3160^2 + 3242^2}$$

$$R = 4527 \text{ N}$$

$$\alpha = \tan^{-1} \left[ \frac{\sum FV}{\sum FH} \right]$$

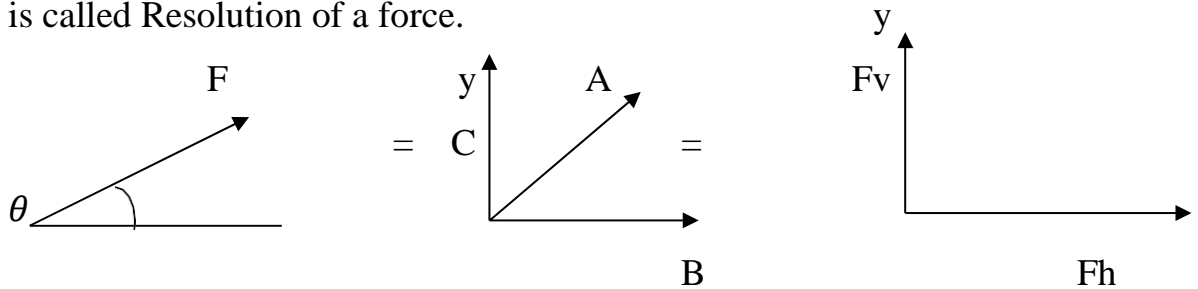
$$\alpha = \tan^{-1} \left[ \frac{\sum 3242}{\sum 3160} \right]$$

$$\alpha = 45^\circ 73'$$

⇒ **Resultant force of more than Two concurrent Forces:**

### **Resolution of Forces:**

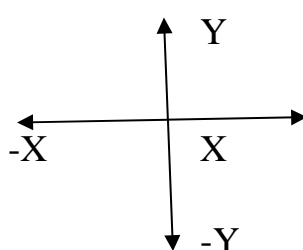
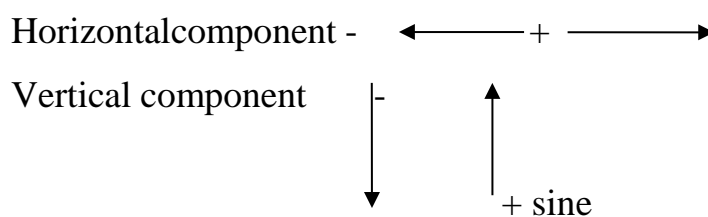
Splitting up a force into components along the fixed reference axis is called Resolution of a force.



$F_h$  = horizontal component =  $+F \cos \theta$

$F_v$  = vertical component =  $+F \sin \theta$

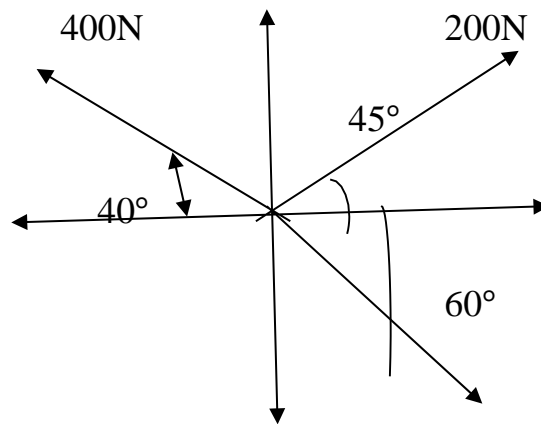
Sign conversion:



$$R = \sqrt{\sum FH^2 + \sum FV^2}$$

$$\alpha = \tan^{-1} \left[ \frac{\sum FV}{\sum FH} \right]$$

1. Three coplanar concurrent forces are acting at a point as shown in fig. Determine the resultant in magnitude of direction.



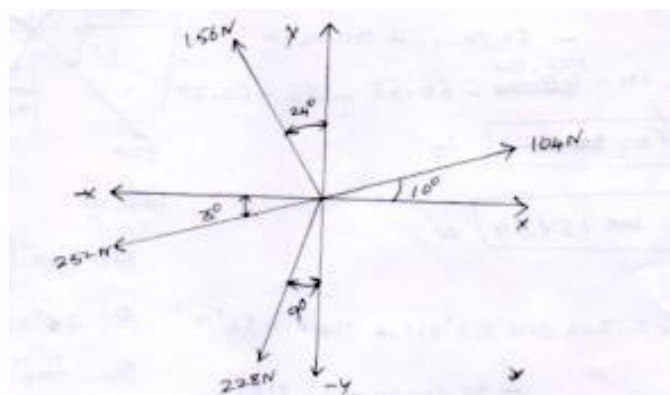
Soln:

Force & magnitude	$\theta$	$F\cos\theta$	$F\sin\theta$
$F_1=200$	$45^\circ$	$200 \cos 45^\circ = 141.42$	$200 \sin 45^\circ = 141.42\text{N}$
$F_2=400$	$150^\circ$	$400 \cos 150^\circ = -326.41$	$400 \sin 150^\circ = 200$
$F_3=600$	$300^\circ$	$600 \cos 300^\circ = 300\text{ N}$	$600 \cos 300^\circ = -519.61$
		$\Sigma FH = 95.01\text{ N}$	$\Sigma FV = -178.19\text{N}$

$$R = \sqrt{\Sigma FH^2 + \Sigma FV^2} = \sqrt{95.01^2 + -178.19^2}$$

$$R = 201.95\text{ N}$$

2. The four coplanar forces acting at a point a as shown in fig. Determine the resultant in magnitude and direction.



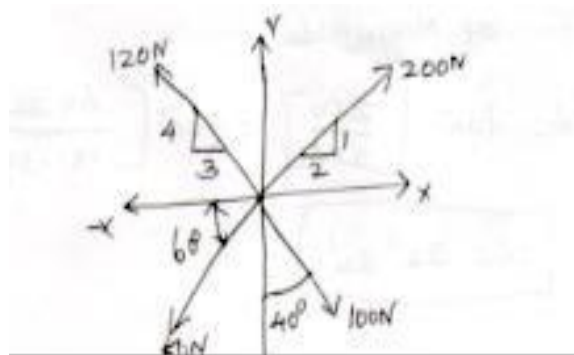
**Soln:2 nd method**

Note:  $\theta_1=10^\circ$   $\theta_2=90-24=66^\circ$   $\theta_3=3^\circ$   $\theta_4=90-9=81^\circ$

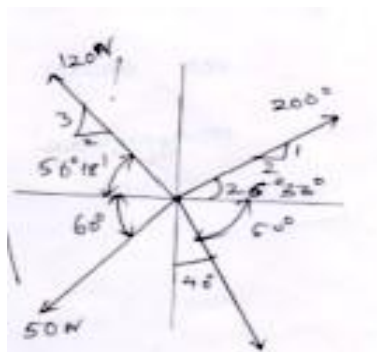
$$\sum FH = -248.36 \quad \sum FV = -77.82$$

$$R=260.26\text{N} \quad \theta=17.39^\circ$$

3. A system of four forces acting on a body is shown in fig below. Determine the resultant force and direction.



**Soln:**



$$\text{Resultant force } R = \sqrt{\sum FH^2 + \sum FV^2}$$

$$\sum FH = 200 \cos 26^\circ 33' - 120 \cos 56^\circ 18' - 50 \cos 60 + 100 \cos 40$$

$$\sum FH = 178.90 - 66.58 - 25 + 64.27$$

$$\sum FH = 151.59$$

$$\Sigma FV = 200\sin 26^\circ 33' + 120\sin 56^\circ 18' - 50\sin 60 - 100\sin 50$$

$$\Sigma FV = 89.39 + 99.83 - 43.30 - 76.60$$

$$\Sigma FV = 69.32\text{N}$$

$$R = \sqrt{(151.59)^2 + (69.32)^2}$$

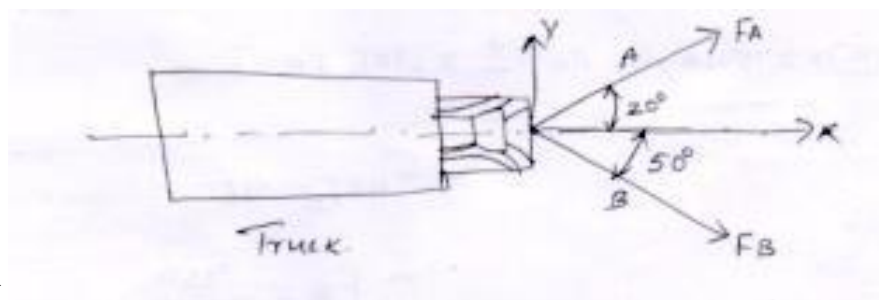
$$R = 166.68\text{N}$$

Direction of magnitude

$$\alpha = \tan^{-1}\left[\frac{\Sigma FV}{\Sigma FH}\right] = \tan^{-1}\left[\frac{69.32}{151.59}\right]$$

$$\alpha = 24^\circ 34'$$

4. The truck shown is to be towed using two ropes. Determine the magnitude of forces  $F_A$  &  $F_B$  acting on each rope in order to develop a resultant force of 950N directed along the positive X axis.



Resultant force  $R = 950\text{ N}$  in positive  $x$  direction

$$\therefore \text{Hence } \Sigma FH = 950$$

$$\Sigma FH = 0$$

Resolving forces horizontally

$$\Sigma FH = F_A \cos 20^\circ + F_B \cos 50^\circ$$

$$FA\cos 20 + FB\cos 50 = 950 \text{-----} (1)$$

Resolving forces vertically

$$\sum FV = F\sin 20 - FB\sin 50 = 0$$

$$F\sin 20 - FB\sin 50 = 0 \text{-----} (2)$$

Solving eq(1) & (2)

$$FA\cos 20 + FB\cos 50 = 950$$

$$F\sin 20 - FB\sin 50 = 0$$

$$0.939FA + 0.642FB = 950 \text{-----} (1)$$

$$0.342FA + 0.766FB = 0 \text{-----} (2)$$

$$0.939FA + 0.642FB = 950 \text{-----} (1)$$

$$(2) \times 2.75 \quad 0.342FA \pm 0.766FB = 0$$

$$2.748FB = 950$$

$$FB = \frac{950}{2.748}$$

$$FB = 345.64 \text{ N}$$

FB value sub in eqn(1)

$$0.94 \times FA + 0.642 \times 345.64 = 950$$

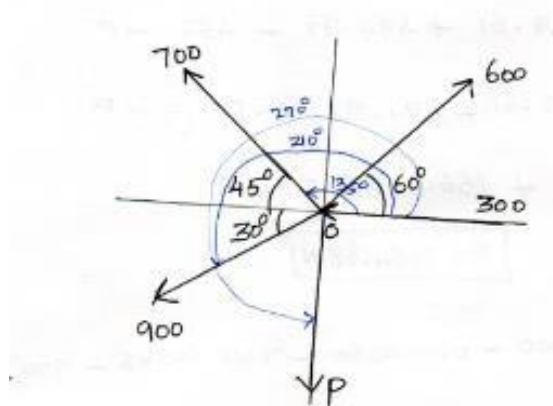
$$0.94 FA = 950 - 221.9 = 728$$

$$FA = \frac{728.09}{0.94}$$

$$FA = 774.57 \text{ N}$$

5. Five forces are acting on a particle. The magnitude of the forces are 300 N, 600N, 700N, 900N and P and their respective angles with the horizontal are  $0^\circ, 60^\circ, 135^\circ, 210^\circ, 270^\circ$ . If the vertical component of all the force is -1000N, Find the value of P. Also calculate the magnitude and the direction of the resultant, assuming that the first force acts towards the point, while all the remaining forces act away from the point.

**Given:**



$$\theta_1=0^\circ \quad \theta_2=60^\circ \quad \theta_3=180-135=45^\circ$$

$$\theta_4=180+[90-60]=210^\circ=30^\circ$$

$$\theta_5=270=90^\circ$$

$$F_1=300, \quad F_2=600 \quad F_3=700 \quad F_4=900 \quad F_5=P$$

$$\sum FV = -1000N$$

**Soln**

$$\text{Resultant force } R = \sqrt{(\sum FH)^2 + (\sum FV)^2}$$

$$\sum FV = -1000N$$

To find the value of 'P'

Algebraic sum of vertical components

$$\sum F_v = 600 \sin 60 + 700 \sin 45 - 900 \sin 30 - P$$

$$-1000 = 519.61 + 494.97 - 450 - P$$

$$-1000 = -519.61 - 494.97 + 450 = -P$$

$$-1564.58 = -P$$

$$P = 1564.58 \text{ N}$$

$$\sum F_H = -300 + 600 \cos 60 - 700 \cos 45 - 900 \cos 30$$

$$\sum F_H = -300 + 300 - 494.97 - 779.42$$

$$\sum F_H = -1274.39 \text{ N}$$

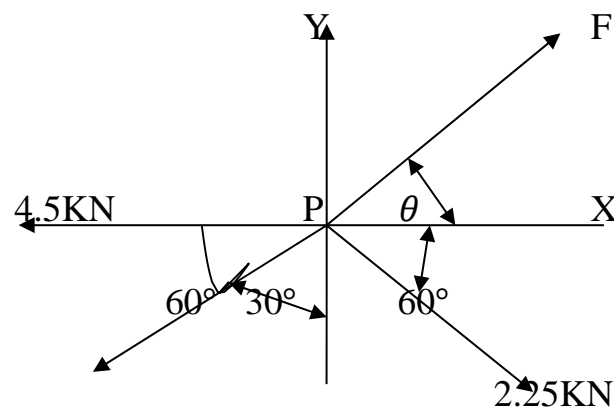
$$\text{Resultant force } R = \sqrt{(-1274.39)^2 + (-1000)^2}$$

$$R = 1619.89 \text{ N}$$

$$\text{Direction } \alpha = \tan^{-1} \left[ \frac{\sum F_v}{\sum F_H} \right] = \tan^{-1} \left[ \frac{1000}{1274} \right]$$

$$\alpha = 38^\circ 7'$$

6. Determine the magnitude and angle of  $f$  so that particle P shown in Fig



**Soln:**

$$\sum F_H = F \cos \theta - 4.5 - 7.5 \cos 60 + 2.25 \cos 60 = 0$$



$$F \cos \theta - 4.5 - 3.75 + 1.125 = 0 \text{ ----- (1)}$$

$$\sum F_V = F \sin \theta - 7.5 \sin 60 - 2.25 \sin 60 = 0$$

$$F \sin \theta - 6.49 - 1.94 = 0 \text{ ----- (2)}$$

Eqn (1) Rearrange

$$F \cos \theta - 7.125 = 0$$

$$F \cos \theta = 7.125 \text{ ----- (3)}$$

Eqn(2) Rearrange

$$F \sin \theta - 8.43 = 0$$

$$F \sin \theta = 8.43 \text{ ----- (4)}$$

$$\frac{(4)}{(3)} = \frac{F \sin \theta}{F \cos \theta} = \frac{8.43}{7.125}$$

$$\tan \theta = 1.183$$

$$\theta = \tan^{-1}[1.183]$$

$$\theta = 49^\circ 47'$$

Substitute  $\theta = 49^\circ 47'$  in eqn(3)

$$F \cos \theta = 7.125 \Rightarrow F \cos 49^\circ 47' = 7.125$$

$$F = \frac{7.125}{\cos 49^\circ 47'}$$

$$F = 11.03 \text{ N}$$